Basis of a vector space :→

Let $V(F)$ be a vector space. And any subset S of $V(F)$ is said to be basis of the vector space $V(F)$ if

(i) S is L.I.

(ii) S is generator of $V(F)$. i.e. Every vector of V is a L.C. of elements of S.

i.e. $V = L(S)$

or Bases S is the set of those L.I. vectors which span the entire vector space of V

Note:→ zero vector never belongs to a basis set. as in this case (i.e. zero vector belongs to basis) the set becomes L.D.

Types of Basis 'S'

Finite Basis

If the no. of vectors in basis set S is finite, then it is called finite Basis.

Infinite Basis

If the no. of vectors (elements) in the basis set S is infinite, then it is called infinite basis.

Dimension of a vector space:-

Let S be a basis of vector space V(F). then the number of elements (vectors) in the basis set is known as dimension of vector space.

$$\dim V(F) = n \text{ (no. of elements in S)}$$

finite dimensional vector space:→ If the basis set S of vector space V(F) is finite, then the vector space is said to be finite Dimensional vector space.

infinite dimensional vector space:- If the basis set S of vector space V(F) is infinite, then the vector space is said to be infinite Dimensional vector space.

Eg:→ $S = \{e_1, e_2, \cdots, e_n\}$ is basis of vector space $V_n(F)$ where

$e_1 = (1, 0, \cdots, 0), \cdots, e_n = (0, 0, \cdots, 1)$

let ∃ scalars $a_1, a_2, \cdots, a_n \in F$ s.t.

$a_1 e_1 + a_2 e_2 + \cdots + a_n e_n = 0$

$a_1(1, 0, \cdots, 0) + a_2(0, 1, \cdots 0) + \cdots + a_n(0, 0, \cdots, 1) = (0, 0, \cdots, 0)$

$\Rightarrow (a_1, a_2, \cdots, a_n) = (0, 0, \cdots, 0)$

$\Rightarrow a_1 = 0, a_2 = 0, \cdots, a_n = 0 \Rightarrow$ S is L.I. set — (1)

let $\alpha = (\alpha_1, \alpha_2, \dots, \alpha_n) \in V_n(F)$

then

$\alpha = \alpha_1 \underbrace{(1, 0, \dots, 0)} + \alpha_2 \underbrace{(0, 1, 0, \dots, 0)} + \dots + \alpha_n \underbrace{(0, 0, \dots, 1)}$

$\alpha = \alpha_1 e_1 + \alpha_2 e_2 + \dots + \alpha_n e_n$

$\Rightarrow$ Every vector of $V_n(F)$ can be expressed as a L.C. of elements of S.

$\Rightarrow$ $\boxed{V_n(F) = L(S)}$ — (2)

by (1) & (2) $\Rightarrow$ S is basis of $V_n(F)$

since the number of elements in S is n (finite). Hence $V_n(F)$ is a finite dimensional vector space.

S is <u>standard</u> <u>Basis</u> of $V_n(F)$

so standard Basis of $V_2(F) \rightarrow \{(1, 0), (0, 1)\}$

" " " $V_3(F) \rightarrow \{(1, 0, 0), (0, 1, 0), (0, 0, 1)\}$

" "

Eg:→ vector space F(F) has basis $S = \{1\}$ where 1 is the unit element of F.

since S contains non-zero vector 1.

$\Rightarrow$ S is L.I.

$\forall a \in F \Rightarrow a = a.1$ where $a \in F$

$\Rightarrow$ every vector of F can be expressed as a L.C. of elements of S

$\Rightarrow$ $S = \{1\}$ is basis of F(F).