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Latin Square Design (L.S.D):— In R.B.D. Whole of the experimental area is divided into relatively homogeneous groups (blocks) and treatments are allocated at random to units within each block i.e. randomisation was one restriction i.e. within blocks.

But in field experimentation it may happen that experimental area (field) exhibits fertility in strips. eg cultivation might result in alternative strips of high or low fertility. R.B.D will be effective if the blocks happen to be parallel to these strips and would be extremely inefficient if the blocks are across the strips. ~~and would limit~~ Intially fertility gradient is seldom known. A useful method of eliminating fertility variation consists in an experimental layout which will control variation in two perpendicular directions. Such a layout is a Latin squaare design (L.S.D)

Layout of Design:— In this design the no. of treatments is equal to the no of replications. Thus in case of m treatments there have to be $m \times m = m^2$ experimental units (plot) arranged in a square so that each row as well as each columns contains m units (plots). The m treatments are then allocated at random to these rows and columns in such a way that every

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say $64 - 16 = 48$ experiment unit obsn.

(3) Statistical analysis is simple even in missing value case

(4) More than one factor can be investigated simultaneously.

Disadvantages of L.S.D :- (1) fundamental interaction assumption of no interaction betn different factors is not true

(2) Unlike R.B.D. in L.S.D the no of treatments is restricted to the no. of replicates. So there is limit of selecting no. of treatments it should be betn. 5 & 10 and more than 10 to 12 is seldom used.

(3) For missing plots techn, the statistical analysis becomes quite complex.

(4) Field layout R.B.D. is much easy to manage than L.S.D..

Statistical Analysis of $m \times m$ L.S.D. :-

(For one obsn per experimental cell)

Let y_{ijk} $(i, j, k = 1, 2, \ldots, m)$ denote the response from the unit (plot as in field experimentation) in the i^{th} row, j^{th} column and receiving the k^{th} treatment. If S represents the set of m^2 values then symbolically $(i, j, k) \in S$. ~~If a~~ For one obsern per experimental unit the linear additive model is

$$y_{ijk} = \mu + \alpha_i + \beta_j + \tau_k + \varepsilon_{ijk} \quad (i, j, k) \in S$$

treatments occurs once and only once in each row and in each column contains m units (plots). ~~The m treatments are then allocated at random to these rows and columns in such a way that every treatment occurs once and only once in each row and in~~ each column. Such a layout is known as m×m Latin Square Design (L.S.D.) ~~an~~ and is extensively used in agricultural experiments.

e.g. If we are interested in studying the effects of m types of fertilizers on the yield of a certain variety of wheat, it is customary to conduct the experiment on a square field with m^2 plots of equal area. and to associate treatments with different fertilisers and row and column effects with variations in fertility of soil.

For an m×m LSD a particular layout in an experiments with four treatments A, B, C, D is given below

A	B	D	C
B	D	C	A
D	C	A	B
C	A	B	D

Advantage of L.S.D.:- (1) control more variation than C.R.D & R.B.D.

(2) L.S.D. is an incomplete 3-way layout

$4^3 = 64$ → complete ~~th~~ 3 way layout — experimental unit

$4^2 = 16$ = ~~48~~ = experimental unit incomplete 3-way layout where μ is the constant mean effect α_i, β_j and τ_k are the effects due to the i^{th} row, j^{th} column and k^{th} treatment respectively and ε_{ijk} is error effect due to random component assumed to be normally distributed with mean zero & variance σ_e^2, i.e. $\varepsilon_{ij} \sim N(0, \sigma_e^2)$. If we write

$G = y_{...}$ = Total of all m^2 obserⁿ.
$R_i = y_{i..}$ = " the m obserⁿ in the i^{th} row
$C_j = y_{.j.}$ = " " " " " " " j^{th} column
$T_k = y_{..k}$ = " " " " " from k^{th} treatment

and we have

$$\sum_{i,j,k\in S}^{m} (y_{ijk} - \bar{y}_{...})^2 = \sum_{i,j,k\in S} \Big[(\bar{y}_{i..} - \bar{y}_{...}) + (\bar{y}_{.j.} - \bar{y}_{...}) + (\bar{y}_{..k} - \bar{y}_{...}) + (y_{ijk} - \bar{y}_{i..} - \bar{y}_{.j.} - \bar{y}_{..k} + 2\bar{y}_{...})\Big]^2$$

$$= m\sum_i (\bar{y}_{i..} - \bar{y}_{...})^2 + m\sum_j (\bar{y}_{.j.} - \bar{y}_{...})^2 + m\sum_k (\bar{y}_{..k} - \bar{y}_{...})^2 + \sum_{i,j,k\in S} (y_{ijk} - \bar{y}_{i..} - \bar{y}_{.j.} - \bar{y}_{..k} + 2\bar{y}_{...})^2$$

The product term vanishes since the algebric sum of deviations from mean is zero

$\therefore$ T.S.S. = S.S.R. + S.S.C. + S.S.T. + S.S.E.

