Ex:- Express the vector $\alpha = (0, 4, 20)$ of $V_3(R)$ as a LC of vectors $\alpha_1 = (2, 1, -1)$, $\alpha_2 = (-1, 0, 3)$ & $\alpha_3 = (0, 1, 5)$

Sol^n:→ let $\alpha = a_1\alpha_1 + a_2\alpha_2 + a_3\alpha_3$

where $a_1, a_2, a_3 \in R$

$$(0, 4, 20) = a_1(2, 1, -1) + a_2(-1, 0, 3) + a_3(0, 1, 5)$$

$$= (2a_1, a_1, -a_1) + (-a_2, 0, 3a_2) + (0, a_3, 5a_3)$$

$(0,4,20) = (2a_1 - a_2,\ a_1 + a_3,\ -a_1 + 3a_2 + 5a_3)$

$\Rightarrow \quad 2a_1 - a_2 = 0$ — (i)

$a_1 + a_3 = 4$ — (ii)

$-a_1 + 3a_2 + 5a_3 = 20$ — (iii)

(i) $\Rightarrow \quad a_2 = 2a_1$ so from eq.^{n} (iii)

$-a_1 + 6a_1 + 5a_3 = 20$

$5a_1 + 5a_3 = 20$

$a_1 + a_3 = 4$ — (iv)

On simplification Eq.^{n} (i) – (iv)

$a_1 = 1,\ a_2 = 2,\ a_3 = 3$

Here $a_1, a_2, a_3 \in R$ so

$$\boxed{\alpha = 1.\alpha_1 + 2.\alpha_2 + 3.\alpha_3}$$

Ex:→ for which value of c will the vector $\alpha = (10, c, -13) \in V_3(R)$ is a LC of vectors $\alpha_1 = (4, 3, -2)$ & $\alpha_2 = (2, -6, 7)$.

Sol^n:→ let $\alpha = a_1\alpha_1 + a_2\alpha_2$ where $a_1, a_2 \in R$

$\Rightarrow (10, c, -13) = a_1(4, 3, -2) + a_2(2, -6, 7)$

$\Rightarrow (10, c, -13) = (4a_1, 3a_1, -2a_1) + (2a_2, -6a_2, 7a_2)$

$\Rightarrow (10, c, -13) = (4a_1 + 2a_2,\ 3a_1 - 6a_2,\ -2a_1 + 7a_2)$

$\Rightarrow \quad 4a_1 + 2a_2 = 10$ — (i)

$\Rightarrow \quad 3a_1 - 6a_2 = c$ — (ii)

$-2a_1 + 7a_2 = -13$ — (iii)

$\Rightarrow$ (i) & (iii)

$16a_2 = -16$

$\Rightarrow a_2 = -1 \in R$

$\Rightarrow 4a_1 = 12 \Rightarrow a_1 = 3 \in R$

put value of a_1 & a_2 in (ii), we have

$$3(3) - 6(-1) = c$$

$$\Rightarrow 9 + 6 = c$$

$$\Rightarrow \boxed{15 = c}$$

Ex:→ In the vector space of 2×2 matrices over the field of R, express the vectors:

$\alpha = \begin{pmatrix} 3 & -1 \\ 1 & 2 \end{pmatrix}$ as LC of following vectors:

$\alpha_1 = \begin{pmatrix} 1 & 1 \\ 0 & -1 \end{pmatrix}$, $\alpha_2 = \begin{pmatrix} 1 & 1 \\ -1 & 0 \end{pmatrix}$, $\alpha_3 = \begin{pmatrix} 1 & -1 \\ 0 & 0 \end{pmatrix}$

solⁿ:→ let $\alpha = a_1\alpha_1 + a_2\alpha_2 + a_3\alpha_3$ where $a_1, a_2, a_3 \in R$

