Hypothesis Testing

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What is a Hypothesis?

- A hypothesis is an assumption about the population parameter.
  - A parameter is a characteristic of the population, like its mean or variance.
  - The parameter must be identified before analysis.

I assume the mean height of this batch is 5.5”

Characteristics of hypothesis

- Clear and precise
- Capable of being tested
- Should state relationship between variables
- Specific
- Simple and understandable
- Consistent with most known facts

The Null Hypothesis, H_0

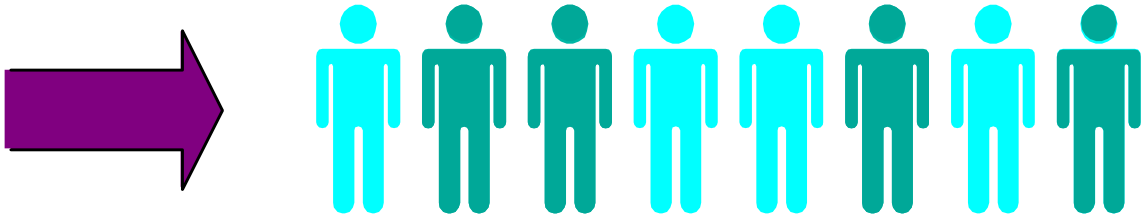
- States the Assumption (numerical) to be tested
  - Refers to the Status Quo
  - Always contains the ' = ' sign
- The Null Hypothesis may or may not be rejected.
- The hypothesis that are proposed with the intent of receiving a rejection denoted as H_0.
- Rejection of (H_0) when it is true, involves great risk is alpha (α) i.e. level of significance,

The Alternative Hypothesis, H_a or H_1

- Is the opposite of the null hypothesis
- Challenges the Status Quo
- Never contains the '=' sign
- The Alternative Hypothesis may or may not be accepted
- Is generally the hypothesis that is believed to be true by the researcher

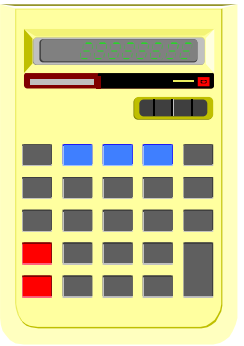
Hypothesis Testing Process

Assume the population mean age is 50. (Null Hypothesis)



Population

Sample



The Sample Mean Is 20

Is $\overline{X} = 20 \cong \mu = 50$?

No, not likely!

Null Hypothesis

Hypothesis Testing: Result Possibilities

	We Reject H_0. (accept H_a)	We Fail to Reject H_0 (not enough evidence to accept H_a)
H_0 is true.	Type I Error	Correct Decision
H_0 is false. (H_a is true)	Correct Decision	Type II Error

Hypothesis Testing

- It is a statistical method that uses sample data to evaluate the validity of a hypothesis about a population parameter.

Level of Significance, α

- Defines Unlikely Values of Sample Statistic if Null Hypothesis Is True
  - Called Rejection Region of Sampling Distribution
- Designated α (alpha)
  - Typical values are 0.01, 0.05, 0.10
- Selected by the Researcher at the Start
- Provides the Critical Value(s) of the Test

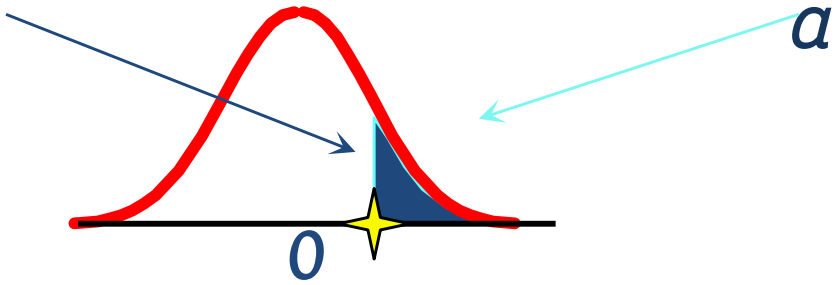
Level of Significance, α and the Rejection Region

