Integral and Differential form of Faraday's law

The induced e.m.f. is equal to the

$$\varepsilon = \oint \vec{E}\cdot\vec{dl} \quad \text{where } \vec{E} \text{ is electric field} \qquad \text{—(1)}$$

$\vec{dl} = \hat{e}_l dl$ displacement.

The magnetic flux through coil is

$$\phi = \int_s \vec{B}\cdot\hat{e}_n ds$$

$$\phi = \int_s \vec{B}\cdot\vec{ds} \quad \text{where } \vec{B} \text{ is magnetic field} \qquad \text{—(2)}$$

$\vec{ds} =$ surface Area of coil

Then we know from Faraday's law

$$\varepsilon = \oint \vec{E}\cdot\vec{dl} = -\frac{d}{dt}\phi = -\frac{d}{dt}\int \vec{B}\cdot\vec{ds} \qquad \text{—(3)}$$

Since surface area 's' does not depend on time then

$$\oint \vec{E}\cdot\vec{dl} = -\int_s \frac{d\vec{B}}{dt}\cdot\vec{ds} \qquad \text{—— (4)}$$

This is the integral form of Faraday's law

Using Stoke's theorem

$$\oint \vec{E}\cdot\vec{dl} = \int_s \text{Curl}\,\vec{E}\cdot\vec{ds} = \int (\nabla\times\vec{E})\cdot\vec{ds} \qquad \text{—(5)}$$

Using eqn (4) and (5), we get

Then $\int_S \text{curl}\,\vec{E}\cdot d\vec{s} = -\int_S \frac{d\vec{B}}{dt}\cdot d\vec{s}$

Then

$$\int \left(\text{curl}\,\vec{E} + \frac{\partial B}{\partial t}\right)\cdot ds = 0 \quad \text{--- (6)}$$

Since $d\vec{s}$ is arbitary so $d\vec{s} \neq 0$, then

$$\text{curl}\,\vec{E} + \frac{\partial B}{\partial t} = 0$$

$$\text{curl}\,\vec{E} = \nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t} \quad \text{--- (7)}$$

This is differential form of Faraday's law of e.m. induction

We also know that

$$\text{div}\,\vec{E} = \nabla\cdot\vec{E} = \rho/\epsilon_0 \quad \text{--- (8)} \quad \text{(Gauss' law in differential form)}$$

Eq^n (7) and (8) show that $\vec{E}$ has a non-conservative part due to changing $\vec{B}$ as well as conservative part due to ρ (charge density)

The sources of e.m. field are two kinds

(i) first kind is associated with a system such as
(ii) Electrostatic in which energy is conserved during cyclic process.

(iii) This source of such type (conservative or irrotational) is described by $\nabla\cdot\vec{E} = \rho/\epsilon_0$

where $\vec{\sigma} = \cancel{\phi}$

$\vec{E} = -\nabla V$ V ϕ → Electric ~~flux~~ potential

$$\text{Curl}(\text{div} E) = 0$$

then

$$\text{Curl grad } V = 0$$

$$\nabla \times (\nabla V) = 0 \quad \text{— (9)}$$

(ii) The second kind of source is associated with a system in which there is energy transferred in a cyclic process (i.e. magnetic field of a solenoid)

and specified by curl source and has no divergence

Therefore any vector field is uniquely determined if its div. and curl sources are given

This is known as Helmholtz theorem

Taking div. of eqn (7)

$$\nabla \cdot (\nabla \times \vec{E}) = -\frac{\partial}{\partial t}(\nabla \cdot \vec{B}) = 0$$

This is true only if $\nabla \cdot \vec{B} = 0$ — (11) {$\nabla \cdot B$ is independent of time at every point}

i.e. $\vec{B}$ is always solenoidal

The Faraday's law has two important consequences: -

(i) $\vec{E}$ is no loger a conservative field when $\vec{B}$ varies with time. It indicates that energy can flow b/w electric and magnetic forms through time varying field.

