Lattice Thm (1)

Thm Let $(A, \leq)$ be a lattice with binary operations $\vee$ and $\wedge$. Then

(1) $a\vee(b\vee c) = (a\vee b)\vee c$ and

$a\wedge(b\wedge c) = (a\wedge b)\wedge c$; $a, b, c \in A$

(2) $a\vee(a\wedge b) = a$ and $a\wedge(a\vee b) = a$; $a, b \in A$

Pf. Let $a\vee(b\vee c) = x$

and $(a\vee b)\vee c = y$

x is the join of a and $b\vee c$ i.e., x is the LUB of a and $b\vee c$,

$\Rightarrow a \leq x$ and $b\vee c \leq x$

Now $b\vee c \leq x$

$\Rightarrow b \leq x$ and $c \leq x$

Since the join of a and b is the LUB of a and b, from $a \leq x$ and $b \leq x$, we get

$a\vee b \leq x$

Further, $a\vee b \leq x$ and $c \leq x$

$\Rightarrow (a\vee b)\vee c \leq x \Rightarrow y \leq x$

Similarly, $x \leq y$ $\therefore x = y$ (by Antisymmetry)

ie, $a\vee(b\vee c) = (a\vee b)\vee c$

By Principle of Duality, $(a\wedge b)\wedge c = a\wedge(b\wedge c)$

$\therefore$ Both join & meet are Associative.

Lattice Thm (2)

(2) $a \leq a \vee (a \wedge b)$

$\because$ $a \vee (a \wedge b)$ is the join of a and $a \wedge b$

Also, $a \leq a$ and $a \wedge b \leq a$

$\therefore$ $a \vee (a \wedge b) \leq a \vee a$

ie, $a \vee (a \wedge b) \leq a$ (By Idempotent property $a \vee a = a$)

$\therefore$ $a \vee (a \wedge b) = a$ (by Antisymmetry)

Other result follows by Principle of Duality.

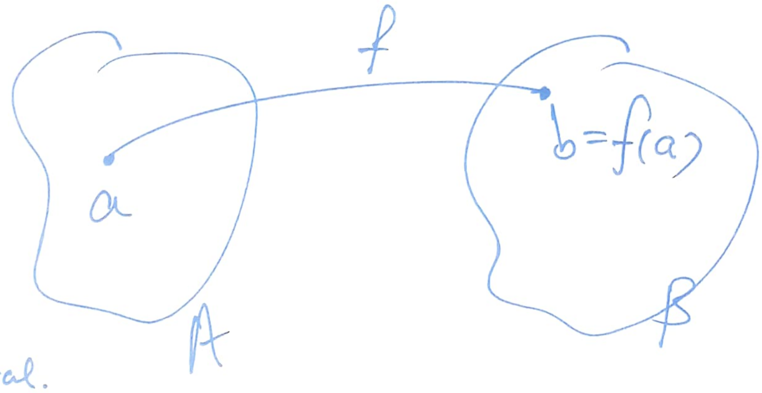
This is <u>Absorption Property</u>.

Kolman p161 (mappings/ Transformations) F^n (1)

Function – Special type of Relation.

A f^n from a set A to set B is a relation from A to B, denoted by $f: A \to B$ s.t. $\forall a \in \text{Domain}(f)$, $f(a)$, the value or image of a contains just one elt of B.

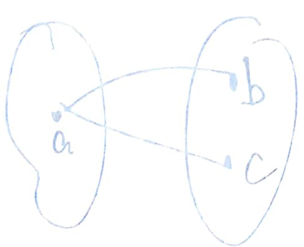
a: argument



P22 Iyengar et al.

ex. The relation $f = \{(1,d), (2,c), (3,a)\}$ from $A = \{1,2,3\}$ to $B = \{a,c,d\}$ is a f^n from A to B, $\text{Domain}(f) = A$, $\text{Range}(f) = B$

ex. Relation $f = \{(a,b), (a,c), (b,d)\}$ from $A = \{a,b\}$ to $B = \{b,c,d\}$ is NOT a f^n.



$\because$ a is not single-valued

ex $A = \{1,2,3\}$, $B = \{x, y, z\}$

Relation $R = \{(1,x), (2,x)\}$

$Dom(R) = \{1,2\}$

$Range(R) = \{x\}$

Special Types of f^ns: Let f be a f^n from A to B

Kolman P164

One-One (1-1) if $f(a) = f(a')$
$\Rightarrow a = a'$

OR if we cannot have $f(a) = f(a')$ for 2 distinct a & a' of A.

Onto :- if $Range(f) = B$.

Invertible f^n: $f: A \to B$ is invertible if its

Kolman P166 inverse f^n, f^{-1} is also a f^n.

ex $A = \{1,2,3,4\}$, $B = \{a,b,c,d\}$

$f = \{(1,a), (2,a), (3,d), (4,c)\}$

Then $f^{-1} = \{(a,1), (a,2), (d,3), (c,4)\}$

f^{-1} is NOT a f^n ! $f^{-1}(a) = \{1,2\}$ (not single valued)

Pigeonhole Principle

If n pigeons fly into K pigeonholes and $n > K$ then there ~~are~~ exist at least one pigeonhole that contains more than 1 pigeon.

Ex If any 5 numbers from 1 to 8 are chosen, then show that 2 of them will add to 9.

Sol. Let us construct 4 different sets, each containing 2 numbers that add upto 9 as follows:

$S_1 = \{1, 8\}$, $S_2 = \{2, 7\}$, $S_3 = \{3, 6\}$, $S_4 = \{4, 5\}$

Each of the 5 numbers chosen must belong to one of ~~these~~ these sets. Since there are only 4 sets, applying the Pigeonhole Principle, 2 of the chosen numbers belong to the same set, that add upto 9. Ans