

There are five basic properties:-

- (i) closure property $\Rightarrow a, b \in G \Rightarrow a * b \in G$
- (ii) Associativity property $\Rightarrow a * (b * c) = (a * b) * c, \forall a, b, c \in G$
- (iii) Existence of identity Element $\Rightarrow$
 $\exists e \in G : a * e = a = e * a \quad \forall a \in G$
↑
left identity
right identity
- (iv) Existence of inverse $\Rightarrow$
 $\forall a \in G \exists b \in G : a * b = b * a = e.$
 then b is said to be inverse of a .
- (v) commutative property:- $a * b = b * a \quad \forall a, b \in G$

$\rightarrow$ An algebraic structure $(G, *)$ is said to be

Name of Group	must hold property	Example
quasi group	(i)	$(\mathbb{N}, +), (\mathbb{N}, \cdot), (\mathbb{Z}, +), (\mathbb{Z}, \cdot)$ $(\mathbb{W}, +), (\mathbb{W}, \cdot), (\mathbb{R}, +), (\mathbb{R}, \cdot)$ etc.
Semi group	(i), (ii)	————— (Same as quasi)
Monoid	(i), (ii), (iii)	$(\mathbb{N}, \cdot), (\mathbb{W}, +), (\mathbb{W}, \cdot),$ $(\mathbb{Z}, +), (\mathbb{Z}, \cdot), (\mathbb{R}, +),$ $(\mathbb{R}, \cdot)$ etc.
Group	(i), (ii), (iii) & (iv)	$(\mathbb{Z}, +), (\mathbb{R}, +), (\mathbb{R}, \cdot),$ $(\mathbb{C}, +), (\mathbb{C}, \cdot)$ etc.
Abelian group	(i), (ii), (iii), (iv) & (v)	$(\mathbb{Z}, +), (\mathbb{R}, +), (\mathbb{R}, \cdot),$ $(\mathbb{C}, +), (\mathbb{C}, \cdot)$ etc.

Rings :- An algebraic structure $(R, +, \cdot)$ is called a ring if

- (i) $(R, +)$ is an abelian group.
- (ii) $(R, \cdot)$ is a semigroup.
- (iii) $\cdot$ is distributive over the operation $+$.

Eg :- $(\mathbb{Z}, +, \cdot)$, $(\mathbb{R}, +, \cdot)$, $(\mathbb{C}, +, \cdot)$
 ↑ ↑ ↑
 Integers real no. complex no.

Smallest ring is null ring or zero ring containing only zero element (additive identity)

Field :- A ring $(R, +, \cdot)$ is called field if

- (i) R is commutative ring
- (ii) with unit element (w.r. to $\cdot$)
- (iii) Each non zero element of R has its multiplicative inverse in R .

Eg :- $(\mathbb{Q}, +, \cdot)$, $(\mathbb{R}, +, \cdot)$, $(\mathbb{C}, +, \cdot)$,
 $(\{0, 1, 2, 3, 4\}, +_5, \times_5)$ are some examples of field.

Internal and External operations:
 (आन्तरिक एवं बाह्य संक्रियाएँ)

Let $V \neq \emptyset$, then the binary composition

' $*$ ' defined as $* : V \times V \rightarrow V$

is known as internal operation in V

i.e. for every $a, b \in V \Rightarrow a * b \in V$

Example $\Rightarrow$ Addition '+' and multiplication

' $\cdot$ ' is internal operations in $\mathbb{R}$ (real no.)

For eg $2.1 \in \mathbb{R}, 3.5 \in \mathbb{R} \Rightarrow (2.1) + (3.5) \in \mathbb{R}$
& $(2.1)(3.5) \in \mathbb{R}$

$\rightarrow$ Let V & F are two non-empty set.
then the composition ' $\cdot$ ' defined as

$$\bullet : F \times V \rightarrow V$$

is known as external binary composition.

— on $\forall$ ^{every} $a \in F, v \in V \Rightarrow a \cdot v \in V$
i.e. for

Ex: — let $F = \mathbb{Z}$ (integers), V is the set of rational

numbers $\mathbb{Q}$ and $\bullet \rightarrow$ multiplication operation,
then multiplication is external binary
composition on $\mathbb{Q}$. As we know that
multiplication of a rational number
by an integer is again a rational no.