H_0: $\mu \geq 3$

H_1: $\mu < 3$

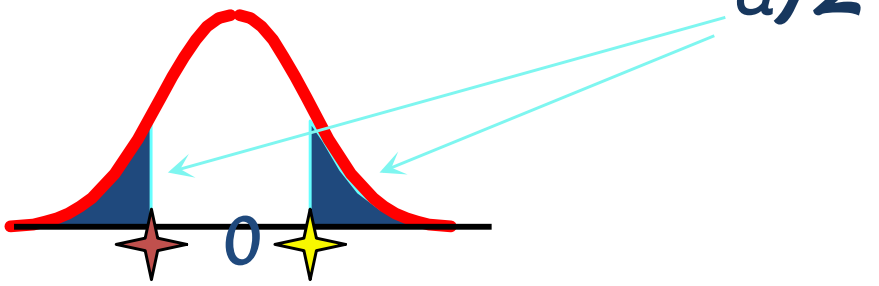
H_0: $\mu \leq 3$

H_1: $\mu > 3$



H_0: $\mu = 3$

H_1: $\mu \neq 3$



Errors in Making Decisions

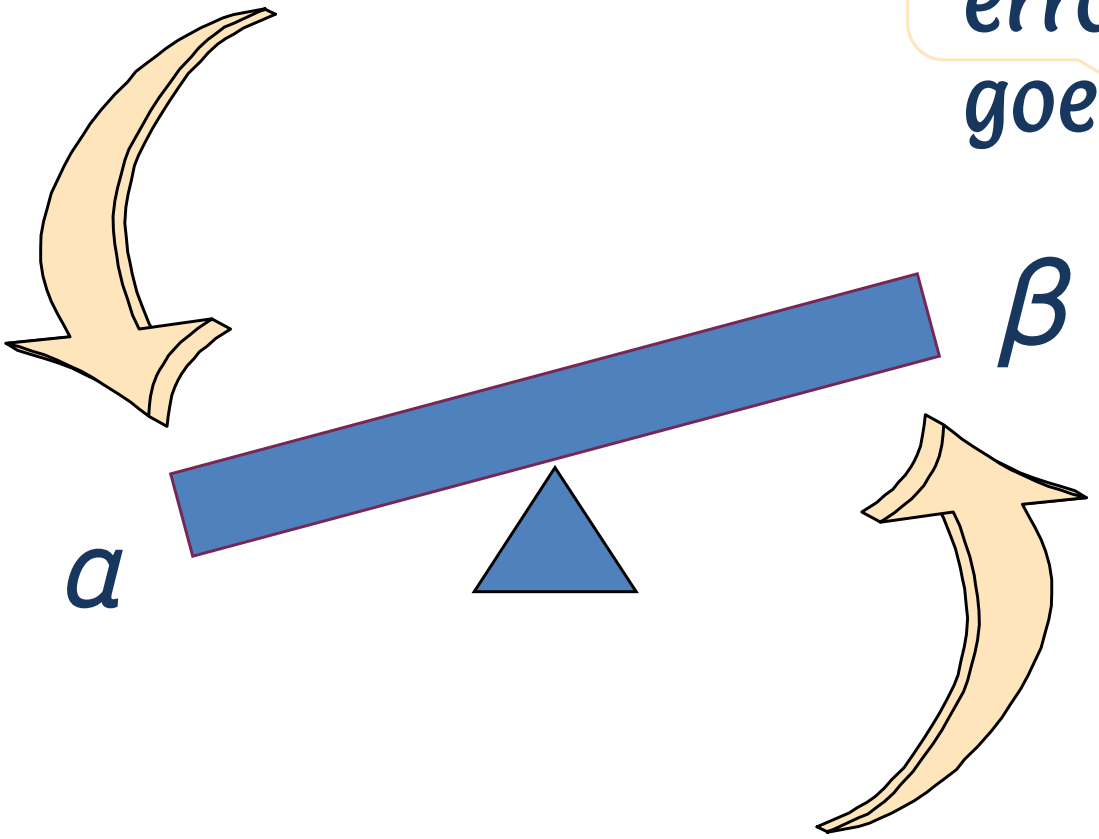
- Type I Error
  - Reject True Null Hypothesis ("False Positive")
  - Has Serious Consequences
  - Probability of Type I Error Is α
    - Called Level of Significance
    - Set by researcher
- Type II Error
  - Do Not Reject False Null Hypothesis ("False Negative")
  - Probability of Type II Error Is β (Beta)

