

Paper-II - ElectrodynamicsUnit-IElectromagnetic Induction

It shows that how a time varying field (magnetic) gives rise to an electric field and vice-versa.

Electromotive force (e.m.f.)

For steady current flow in conductors, a potential difference must exist b/w its two ends.

This is maintained by a source of e.m.f.

The devices provide the e.m.f. are

- (i) Battery,
- (ii) Dynamo etc.

They convert some other kind of energy into electrical energy; example

Battery $\rightarrow$ chemical energy $\rightarrow$ electrical energy

Dynamo $\rightarrow$ mechanical energy $\rightarrow$ electrical energy

Let if Battery maintains the potential difference of V volt b/w terminals then its e.m.f. is V volt.

The line integral of the field over path b/w two points (A, B) is equal to the potential difference the points (if it is terminals of the battery) the e.m.f. of battery

$$V = \int_A^B \vec{E} \cdot d\vec{l} \quad \text{--- (1)}$$

when current flow in conductor, the battery must do work to keep the potential difference b/w its terminal constant. If a charge q moves from one terminal of battery to other through the conductor.

The work done by battery is

$$W = \int_A^B q \vec{E} \cdot d\vec{l}$$

The e.m.f. of battery

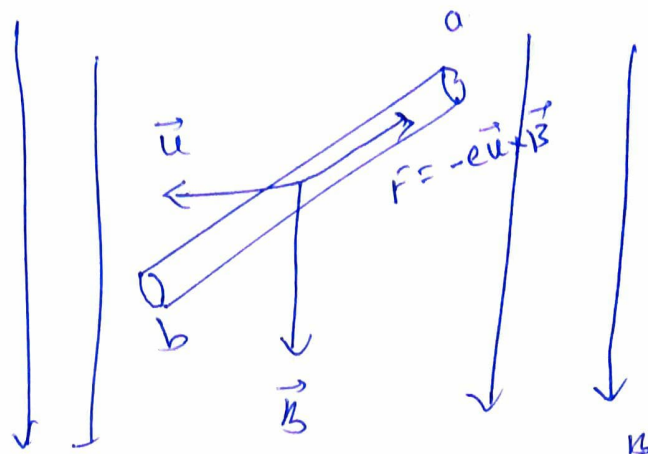
$$V = \frac{1}{q} \int_A^B q \vec{E} \cdot d\vec{l} \quad \text{--- (2)}$$

The e.m.f. is not force but it is a quantity which has the dimensions energy per charge.

Consider a metal bar of length l moving with velocity ' v ' in perpendicular direction of uniform magnetic field,

B .

(2)



Then electrons of bar move towards the end of bar and collect there setting a field given by

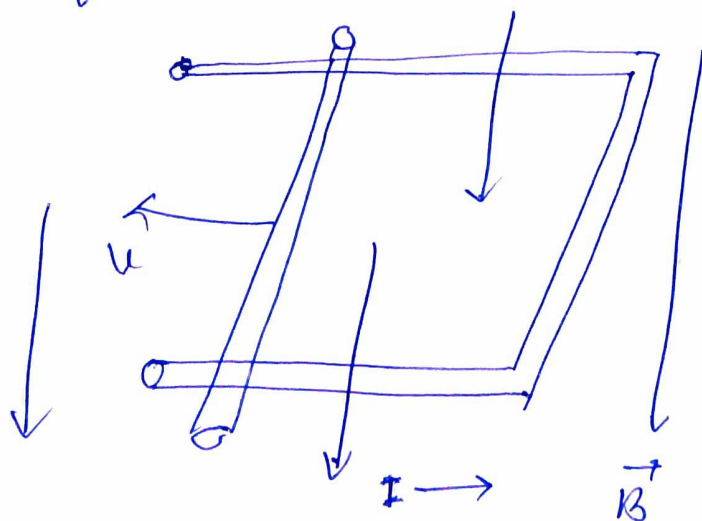
$$\vec{E} = -\vec{u} \times \vec{B} \quad \text{--- (3)}$$

The potential difference b/w end of bar is

$$V_{ba} = \int_b^a \vec{E} \cdot d\vec{l} = uBL \quad \text{--- (4)}$$

The potential difference produces no electric current flow.

