Linear combination of vectors :→

let V(F) be a vector space. & if.

$a_1, a_2, \dots a_n \in F$; $\alpha_1, \alpha_2 \dots \alpha_n \in V$

then $a_1\alpha_1 + a_2\alpha_2 + \dots + a_n\alpha_n$ or $\sum_{i=1}^{n} a_i\alpha_i$

is known as linear combination (LC) of vectors $\alpha_1, \alpha_2, \dots, \alpha_n$.

Linear span :→ let S be a non empty subset of vector space V(F) then any set that contains all possible ~~finite~~ LC of elements of S is known as linear span of S. It is denoted by L(S)

$$L(S) = \{ \sum a_i\alpha_i \mid a_i \in F, \alpha_i \in S \}$$

Eg :→ (i) If $S = \{(0,1)\} \subset V_2(R)$

then $L(S) = \{(0,a) \mid a \in R\}$

which is collection of all possible points on y-axis

Linear span of empty set ϕ is defined by $\{0\}$ i.e. $\boxed{L(\phi) = \{0\}}$ — (*) (27)

Eg:→ of LC :-

(i) In vector space $V_3(R)$, express the vector $\alpha = (2,4,5)$ as a linear combination of vectors $\alpha_1 = (1,0,0)$, $\alpha_2 = (0,2,0)$ and $\alpha_3 = (0,0,1)$

Solⁿ:- By the defⁿ of LC (linear combination)

let $\alpha = a_1\alpha_1 + a_2\alpha_2 + a_3\alpha_3$ — (i)

where $a_1, a_2, a_3 \in R$

$(2,4,5) = a_1(1,0,0) + a_2(0,2,0) + a_3(0,0,1)$

$\Rightarrow (2,4,5) = (a_1,0,0) + (0,2a_2,0) + (0,0,a_3)$

$\Rightarrow (2,4,5) = (a_1+0+0,\ 0+2a_2+0,\ 0+0+a_3)$

$\Rightarrow (2,4,5) = (a_1,\ 2a_2,\ a_3)$

$\Rightarrow a_1 = 2,\ 2a_2 = 4,\ a_3 = 5$

$\Rightarrow a_1 = 2,\ a_2 = 2,\ a_3 = 5$ put this in (i),

$\boxed{\alpha = 2\alpha_1 + 2\alpha_2 + 5\alpha_3}$ is required LC.

(ii) In a vector space $V_3(R)$, Express $\alpha = (1,-2,5)$ as a LC of vectors $v_1 = (1,2,3)$, $v_2 = (2,-1,1)$ and $v_3 = (1,1,1)$

Solⁿ:→ By the defⁿ of LC

let $\alpha = a_1\alpha_1 + a_2\alpha_2 + a_3\alpha_3$ where $a_1, a_2, a_3 \in R$ — (i)

$(1,-2,5) = a_1(1,2,3) + a_2(2,-1,1) + a_3(1,1,1)$

$\Rightarrow (1,-2,5) = (a_1, 2a_1, 3a_1) + (2a_2, -a_2, a_2) + (a_3, a_3, a_3)$

$\Rightarrow (1,-2,5) = (a_1 + 2a_2 + a_3,\ 2a_1 - a_2 + a_3,\ 3a_1 + a_2 + a_3)$

$\Rightarrow \quad a_1 + 2a_2 + a_3 = 1$ —— (ii)

$2a_1 - a_2 + a_3 = -2$ —— (iii)

$3a_1 + a_2 + a_3 = 5$ —— (iv)

(ii) + 2(iii) $\Rightarrow \quad 5a_1 + 3a_3 = -3$ —— (V)

(iii) + (iv) $\Rightarrow \quad 5a_1 + 2a_3 = 3$ —— (vi)

(V) — (vi) $\Rightarrow \quad a_3 = -6$

put value of a_3 in (V) $\Rightarrow 5a_1 - 18 = -3$

$\Rightarrow \quad 5a_1 = 15$

$\Rightarrow \quad a_1 = 3$

put value of a_1, a_3 in (ii) $\Rightarrow 3 + 2a_2 - 6 = 1$

$\Rightarrow \quad 2a_2 = 4$

$\Rightarrow \quad a_2 = 2$

put value of a_1, a_2, a_3 in (i), we have

$\alpha = 3\alpha_1 + 2\alpha_2 + (-6)\alpha_3$

(viii) Express $\alpha = (2,-5,3)$ as a LC of vectors. $\alpha_1 = (1,-3,2)$, $\alpha_2 = (2,-4,-1)$ and $\alpha_3 = (1,-5,7)$

solⁿ:→ by the defⁿ of LC

$\alpha = a_1\alpha_1 + a_2\alpha_2 + a_3\alpha_3$ —— (i) where $a_1, a_2, a_3 \in R$

$(2,-5,3) = a_1(1,-3,2) + a_2(2,-4,-1) + a_3(1,-5,7)$

$= (a_1 + 2a_2 + a_3,\ -3a_1 - 4a_2 - 5a_3,\ 2a_1 - a_2 + 7a_3)$

$\Rightarrow a_1 + 2a_2 + a_3 = 2$ —— (ii)

$-3a_1 - 4a_2 - 5a_3 = -5$ —— (iii)

$2a_1 - a_2 + 7a_3 = 3$ —— (iv)

$2 \times (ii) + (iii) \Rightarrow -a_1 - 3a_3 = -1$ —— (v)

~~2 x (iii) +~~ $(ii) + 2(iv) \Rightarrow 5a_1 + 15a_3 = 8$

$\Rightarrow a_1 + 3a_3 = \frac{8}{3}$ —— (vi)

from (v) & (vi), $a_1 + 3a_3 = 1$ as well as $8/3$ which is not possible. Hence eq.^{n} (ii), (iii) & (iv) is inconsistent system of equations.

$\Rightarrow$ vector α cannot be expressed as a linear combinations of vectors $\alpha_1, \alpha_2, \alpha_3$.

Eg. of Linear span (LS)

(i) If $S = \{(1,0,0), (0,1,0), (0,0,1)\}$ is a subset of vector space $V_3(R)$.

then $L(S) = \{a(1,0,0) + b(0,1,0) + c(0,0,1) \mid a, b, c \in R\}$

$\Rightarrow L(S) = \{(a,b,c) \mid a, b, c \in R\}$

~~Hence for~~ Hence for $\alpha = (a,b,c) \in V_3(R) \Rightarrow \alpha \in L(S)$

$\Rightarrow \quad V_3(R) \subset L(S)$ —— (i)

But $\quad L(S) \subset V_3(R)$ —— (ii)

by (i) & (ii) $\Rightarrow \boxed{V_3(R) = L(S)}$