$$\begin{pmatrix} 3 & -1 \\ 1 & 2 \end{pmatrix} = a_1\begin{pmatrix} 1 & 1 \\ 0 & -1 \end{pmatrix} + a_2\begin{pmatrix} 1 & 1 \\ -1 & 0 \end{pmatrix} + a_3\begin{pmatrix} 1 & -1 \\ 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 3 & -1 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} a_1 & a_1 \\ 0 & -a_1 \end{pmatrix} + \begin{pmatrix} a_2 & a_2 \\ -a_2 & 0 \end{pmatrix} + \begin{pmatrix} a_3 & -a_3 \\ 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 3 & -1 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} a_1 + a_2 + a_3 & a_1 + a_2 - a_3 \\ -a_2 & -a_1 \end{pmatrix}$$

$$\Rightarrow \left.\begin{aligned} a_1 + a_2 + a_3 &= 3 \\ a_1 + a_2 - a_3 &= -1 \\ -a_2 &= 1 \\ -a_1 &= 2 \end{aligned}\right] \Rightarrow \begin{aligned} a_1 &= -2 \\ a_2 &= -1 \\ a_3 &= 2, 6 \end{aligned}$$

$$\Rightarrow \boxed{\alpha = -2\alpha_1 - \alpha_2 + 2\alpha_3}$$

Ex: → Show that the following vectors span the vector space $R^3(R)$

$\alpha_1 = (1,1,1),\ \alpha_2 = (2,2,0),\ \alpha_3 = (3,0,0)$

Solⁿ: → let $\alpha = (v_1, v_2, v_3) \in R^3(R)$

let $S = \{\alpha_1, \alpha_2, \alpha_3\}$

If S span R^3 than ~~prove that~~ every vector of R^3 is LC of elements of S.

$$\alpha = a_1\alpha_1 + a_2\alpha_2 + a_3\alpha_3$$

$$(v_1, v_2, v_3) = a_1(1,1,1) + a_2(2,2,0) + a_3(3,0,0)$$

$$(v_1, v_2, v_3) = (a_1 + 2a_2 + 3a_3,\ a_1 + 2a_2 + 0,\ a_1 + 0 + 0)$$

$$\Rightarrow \quad a_1 + 2a_2 + 3a_3 = v_1$$

$$a_1 + 2a_2 = v_2$$

$$a_1 = v_3$$

$$\Rightarrow \quad a_1 = v_3,\quad a_2 = \frac{v_2 - v_3}{2},\quad a_3 = \frac{v_1 - v_2}{3}$$

$$\Rightarrow \quad \alpha = v_3\alpha_1 + \left(\frac{v_2 - v_3}{2}\right)\alpha_2 + \left(\frac{v_1 - v_2}{3}\right)\alpha_3$$

where $a_1, a_2, a_3 \in R$.

⇒ Any element of R^3 can be expressed as a LC of elements of S

⇒ $R^3 = L(S)$ or R^3 is generated/span by $\alpha_1, \alpha_2, \alpha_3$

Q: If $\alpha_1 = (1, 2, -1)$, $\alpha_2 = (2, 3, -2)$, $\alpha_3 = (4, 1, 3)$ and $\alpha_4 = (-3, 1, 2)$ be the vectors in $V_3(R)$, show that

$$L(\{\alpha_1, \alpha_2\}) \neq L(\{\alpha_3, \alpha_4\})$$

हल:- सम्भव हो, तो माना कि

$$L(\{\alpha_1, \alpha_2\}) = L(\{\alpha_3, \alpha_4\})$$

तब प्रत्येक स्वेच्छ अदिश $a_1, a_2 \in R$ के लिये $a_3, a_4 \in R$ ऐसे विद्यमान होंगे कि

$$a_1\alpha_1 + a_2\alpha_2 = a_3\alpha_3 + a_4\alpha_4 \quad \text{— (*)}$$

$$\Rightarrow a_1(1, 2, -1) + a_2(2, -3, 2) = a_3(4, 1, 3) + a_4(-3, 1, 2)$$

$$\Rightarrow (a_1 + 2a_2, 2a_1 - 3a_2, -a_1 + 2a_2) = (4a_3 - 3a_4, a_3 + a_4, 3a_3 + 2a_4)$$

