Change in the component a_{ij} of matrix A

Let us assume that the component a_{ij} of matrix A is changes to $a_{ij} \rightarrow a_{ij} + \Delta a_{ij}$

Such variation can be make in two possible way.

I) Only when a_{ij} is change to $a_{ij} + \Delta a_{ij}$ which doesn't belongs to basis matrix B

2) Only that a_{ij} is change to $a_{ij} + \Delta a_{ij}$ is belongs to basis matrix B

case (i) $a_{ij} \notin B$

Then the range for a_{ij} is given by

$$\underset{(c_B\beta_i > 0)}{\underset{j}{\text{Max}}} \left[\frac{-\Delta_j}{c_B\beta_i} \right] \le \Delta a_{ij} \le \underset{(c_B\beta_i < 0)}{\underset{j}{\text{Min}}} \left[\frac{-\Delta_j}{c_B\beta_i} \right] \quad -(1)$$

also Δa_{ij} is unrestricted if $c_B\beta_i = 0$

In this case neither the optimum solution $X_B = B^{-1}b$ ~~and~~ nor the value of objective function changes

	c_j		3	5	4	0	0	0
	c_B	X_B	x_1	x_2	x_3	$x_4(\beta_1)$	x_5	x_6
x_2	5	50/41	0	1	0	15/41	8/41	-10/41
x_3	4	62/41	0	0	1	-6/41	5/41	4/41
x_1	3	89/41	1	0	0	-2/41	-12/41	15/41
	$z_j - c_j$		0	0	0	45/41	24/41	11/41

$B = (X_2, X_3, X_1)$

$$B^{-1} = \begin{array}{ccc} \beta_1 & \beta_2 & \beta_3 \end{array}$$
$$B^{-1} = \begin{bmatrix} 15/41 & 8/41 & -10/41 \\ -6/41 & 5/41 & 4/41 \\ -2/41 & -12/41 & 15/41 \end{bmatrix}$$

$B^{-1} = (X_4, X_5, X_6)$
$= (\beta_1, \beta_2, \beta_3)$

$c_B = (5, 4, 3)$

$c_B\beta_1 = \frac{45}{41}$ $\quad c_B\beta_2 = \frac{24}{41}$ $\quad c_B\beta_2 = \frac{11}{41}$

$$\begin{bmatrix} 5 \\ 4 \\ 3 \end{bmatrix} \begin{bmatrix} 15/41 \\ -6/41 \\ -2/41 \end{bmatrix} = \frac{75}{41} - \frac{24}{41} - \frac{6}{41} = \frac{45}{41}$$

Now, using the result ①, ~~for a_4~~ for $j=4$, we have

$$\underset{\substack{j \\ c_B\beta_i>0}}{\text{Max}}\left[\frac{-\Delta_4}{c_B\beta_i}\right] \leq \Delta a_{i4} \leq \underset{\substack{j \\ c_B\beta_i<0}}{\text{Min}}\left[\frac{-\Delta_4}{c_B\beta_i}\right]$$

for $i=1$

$$\underset{\substack{j \\ c_B\beta_1>0}}{\text{Max}}\left[\frac{-\Delta_4}{c_B\beta_1}\right] \leq \Delta a_{14} \leq \underset{\substack{j \\ c_B\beta_1<0}}{\text{Min}}\left[\frac{-\Delta_4}{c_B\beta_1}\right]$$

$$\text{Max}\left[\frac{-45/41}{45/41}\right] \leq \Delta a_{14}$$

$$-1 \leq \Delta a_{14} < \infty$$

for $i=2$

$$\underset{\substack{j \\ c_B\beta_2>0}}{\text{Max}}\left[\frac{-\Delta_4}{c_B\beta_2}\right] \leq \Delta a_{24} \leq \underset{\substack{j \\ c_B\beta_2<0}}{\text{Min}}\left[\frac{-\Delta_4}{c_B\beta_2}\right]$$

$$\underset{j}{\text{Max}}\left[\frac{-45/41}{24/41}\right] \leq \Delta a_{24} \leq$$

$$-\frac{45}{24} \leq \Delta a_{24} < \infty$$

for $i=3$

$$\underset{\substack{j \\ c_B\beta_3>0}}{\text{Max}}\left[\frac{-\Delta_4}{c_B\beta_3}\right] \leq \Delta a_{34} \leq \underset{\substack{j \\ c_B\beta_3<0}}{\text{Min}}\left[\frac{-\Delta_4}{c_B\beta_3}\right]$$

$$\text{Max}\left[\frac{-45/41}{11/41}\right] \leq \Delta a_{34}$$

$$-\frac{45}{11} \leq \Delta a_{34} < \infty$$