Again if bar forms a part of a circuit then there will flow of current.



the line integral of force on charge q round the circuit

$$\oint q \vec{E} \cdot d\vec{l} = q u B L$$

the e.m.f. induced in the closed circuit due to motion of conductor is

$$V = \frac{1}{q} \int q (\vec{E} \cdot d\vec{l}) = u B L \quad \text{--- (5)}$$

this is called motional e.m.f. since it depends on velocity of conductor.

we have two sorts of voltage

- (i) electrostatic potential difference due to stationary charges
- (ii) electromotive force due to moving charge.

Here $u B L$ is the magnetic flux through the area swept by the bar in unit time

the rate of change of flux through the circuit is $u B L$;

$$|e.m.f.| = u B L = \frac{d\phi}{dt} \quad \text{--- (6)}$$

where ϕ is total magnetic flux.

Faraday's Law of Electromagnetic Induction

(3)

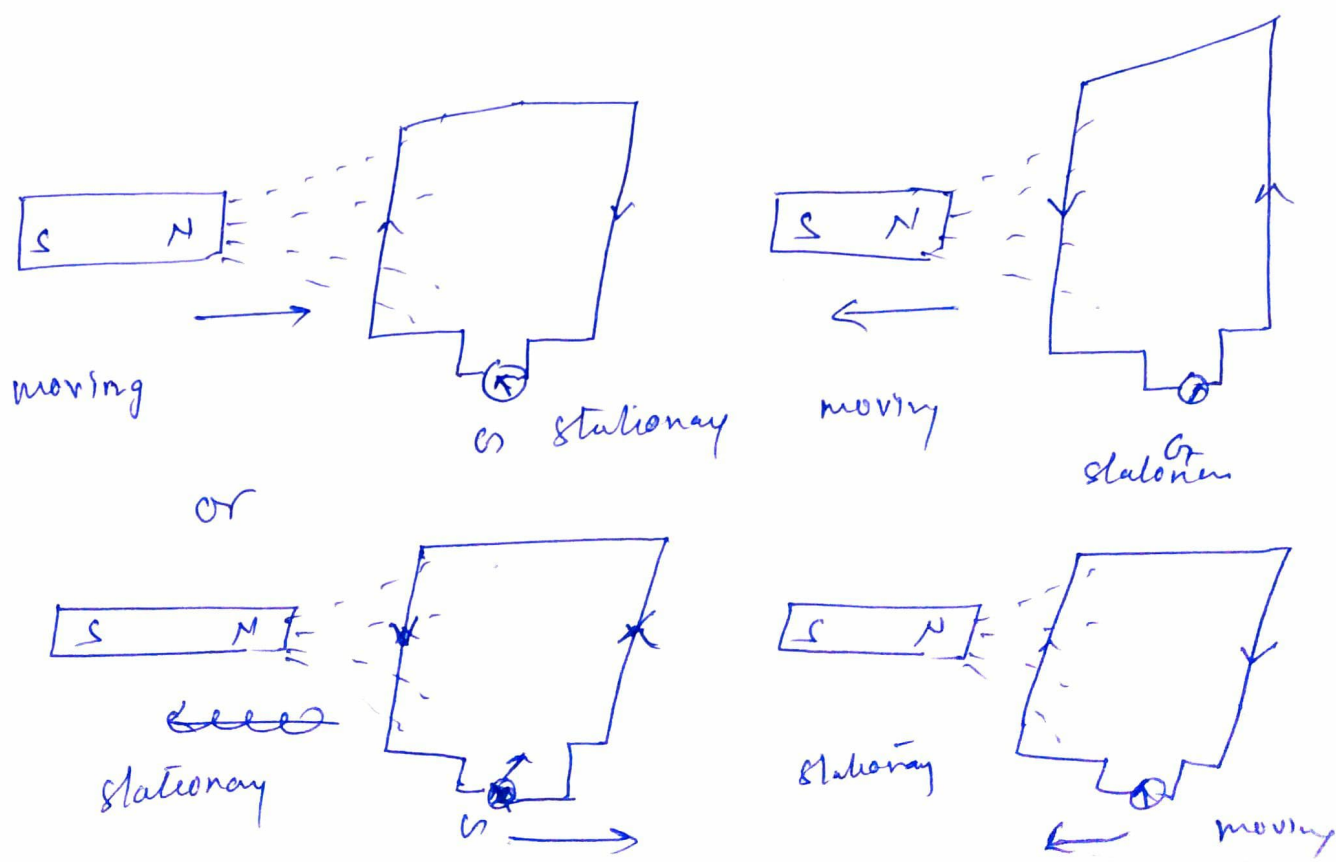
Faraday and Henry observed that

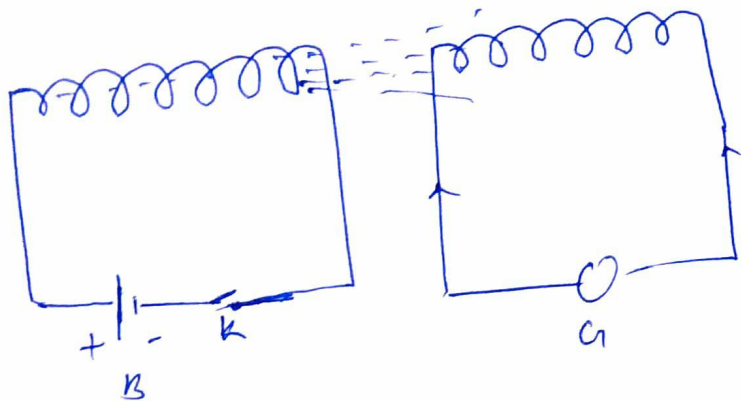
(i) If a magnet is moved about to the neighbourhood of a wire formed closed circuit (with no battery), then a current is produced in the wire so long as the movement lasts.

(ii) The same effect is observed if the magnet is kept still and the circuit is moved.

This gives ~~that~~ an impression that for production of current in the circuit, there must be a relative movement.

(iii) A transient current is induced in a loop of wire if the stationary current in an adjacent circuit is turned on or off.





Either on or off the switch
or movement of coil.

The changing flux induces an electric field and hence an e.m.f. in the circuit which causes the current to flow. Faraday called this phenomenon electromagnetic induction. Faraday's results were summed up in what is known as the flux rule.

The changing magnetic flux induces e.m.f. in circuit.
~~Some flux is proportional to the magnetic field then~~
The magnitude of e.m.f. is given by
If $\mathcal{E}$ is e.m.f. and ϕ is flux then