$$\Rightarrow a_1 + 2a_2 = 4a_3 - 3a_4 \quad \text{— (i)}$$

$$2a_1 - 3a_2 = a_3 + a_4 \quad \text{— (ii)}$$

$$-a_1 + 2a_2 = 3a_3 + 2a_4 \quad \text{— (iii)}$$

(i) तथा (iii) से

$$4a_2 = 7a_3 - a_4$$

$$\Rightarrow a_2 = \frac{1}{4}(7a_3 - a_4) \quad \text{— (iv)}$$

a_2 का मान (i) में रखने पर

$$a_1 = \frac{1}{2}(a_3 - 5a_4) \quad \text{— (v)}$$

(iv) तथा (v) से मान (ii) में प्रतिस्थापित करने पर

$$2a_1 - 3a_2 = a_3 - 5a_4 - \frac{3}{4}(7a_3 - a_4)$$

$$= \frac{-17a_3 - 17a_4}{4}$$

$$= -\frac{17}{4}(a_3 + a_4) \neq a_3 + a_4$$

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$\Rightarrow$ समीकरण (ii) सतुंष्ट नहीं हो रहा है।

$\Rightarrow$ अतः संबंध (✱) के लिये $a_1, a_2, a_3, a_4 \in R$ संभव नहीं हैं।

अतः $L(\{\alpha_1, \alpha_2\}) \neq L(\{\alpha_3, \alpha_4\})$

उदाहरण 6 :→ सदिश समष्टि $V_3(R)$ के सदिश $\alpha_1 = (1,2,1)$, $\alpha_2 = (3,1,5)$, $\alpha_3 = (3,-4,7)$ के लिये सिद्ध करिये कि $K = \{\alpha_1, \alpha_2\}$ तथा $W = \{\alpha_1, \alpha_2, \alpha_3\}$ द्वारा जनित सदिश उपसमष्टियाँ समान होंगी।

For the ~~Elements~~ vectors $\alpha_1 = (1,2,1)$, $\alpha_2 = (3,1,5)$, $\alpha_3 = (3,-4,7)$ of the vector space $V_3(R)$, prove that the subspaces spanned by $K = \{\alpha_1, \alpha_2\}$ and $W = \{\alpha_1, \alpha_2, \alpha_3\}$ are the same.

हल :→ एकघात विस्तृति की परिभाषा से

$L(W) = \{a_1\alpha_1 + a_2\alpha_2 + a_3\alpha_3 \mid a_1, a_2, a_3 \in R\}$

$= \{a_1(1,2,1) + a_2(3,1,5) + a_3(3,-4,7)\}$ —(i)

पुनः माना कि

$(3,-4,7) = b_1(1,2,1) + b_2(3,1,5)$

$= (b_1 + 3b_2, 2b_1 + b_2, b_1 + 5b_2)$ —(ii)

$$\Rightarrow \left.\begin{aligned} b_1 + 3b_2 &= 3 \\ 2b_1 + b_2 &= -4 \\ b_1 + 5b_2 &= 7 \end{aligned}\right\} \Rightarrow b_1 = -3, \; b_2 = 2$$

b_1 तथा b_2 के मान (ii) में रखने पर

$(3,-4,7) = -3(1,2,1) + 2(3,1,5)$

$\Rightarrow a_3(3,-4,7) = -3a_3(1,2,1) + 2a_3(3,1,5)$ —(iii)

~~अब~~ (iii) से मान (i) में प्रतिस्थापित करने पर

$$L(W) = \{a_1(1,2,1) + a_2(3,1,5) - 3a_3(1,2,1) + 2a_3(3,1,5)\}$$

$$= \cancel{\{a_1(1,2,1) + a_3(}$$

$$= \{(a_1 - 3a_3)(1,2,1) + (a_2 + 2a_3)(3,$$

$$= \{a(1,2,1) + b(3,1,5) \mid a, b \in$$

$$\left\{ \begin{aligned} \because a &= a_1 - 3a_3 \in R \\ b &= a_2 + 2a_3 \in R \end{aligned} \right\}$$

$$= L\{K\}$$