How to choose between Type I and Type II errors

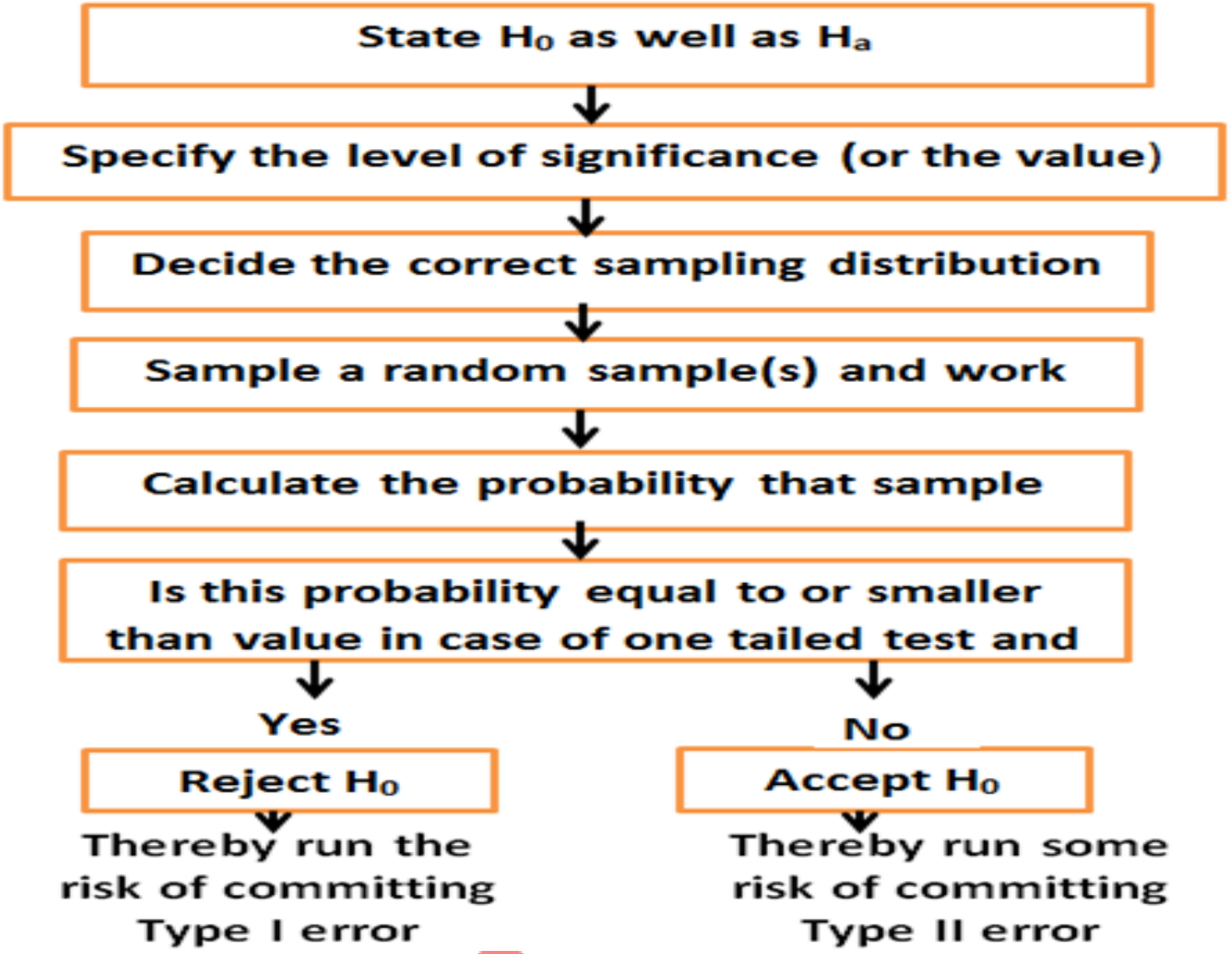
- Choice depends on the cost of the error
- Choose little type I error when the cost of rejecting the maintained hypothesis is high
  - A criminal trial: convicting an innocent person
- Choose large type I error when you have an interest in changing the status quo
  - A decision in a startup company about a new piece of software

α & β Have an Inverse Relationship

Reducing probability of one error and the other one goes up.

