$$M\vec{V}_{cm} = m_1\vec{V}_1 + m_2\vec{V}_2 + m_3\vec{V}_3 + \cdots + m_n\vec{V}_n$$

Here M → particle mass

V_{cm} → वस्तु के द्रव्यमान केन्द्र का linear velocity

P_{cm} value put in eq. 1

$$F_{ext.} = \frac{d}{dt}(MV_{cm})$$

$$= M\frac{dV_{cm}}{dt}$$

$$\boxed{F_{ext.} = M a_{cm}}$$

Law of conservation of Momentum :- The total momentum before the collision is the same as the total momentum after the collision.

Ex. friction - act on the system

If external force worke is zero $(F_{ext.} = 0)$

$$F_{ext.} = \frac{dP_{cm}}{dt} = 0$$

or $\boxed{P_{cm} = const}$

$\boxed{MV_{cm} = const}$

when linear momentum is const → they ~~we say~~.

$$\vec{P}_1 + \vec{P}_2 + \vec{P}_3 + \cdots + \vec{P}_n = const.$$

✓ **Law of Conservation of Angular Momentum :-**

'The angular momentum of a particle is defined as the moment of its linear momentum.

Angular momentum . $\vec{L}_{cm} = \vec{r}_{cm} \times m\vec{V}_{cm}$

$$\vec{L}_{cm} = \vec{r}_{cm} \times \vec{P}_{cm} \quad -(1)$$

diffⁿ करने पर

$$\frac{d\vec{L}_{cm}}{dt} = \frac{d}{dt}(\vec{r}_{cm} \times \vec{P}_{cm})$$

$$\frac{dL_{cm}}{dt} = \frac{d\vec{r}_{cm}}{dt} \times \vec{P}_{cm} + \vec{r}_{cm} \times \frac{d\vec{P}_{cm}}{dt}$$

$\vec{v}$

$$= \frac{d\vec{r}}{dt} \times m\vec{v} + \vec{r}_{cm} \times m\frac{d\vec{v}_{cm}}{dt} \qquad [\because \vec{v}\times\vec{v} = 0]$$

$\vec{v}\times\vec{v}$ $\quad \{ \frac{dr}{dt} = 0 \}$

$$= 0 + \vec{r}_{cm} \times m\vec{a}$$

$\{ \mu = \frac{m_1 m_2}{m_1 + m_2}$

$$\text{or } \frac{d\vec{L}_{cm}}{dt} = \vec{r}_{cm} \times \vec{F}_{ext}.$$

ML^2T^{-2}

$$\therefore \frac{d\vec{L}_{cm}}{dt} = \vec{\tau}_{ext}.$$

Torque → S.I. - N.M, J/Rad.

by external force वस्तु पर कार्यरत बल आघूर्ण

$(\tau_{ext.} = 0)$

$I\omega$ - I moment of inertia, ω Angular velocity

$$\frac{d\vec{L}_{cm}}{dt} = 0$$

or $\vec{L}_{cm} = \text{const.}$, or $\vec{L} = \text{const.}$ } This is show of the law of conservation of Angular momentum

In a system different^2 particle's total angular ~~Particle~~ momentum vector is const.

$$\vec{L}_1 + \vec{L}_2 + \vec{L}_3 + - - + \vec{L}_n = \text{const}.$$

→ The law of conservation of Angular momentum of a body states that the angular momentum of its momentum of inertia that is the product of its rotation and its angular velocity about the axis of rotation and its angular velocity about the same axis, cannot change unless an external torque acts on the system.

Friction: friction is the force that opposes the relation motion b/w the two surfaces of objects in contact. The force of friction always acts in the direction opposite to that of the applied force.

