(9)

Ex:-1: Prove that the following vectors of $V_3(R)$ are L.D. or L.I.

$\alpha_1 = (1,3,2)$, $\alpha_2 = (1,-7,-8)$, $\alpha_3 = (2,1,-1)$

Sol^n:- let scalars $\exists\, a_1, a_2, a_3 \in R$ s.t.

$$a_1\alpha_1 + a_2\alpha_2 + a_3\alpha_3 = 0$$

$$a_1(1,3,2) + a_2(1,-7,-8) + a_3(2,1,-1) = (0,0,0)$$

$$(a_1, 3a_1, 2a_1) + (a_2, -7a_2, -8a_2) + (2a_3, a_3, -a_3) = (0,0,0)$$

$$(a_1 + a_2 + 2a_3,\ 3a_1 - 7a_2 + a_3,\ 2a_1 - 8a_2 - a_3) = (0,0,0)$$

$$a_1 + a_2 + 2a_3 = 0 \quad \text{—(i)}$$

$$3a_1 - 7a_2 + a_3 = 0 \quad \text{—(ii)}$$

$$2a_1 - 8a_2 - a_3 = 0 \quad \text{—(iii)}$$

by (i) & (iii) $\Rightarrow 5a_1 - 15a_2 = 0 \Rightarrow a_1 = 3a_2$

by (ii) & (iii) $\Rightarrow 5a_1 - 15a_2 = 0$

if we let $a_2 = 1$, $\Rightarrow a_1 = 3$

put this in Eq^n (i) $\Rightarrow a_3 = -2$

$\Rightarrow 3\alpha_1 + 1\cdot\alpha_2 - 2\alpha_3 = 0$ where $3, 1, -2 \in R$

$\Rightarrow$ vectors $\alpha_1, \alpha_2, \alpha_3$ are L.D.

Ex:-2. Show that the following vectors of $V_3(R)$ are L.I. $\alpha_1 = (1,1,0)$, $\alpha_2 = (1,1,1)$, $\alpha_3 = (2,1,3)$

Sol^n:- let $\exists$ scalars $a_1, a_2, a_3 \in R$ s.t.

$$a_1\alpha_1 + a_2\alpha_2 + a_3\alpha_3 = 0$$

$$\Rightarrow a_1(1,1,0) + a_2(1,1,1) + a_3(2,1,3) = (0,0,0)$$

$$\Rightarrow (a_1 + a_2 + 2a_3,\ a_1 + a_2 + a_3,\ 0 + a_2 + 3a_3) = (0,0,0)$$

$$\Rightarrow \left.\begin{array}{l} a_1 + a_2 + 2a_3 = 0 \text{ --- (i)} \\ a_1 + a_2 + a_3 = 0 \text{ --- (ii)} \\ a_2 + 3a_3 = 0 \text{ --- (iii)} \end{array}\right] \Rightarrow \begin{array}{l} \text{Eq.}^n \text{ (i) } - \text{(ii)}, \\ a_3 = 0 \end{array}$$

put value of a_3 in eq.^{n} (iii) $\Rightarrow a_2 = 0$

Eq.^{n} (i) $\Rightarrow a_1 = 0$

Hence $a_1 = 0,\ a_2 = 0,\ a_3 = 0$

$\Rightarrow$ vectors $\alpha_1, \alpha_2, \alpha_3$ are L.I.

Ex:- Show that the following matrices in the vector space V(R) of all 2×2 matrices are L.I.

$$\alpha_1 = \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix},\quad \alpha_2 = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix},\quad \alpha_3 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Sol^n:→ let $\exists$ scalars $a_1, a_2, a_3 \in R$. s.t.

$$a_1\begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix} + a_2\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} + a_3\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} a_1 + a_2 + a_3 & a_1 + a_2 + 0 \\ 0 + a_2 + 0 & 0 + a_2 + a_3 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$\Rightarrow \left.\begin{array}{l} a_1 + a_2 + a_3 = 0 \\ a_2 = 0 \\ a_2 + a_3 = 0 \end{array}\right] \Rightarrow \begin{array}{l} a_1 = 0, a_2 = 0, a_3 = 0 \\ \Rightarrow \alpha_1, \alpha_2, \alpha_3 \text{ are L.I.} \end{array}$$

Ex: → Show that in polynomial vector space $V_3(R)$, vectors $1+x+2x^2$, $2-x+x^2$, $-4+5x+x^2$ are L.D.

