Continuity

The primary object of Differential Calculus is to deal with the way in which one quantity varies when the other, on which it depends is made to vary and with the allied topics. when two quantities change in relation to eachother, the ratio

$$= \frac{\text{change in quantity during a certain interval}}{\text{Change in the other quantity during the same interval.}}$$

is known as the "rate of change of one w.r.to other" over the interval. Then we further define & discuss the concept of rate of change "at a particular point".

The concept of set is also very important in number system.

__Quantities__: Any thing which can be measured is called quantity, like length, area, volume etc.

__Quantities__

Constants

Variables

Absolute constants | Arbitrary constants | Independent variables | Dependent variables

(a) <u>constant</u>: A quantity which has a fixed value throughout a set of mathematical operations is called a constant.

(i) <u>Absolute constant</u>: A quantity which has the same value in every mathematical operation is called absolute constants.
like 1, 2, $\sqrt{2}$, π, $1/3$, e etc.

(ii) <u>Arbitrary constant</u>:- A quantity, which is considered fixed, throughout a particular operation, but different for different operations.
For example, in the equation of ~~[illegible]~~ straight line $\frac{x}{a} + \frac{y}{b} = 1$, the quantities a, & b are fixed for same straight line, but different for different lines.

<u>variables</u>: A quantity which is capable of assuming any of a set of numerical values during any set of mathematical operation.

(1) <u>Independent variables</u>:- A variable which can take up any arbitrary value assigned to it.

(ii) Dependent variables: A variable whose value depends on the other arbitrary chosen value of some other variable.
In short we can say" A variable, which depends on the other variable".

Function Let A and B are two sets, then a function from A to B is a rule which associates each element of the set A to a unique element of the set B. We denote it as $f: A \rightarrow B$
Set A is called domain & B is called Co-domain of f
A may be finite, infinite or an interval.
Let $a \in A$ is associated with $b \in B$, under the function 'f', then b is called f image of a and it is denoted by $b = f(a)$.

Ex:- ⓐ Let r be the radius of a circle, then it's area $\pi r^2 = A$ is called a function of r
(b) If $y = 2x^2 + x + 1$, then y is a function of x.
(c) If $y = 2u + v$, then y is a function of u & v both.

If the domain of the function is a subset of real ~~real~~ numbers and range is real, then function is said to be a real valued function or function of real ~~vari~~ variable or real function.

Nature of functions

(a) Positive and Negative functions: A function f, defined on closed interval $[a, b]$ is called positive or negative if $f(x) \geq 0$ or $f(x) < 0$ $\forall x \in [a, b]$ respectively.

(b) Even or odd function: A function f, defined on a closed interval $[a, b]$ is called even or odd function if $f(-x) = f(x)$ or $f(-x) = -x$ $\forall x \in [a, b]$ respectively.

Ex Sinx is an odd function & cosx is an even function.

Because $\underbrace{\sin(-x) = -\sin x}_{\text{odd}}$; $\underbrace{\cos(-x) = \cos x}_{\text{even}}$

(c) Bounded function: A function f, defined on D is called bounded if it's range is bounded, otherwise unbounded.

i.e. if $\exists$ k & K such that $k \leq f(x) \leq K$ $\forall x \in D$, then f is called bounded function.

Here k is called lower bound & K is called upper bound of f in D.
All the numbers which are less than k are also called lower bounds & numbers which are greater than K are also called upper bounds.

G.L.B :- The greatest among the lower bounds of f is called greatest lower bound. and it is called infimum of f. ~~The~~

L.U.B :- The least among the upper bounds of f is called least upper bound and it is called maxima of f.

Ex. 1. Let $f(x) = \frac{1}{x}$, $0 < x < \infty$, then '0' is the infimum, but it doesnot has maxima. (Because it is unbounded)
If we take ~~range~~ Domain (a, ∞) in place of $(0, \infty)$, where $a \neq 0$, then it is bounded. It's lower bound is 0 & upper bound is $\frac{1}{a}$.

Monotonic function :- A function f defined on a domain D is said to be monotonic if it is either monotonically increasing or monotonically decreasing.

<u>Monotonically increasing</u> A function f, defined on D, is said to be monotonically increasing if

$$f(x) \geq f(y) \text{ if } x > y \quad \forall x, y \in D.$$

It is called strictly increasing if

$$f(x) > f(y) \text{ if } x > y \quad \forall x, y \in D.$$

<u>Monotonically decreasing</u> A function f, defined on D, is said to be monotonically decreasing if

$$f(x) \leq f(y) \text{ if } x > y \quad \forall x, y \in D$$

It is called strictly decreasing if

$$f(x) < f(y) \text{ if } x > y \quad \forall x, y \in D.$$

<u>Ex 1.</u> $f(x) = \frac{1}{x} \ \forall \ x \in (0, 1)$ is strictly decreasing function

Ex 2. $f(x) = e^{-\frac{1}{x}} \quad \forall x \in (-\infty, 0)$ is strictly increasing function.

<u>Limit of a function:</u>

<u>Concept of Limit:</u> Let f is a function defined on the domain D. If we are able to get finite value of f(x) at $x=a$ i.e. $f(a)$ then it is called value of the function at $x=a$. If at $x=a$

f(x) takes ~~indet~~ indeterminate form i.e. function is not defined at $x=a$, then we try to find out the value of f(x), very near to $x=a$. If by doing so we get a definite value of f(x) (unique number 'l') as x tends to a, then this unique number is called limit of f(x) as 'x' tends to 'a'. We write it as

$$\lim_{x \to a} f(x) = l$$

Ex: Let $f(x) = \frac{(x-2)}{(x^2-4)}$, then $f(2) = \frac{0}{0}$ form, which is indeterminate. Therefore it is not defined at $x=2$, so we try to find out the value of f(x) near to $x=2$.

$$\Rightarrow \lim_{x \to 2} \frac{(x-2)}{(x^2-4)} = \lim_{x \to 2} \frac{(x-2)}{(x-2)(x+2)}$$

But if $x \neq 2$ then $(x-2)$ can be cancill out.

$\Rightarrow$ value becomes $\lim_{x \to 2} \frac{\cancel{(x-2)}}{\cancel{(x-2)}(x+2)} = \frac{1}{2+2}$

$= \frac{1}{4}$

$$\Rightarrow \lim_{x \to 2} \left[\frac{(x-2)}{(x^2-4)}\right] = \frac{1}{4}$$

Scanned with CamScanner