Where T.S.S. is the total sum of squares and S.S.R.

S.S.C., S.S.T. and S.S.E. represent sum of squares due to rows, columns, treatments and error respectively, given by

$$T.S.S. = \sum_{ijk \in S}^{m} (y_{ijk} - \bar{y}_{...})^2$$

$$S.S.R. = S_R^2 = m \sum_i (\bar{y}_{i..} - \bar{y}_{...})^2$$

$$S.S.C. = S_c^2 = m \sum_j (\bar{y}_{.j.} - \bar{y}_{...})^2$$

$$S.S.T. = S_T^2 = m \sum_k (\bar{y}_{..k} - \bar{y}_{...})^2$$

$$S.S.E. = T.S.S - S.S.R - S.S.C. - S.S.T.$$

ANOVA TABLE FOR m×m L.S.D.

Sources of variation	d.f.	S.S.	M.S.S.	Variance Ratio
Rows	$m-1$	S_R^2	$s_R^2 = S_R^2/m-1$	$F_R = s_R^2/s_e^2$
columns	$m-1$	S_c^2	$s_c^2 = S_c^2/m-1$	$F_c = s_c^2/s_e^2$
Treatments	$m-1$	S_T^2	$s_t^2 = S_T^2/m-1$	$F_T = s_t^2/s_e^2$
Error or Residual	$(m-1)(m-2)$	S_E^2	$s_e^2 = S_E^2/m-1$	
Total	m^2-1			

Let us set up the null hypothesis

for row effect $H_\alpha : \alpha_1 = \alpha_2 = \cdots = \alpha_m = 0$

for column effect $H_\beta : \beta_1 = \beta_2 = \cdots = \beta_m = 0$

for treatment effect $H_\tau : \tau_1 = \tau_2 = \cdots = \tau_m = 0$

The variance ratio F_R, F_c & F_T follows (central) F dist^n with $(m-1)$, $(m-1)(m-2)$ d.f. under the null hypothesis H_α, H_β & H_τ respectively.

If ~~cal~~ $F_R \geq F_\alpha (m-1)(m-2)$ d.f. at α% l.o.s. we reject H_0 otherwise accept. Same for H_β & H_τ.

Remark :- 1 For Practical point of view we write

$$T.S.S. = \sum_{ijk \in S}^{m} (y_{ijk} - \bar{y}_{...})^2$$

$$= \sum y_{ijk}^2 - m^2(\bar{y}_{...}^2) = \sum y_{ijk}^2 - m^2\left(\frac{y_{...}}{m^2}\right)^2$$

$$= \sum y_{ijk}^2 - \frac{G^2}{(m^2)^2} \times m^2 \quad ; \; m^2 = N$$

$$= R.S.S. - \frac{G^2}{N}$$

$$S.S.R. = S_R^2 = m \sum R_i^2 - C.F.$$

$$S.S.C. = S_C^2 = m \sum C_j^2 - C.F.$$

$$S.S.T. = S_T^2 = m \sum T_k^2 - C.F.$$

$$S.S.E = T.S.S. - S.SR. - S.S.C. - S.S.T.$$

~~2~~

2 Standard error of differnce betn any two treatment means is ~~$\left[s_e^2 \left(\frac{1}{m} + \frac{1}{m} \right) \right]^{1/2}$~~

$$S.E.(\cancel{t_1 - t_2}) = \sqrt{s_e^2\left(\frac{1}{m} + \frac{1}{m}\right)} = \sqrt{\frac{2 s_e^2}{m}}$$

and the critical differnce (C.D) for significance of difference betn any two treatment means at α% l.o.s. is:

$$C.D. = \cancel{t_\alpha}\; S.E. \times t_\alpha \text{ (for err d.f.)}$$

$$= \sqrt{\frac{2 s_e^2}{m}} \times t_\alpha \text{ for } (m-1)(m-2) \text{ d.f.}$$