$$|\mathcal{E}| \propto \frac{d\phi}{dt} \quad \text{--- (1)}$$

The direction of induced e.m.f. is provided by Lenz's law

Lenz's law :-

(4)

The direction of induced e.m.f is such that the magnetic flux associated with the current generated by it, ~~opposite~~ opposes the original change of flux causing the e.m.f.

$$\mathcal{E} = -\frac{d\phi}{dt} \propto -\frac{dB}{dt}$$

Induction law of moving circuit

Consider a circuit of an arbitrary ~~shape~~ shape moving in a time-independent magnetic field ' $\vec{B}$ ' with velocity ' $\vec{v}$ ' (not uniform).
~~for~~ An element ' $d\vec{l}$ (PQ)' will move in time dt a distance $v dt$ to position P'Q'.



Let ' u ' is velocity of conducting electrons relative to wire.
The velocity relative to field is ' $\vec{v} + \vec{u}$ '

the force exerted on each electron is

$$e(\vec{v} + \vec{u}) \times \vec{B}$$

the component of this force along the element dl is

$$\{ e(\vec{v} + \vec{u}) \times \vec{B} \} \cdot \hat{e}_l$$

where $\hat{e}_l$ is unit vector in PQ direction

Since $\hat{e}_l$ is parallel to ' u ' then $(u \times B) \cdot \hat{e}_l = 0$

then

$$(e(\vec{v} + \vec{u}) \times \vec{B}) \cdot \hat{e}_l = (e\vec{v} \times \vec{B}) \cdot \hat{e}_l \quad \text{--- (1)}$$

This shows that there is an electric field $\vec{E} = \vec{v} \times \vec{B}$

induced in the wire,

its component along the wire is $(\vec{v} \times \vec{B}) \cdot \hat{e}_l$

The induced e.m.f is

$$\mathcal{E} = \oint (\vec{v} \times \vec{B}) \cdot \hat{e}_l dl \quad \text{--- (2)}$$

Since circuit moves, the element of circuit dl sweeps in dt an area ~~area~~ $PQ \& P' = v dt \times \hat{e}_l dl$

the flux across this element is $(v dt \times \hat{e}_l dl) \cdot \vec{B}$

then total flux $d\phi = \oint \vec{B} \cdot (v dt \times \hat{e}_l dl)$

$$\frac{d\phi}{dt} = \oint \vec{B} \cdot (v \times \hat{e}_l dl) = - \oint (\vec{v} \times \vec{B}) \cdot \hat{e}_l dl \quad \text{--- (3)}$$

Comparing eqn (2) and (3) $\mathcal{E} = - \frac{d\phi}{dt}$ --- (4)