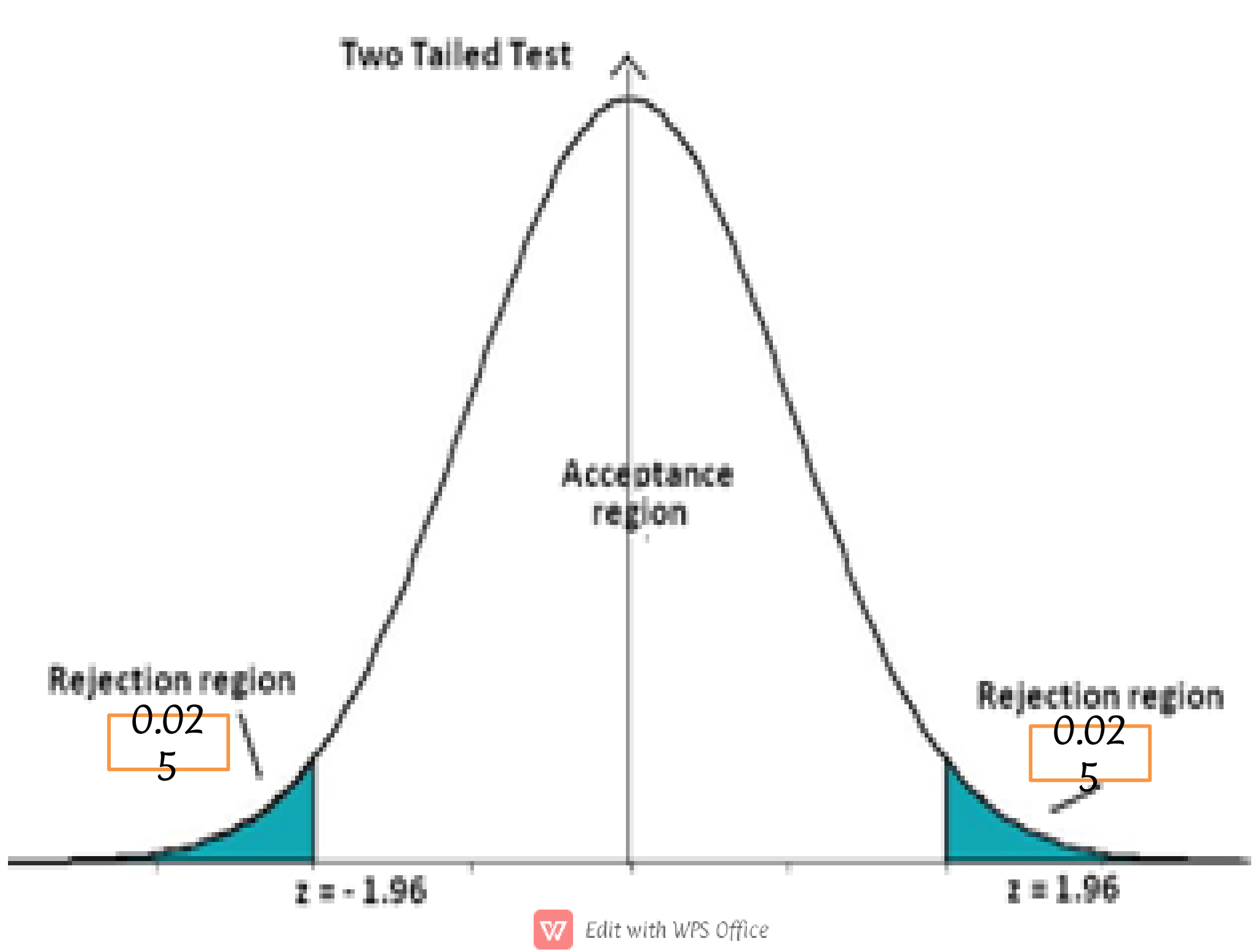


Flow Diagram for Hypotheses Testing



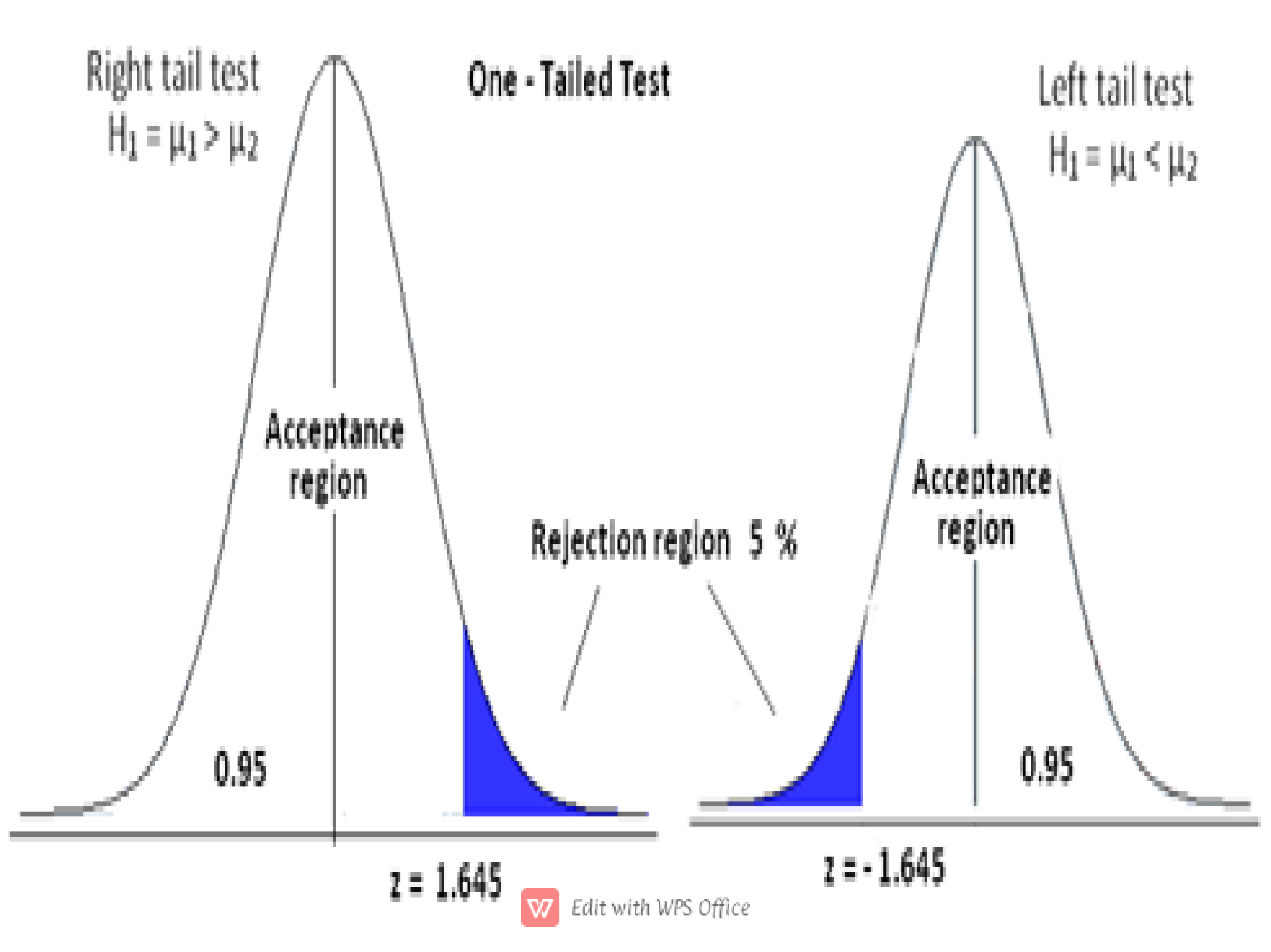
Two tailed Test

- **A two tailed test** rejects the null hypotheses, if say the sample mean is significantly higher or lower than the hypothesized value of the mean of the population.
- Symbolically the two tailed is appropriate when we have $H_0 : \mu = \mu_{H0}$ & $H_a : \mu \# \mu_{Ha}$ which may mean $\mu < \mu_{H0}$ or $\mu > \mu_{H0}$
- Thus in a two tailed test there are two rejection regions, one on each tail.



Two tailed & One tailed tests

- A **one tailed test** would be used when we are to test, say whether the population mean is either lower than or higher than some hypothesized value.
- Symbolically the one tailed is appropriate when we have $H_0 : \mu > \mu_{HO}$ or $\mu < \mu_{HO}$
- & $H_a : \mu < \mu_{HO}$ or $\mu > \mu_{HO}$



Test of Hypotheses

Difference between Parametric & Non-parametric Test

	Parametric test	Non-parametric Test
Meaning	A statistical test, in which specific assumptions are made about the population parameter	A statistical test used in the case of non-metric independent variables.
Basis of test statistic	Distribution	Arbitrary
Measurement level	Interval or Ratio	Nominal or Ordinal
Information about population	Completely known	Unavailable
Various types	z- test	Chi-square (χ^2) test