$F = \mu N$ $\qquad$ $F = \mu mg$

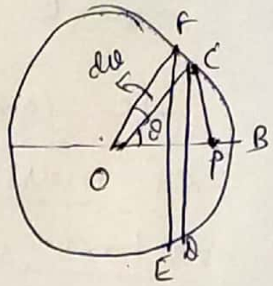
μ → friction coefficient

N → Normal force (equal to weight of object)

$N = mg$

(3) When P lies inside the shell $r < a$

limits $x = a - r$ to $x = a + r$



$\therefore$ The Potential due to the whole shell

$$V = \int_{a-r}^{a+r} \left(-\frac{GM}{2ar}\right) dx$$

$$= -\frac{GM}{2ar} (x)_{a-r}^{a+r}$$

$$= -\frac{GM}{2ar} [a+r-(a-r)]$$

$$\boxed{V = -\frac{GM}{a}}$$

Gravitational field intensity $\rightarrow$

$$E = -\frac{dV}{dr} = -\frac{d}{dr}\left(-\frac{GM}{a}\right)$$

$$\boxed{E = 0}$$

Inside the spherical shell gravitational field intensity is zero.

* Laplace's and Poisson's equation for Gravitational Potential :-

$$\vec{E} = -\vec{\nabla} V \quad -(1)$$

Here $\vec{E} \rightarrow$ gravitational field intensity
$V \rightarrow$ gravitational potential

Here $\vec{\nabla}$ represents the laplacian operator

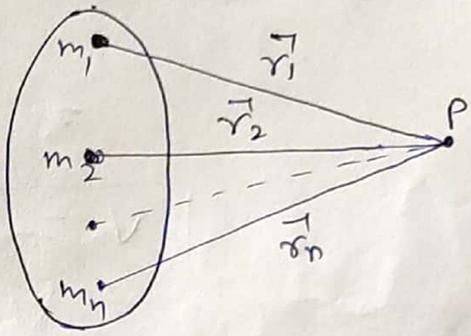
since gravitational potential at any point at a distance r is given by $\rightarrow$

$$V = -\frac{GM}{r} \quad -(2)$$

By eq^n (1) & (2)

$$\vec{E} = \vec{\nabla}\left(\frac{GM}{r}\right) \quad -(3)$$

We consider the system is made up of n particles of masses $m_1, m_2, \ldots m_n$ and their position vectors with respect to P are $\vec{r_1}, \vec{r_2}, \ldots \vec{r_n}$, the force experienced by unit mass at P due to n particles. –



If $\vec{E_1}, \vec{E_2}, \ldots \vec{E_n}$ be the intensities at P due to mass $m_1, m_2, \ldots m_n$ respectively, total intensity at P –

$$\vec{E} = \vec{E_1} + \vec{E_2} + \ldots + \vec{E_n}$$

$$= \vec{\nabla}\left(\frac{Gm_1}{r_1}\right) + \vec{\nabla}\left(\frac{Gm_2}{r_2}\right) + \ldots + \vec{\nabla}\left(\frac{Gm_n}{r_n}\right)$$

$$\vec{E} = \vec{\nabla}\sum_{i=1}^{n}\frac{Gm_i}{r_i} \quad -(4)$$

The potential at P in the presence of n-particles is the amount of work done in bringing a particle of unit mass from infinity to P –

$$-\int_{\infty}^{P}\vec{E}\cdot d\vec{r} = -\sum_{i=1}^{n}\frac{Gm_i}{r_i} = V \quad -(5)$$

Thus the total potential at any point P will be sum of the potentials at that point due to all the particles. since Potential is scalar quantity and therefore if no mass is present (for mass free region) then it satisfy the following relation ug 21

$$\nabla^2 V = -\nabla^2 \sum_{i=1}^{n} \frac{G m_i}{r_i}$$

$$\nabla^2 V = -\sum_{i=1}^{n} \nabla^2 \frac{G m_i}{r_i} = 0$$

$$\boxed{\nabla^2 \cdot V = 0}$$

This relation is known as Laplace eqⁿ for potential.

Poisson's eqⁿ for gravitational potential :-

$$\boxed{\nabla^2 \cdot V = -4\pi\rho = \vec{\nabla} \cdot \vec{E}}$$

प्वासो का eqⁿ