Sol^n: → let ∃ scalars $a_1, a_2, a_3 \in R$ s.t.

$$a_1(1+x+2x^2) + a_2(2-x+x^2) + a_3(-4+5x+x^2) = (0+0.x+0.x^2)$$

$$\Rightarrow a_1 + a_1 x + 2a_1 x^2 + 2a_2 - a_2 x + a_2 x^2 - 4a_3 + 5a_3 x + a_3 x^2 = 0 + 0.x + 0.x^2$$

$$\Rightarrow (a_1 + 2a_2 - 4a_3) + (a_1 - a_2 + 5a_3)x + (2a_1 + a_2 + a_3)x^2 = 0 + 0.x + 0.x^2$$

$$\Rightarrow \left.\begin{array}{l} a_1 + 2a_2 - 4a_3 = 0 \quad \text{—(i)} \\ a_1 - a_2 + 5a_3 = 0 \quad \text{—(ii)} \\ 2a_1 + a_2 + a_3 = 0 \quad \text{—(iii)} \end{array}\right] \Rightarrow \begin{array}{l} \text{by (i) \& (ii)} \\ \Rightarrow 3a_1 + 6a_3 = 0 \\ \Rightarrow a_1 = -2a_3 \end{array}$$

from (i) & (iii) $\Rightarrow 3a_2 - 9a_3 = 0 \Rightarrow a_2 = 3a_3$

taking $a_3 = 1 \Rightarrow a_1 = -2,\ a_2 = 3$

$\Rightarrow -2.\alpha_1 + 3\alpha_2 + 1.\alpha_3 = 0$

$\Rightarrow \alpha_1, \alpha_2, \alpha_3$ are L.D.

Ex:- Show that the vectors (α_1, α_2) and (β_1, β_2) of the vector space $V_2(R)$ are L.D. iff. $\alpha_1\beta_2 - \alpha_2\beta_1 =$

Sol^n:- let the vectors (α_1, α_2) & (β_1, β_2) are L.D. Then ∃ scalars $a_1, a_2 \in R$ s.t.

$$a_1(\alpha_1, \alpha_2) + a_2(\beta_1, \beta_2) = (0, 0)$$

$\Rightarrow (a_1\alpha_1, a_1\alpha_2) + (a_2\beta_1, a_2\beta_2) = (0,0)$

$\Rightarrow a_1\alpha_1 + a_2\beta_1 = 0$ — (i)

$a_1\alpha_2 + a_2\beta_2 = 0$ — (ii)

since the vectors (α_1, α_2) & (β_1, β_2) are L.D. therefore from Eq^n (i) & (ii) we must get non-zero values of scalar a_1, a_2 for non-zero sol^n.

$$\begin{vmatrix} \alpha_1 & \beta_1 \\ \alpha_2 & \beta_2 \end{vmatrix} = 0$$

$\Rightarrow \alpha_1\beta_2 - \alpha_2\beta_1 = 0$ H.P.

Ex:→ Show that vectors $\alpha_1 = (1+i, 2i)$, $\alpha_2 = (1, 1+i)$ are LD in the vector space $V_2(C)$ but they are L.I. in the vector space $V_2(R)$.

Sol^n– let $\alpha_1, \alpha_2 \in V_2(C)$. vectors α_1, α_2 are L.D. if one vector is scalar multiple of others. Since $\exists$ $(1+i) \in C$ s.t.

$$\begin{aligned} (1+i)\alpha_2 &= (1+i)(1, 1+i) \\ &= (1+i, (1+i)^2) \\ &= (1+i, 1-1+2i) \\ &= (1+i, 2i) = \alpha_1 \end{aligned}$$

$\Rightarrow \alpha_1, \alpha_2$ are L.D.

Since $\exists$ no scalars $(\in R)$ s.t. either vector α_1, α_2 can be expressed as a L.C. of other.

Hence. α_1, α_2 are L.I. in $V_2(R)$