(ii) No free magnetic pole can exist. All magnetic poles occur in pairs (positive and negative)

Faraday's law of induction show that $\vec{E}$ and $\vec{B}$ are not independent when we consider their time-dependence. Therefore these two field would be considered as a single field:- e.m. field.

self-and Mutual inductances

when current I flows in a circuit, there will be a magnetic flux(ϕ)

the flux is proportional to I

$$\phi \propto I$$

$$\phi = LI \quad \text{--- (1)}$$

where L is constant

Consider if ϕ changes with time i.e. I is time dependent

then $\phi = \phi(t)$ function of time

then

Therefore

$$\frac{d\phi}{dt} = \frac{d\phi}{dI}\cdot\frac{dI}{dt} = L\frac{dI}{dt} \qquad \text{from eq}^n \text{ 1}$$

then

$$L = \frac{d\phi}{dI} \quad \text{— 2}$$

The L is called self inductance

induced e.m.f.

$$\mathcal{E} = -\frac{d\phi}{dt} = -L\frac{dI}{dt} \quad \text{— 3}$$

L depends on the geometry of the circuit as well as permeability of the medium.

self induction is defined as: a total flux through the circuit when unit current is flowing

$\phi = L$ when I is unit

$L = \phi$ (total flux)

If $I = 1$ Ampere

when $L = \phi$ (weber) — 4

Again when rate of change of current is 1 Ampere/sec then

$\mathcal{E} = -L$

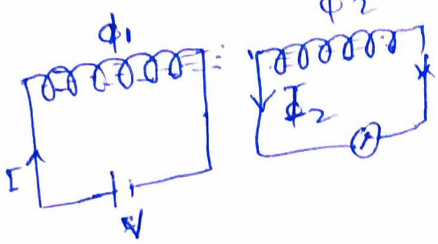
$L = -\mathcal{E} \Rightarrow L = \mathcal{E}$ ~~(volts)~~ (Henry) — 5

The circuit has unit self inductance if $\mathcal{E} = 1$ volt

Let first coil carries a ~~constant~~ current I_1

and second coil is brough near this coil, then will be ϕ_2

through second coil due to the current in first coil



Since $\phi_2 \propto I_1$

$$\phi_2 = L_{21} I_1 \quad \text{—— (6)}$$

where L_{21} is constant

There will also be a flux through first circuit due to I_2

in second coil. then

$$\phi_1 = L_{12} I_2 \quad \text{—— (7)}$$

where L_{12} is again a constant

we know that potential energy of system is given as

$$U_p = -\phi_2 I_2 = -L_{21} I_1 I_2$$
$$= -\phi_1 I_1 = -L_{12} I_2 I_1$$

$$\left\{ \begin{array}{l} U_e = -\vec{p}\cdot\vec{E} \\ U_B = -\vec{M}\cdot\vec{B} \\ U_B = I\phi \Rightarrow \phi = M\cdot ds \end{array} \right.$$

$$\text{Therefore} \quad L_{12} = L_{21} \quad \text{—— (8)}$$

then $L_{12} = L_{21} = M$ is called mutual inductance

b/w two coils

we know that

$$U_p = -\phi_1 I_1 = -I_1 \int \vec{B_1}\cdot d\vec{s}$$

$$U_p = -I_1 \int (\nabla \times A_{12}) \cdot ds$$

$\{ B = \nabla \times A$

$$= -I_1 \int \vec{A}_{12} \cdot \vec{dl}_1$$

using stokes's theorem

$A \rightarrow$ magnetic vector potential

$$= -I_1 \int \left(\frac{\mu}{4\pi} \int \frac{I_2 dl_2}{|r|} \right) \cdot \vec{dl}_1$$

$$= -\frac{\mu}{4\pi} I_1 I_2 \int\int \frac{\vec{dl}_1 \cdot \vec{dl}_2}{|r|}$$

$$U_p = -L_{12} I_1 I_2$$

This is known as Neumann formula

Unit of M is Henry.