Sequence

Sequences:- let A be a non empty set, then every function, which is defined from set of natural numbers ~~A~~ N to A is a sequence.
i.e. A sequene is nothing but a function whose domain is a set of natural numbers N and range is subset of real numbers R.

let, if $x : N \to R$ is a function defined from N to R and let we denoted image of $n \in N$ by $x(n) \in R$, if we write it as x_n, then sequne of x_n is represented by $x = \{x(1), x(2), x(3) --- x(n) --- \}$ in short we represent it by $\langle x_n \rangle$

Constant sequne:- A sequence $\langle x_n \rangle$, where $x_n = c \in R \ \forall n \in N$ is called constant sequne.

Ep. 1. If $x : N \to R$, where $x_n = 1 - (-1)^n$, then the seque will be $\{2, 0, 2, 0, 2, 0, 2 ---\}$

$$\Rightarrow x_n = \begin{cases} 2 & \text{if } n \text{ is odd} \\ 0 & \text{if } n \text{ is even} \end{cases}$$

Ep. 2 Range of sequne: let $\{-1, +1, -1, +1, -1, +1 ---\}$ be a sequence or $\langle (-1)^n \rangle$, then Range of $\langle x_n \rangle$
$= \{-1, 1\}$ which is ~~set~~ finite set.

Bounded sequence: A sequence $\langle x_n \rangle$ is said to be bounded above if it's range is bounded from above i.e. if $\exists$ a real number K s.t.

$$x_n \leq K \quad \forall n \in \mathbb{N}.$$

K is called upper bound of $\langle x_n \rangle$.

Similarly $\langle x_n \rangle$ is said to bounded below if its range is bounded from below.
i.e. if $\exists$ a real number k s.t.

$$k \leq x_n \quad \forall n \in \mathbb{N}.$$

k is called lower bound of $\langle x_n \rangle$.

A sequence $\langle x_n \rangle$ is said to be bounded if it is bounded from below & above.

i.e. if $\exists$ $k, K \in \mathbb{R}$ st.

$$k \leq x_n \leq K \quad \forall n \in \mathbb{N}.$$

It can also be written as, A sequence $\langle x_n \rangle$ is said to be bounded if $\exists$ a real number $M > 0$ st $|x_n| \leq M \quad \forall n \in \mathbb{N}.$

Supremum:- Least upper bound is called Supremum.

Infimum:- Greatest lower bound is called infimum.

Eg1. $\langle \frac{1}{n} \rangle$ sequence has 0 infimum & 1 as supremum because $0 < x_n \leq 1$

Eg 2 $\{1 + \frac{1}{2} + \frac{1}{2^2} + \cdots + \frac{1}{2^{n+1}}\}$ is bounded sequence whose infimum is 1 & supremum is 2.

Monotonic sequence :- A sequence $\langle x_n \rangle$ is said to be monotonically increasing if

$$x_1 \leq x_2 \leq x_3 \leq x_4 \cdots \leq x_n \leq \cdots$$

and monotonically decreasing if

$$x_1 \geq x_2 \geq x_3 \geq x_4 \cdots \geq x_n \geq \cdots$$

i.e. if $x_n \leq x_{n+1}$ $\forall n \in N \Rightarrow$ Monotonically increasing

if $x_n \geq x_{n+1}$ $\forall n \in N \Rightarrow$ Monotonically decreasing

For strictly increasing & decreasing sequence

$$x_n < x_{n+1} \quad \& \quad x_n > x_{n+1} \quad \forall n \in N$$

respectively.

Limit point of a sequence :- A real number l is said to be limit point of sequence $\langle x_n \rangle$ if every neighbourhood (n.b.d.) of l contains infinite number of members of $\langle x_n \rangle$.

$\Rightarrow$ $(\alpha - \epsilon, \alpha + \epsilon)$ contains infinite member of $\langle x_n \rangle$

$\Rightarrow$ $|x_n - \alpha| < \epsilon$ for infinite $n \in N$.