<u>Inference</u> - It denotes a set of premises followed by a Conclusion. Each inference can be written as an implication as follows:

$$\underbrace{[p_1 \wedge p_2 \wedge p_3 \cdots \wedge p_i]}_{\text{Conjunction (AND) of premises}} \rightarrow q \text{ — Conclusion}$$

This inference is valid if the implication is a Tautology, else invalid.

Also, written as

$$\begin{array}{l} p_1 \\ p_2 \\ \vdots \\ p_i \\ \hline \therefore q \end{array}$$

In a valid inference - whenever all premises (hypotheses) are True, the conclusion is also True.

<u>Note</u>: The conclusion of a valid inference is not necessarily True (~~See following ex.~~)

<u>Valid Rules of Inference</u> - <u>Rule 1 (Rule of Detachment)</u>

<u>Rule 1</u>: If ~~p~~ statement p is assumed as True and also the statement $p \rightarrow q$ as True then we must accept q as True.

$$\begin{array}{l} p \\ p \rightarrow q \\ \hline \therefore q \end{array}$$

ex. Write the conclusion of the following argument by Rule of Detachment.

Given: It is 10 AM in Delhi
If it is 10 AM in Delhi then it is 10 PM in New York.

Sol. Let the propositional variables be as follows:

p: It is 10 AM in Delhi

q: It is 10 PM in New York.

Then the given argument in symbolic notation becomes

$$\begin{array}{l} p \\ p \to q \end{array}$$

& the complete argument with conclusion as per Rule of Detachment becomes

$$\begin{array}{l} p \\ p \to q \\ \hline \therefore q \end{array}$$ (It is 10 PM in New York).

Note: Conclusion q is as per Rule of Detachment.
These rules are like formulas. If the arguments fit the rules then the argument is valid else invalid.

~~An ex. of~~

ex. Test the given argument for its validity & ~~Truth~~ truth of its conclusion.

Smoking is healthy

If smoking is healthy then cigarettes are prescribed by the doctor.

$\therefore$ cigarettes are prescribed by the doctor.

Sol. Let p: Smoking is healthy

q: Cigarettes are prescribed by doctor.

2 ways to look at this argument:

① The argument fits in the form of Rule of Detachment $\therefore$ argument is valid. But the conclusion is False $\because$ cigarettes are NOT prescribed by doctor.

② p is False, As per Truth Table of $p \to q$ from 3rd & 4th row, if p is False then $p \to q$ is True & $\therefore$

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

$$p \wedge (p \to q) \Rightarrow F \wedge T = F$$

$\Rightarrow$ Conclusion is F

$\wedge$ AND

(definition of inference $\Rightarrow$ conjunction (AND) of premises)

Inference (i)

Rule 2 (Transitive Rule) -

$p \to q$

$q \to r$

$\therefore p \to r$

Any no. of premises can exist

For ex.

$p \to q$

$q \to r$

$r \to s$

$\therefore p \to s$

p: premise is the 1st

s: .. is the last one.

ex. Test the validity of the argument & discuss the conclusion.

If you invest in stock market then you will get rich.

If you get rich then you will be happy

If you invest in stock market then you will be happy.

Sol. Let p: you ~~get rich~~ invest in stock market, ~~q: you will be happy~~

q: You will get rich

r: you will be happy.

Argument is

$p \to q$

$q \to r$

$\therefore p \to r$

This is of form of Transitive Rule so valid.

But conclusion may be False $\because$ happiness may be due to any factor, not necessarily stock market.

Rule 3 (De Morgan's Laws)

$$\sim(p \vee q) \equiv (\sim p) \wedge (\sim q)$$

$$\sim(p \wedge q) \equiv (\sim p) \vee (\sim q)$$

Rule 4 (Law of Contrapositive)

$$(p \to q) \equiv (\sim q \to \sim p)$$

Ex. Establish validity of the following argument:

Nita is baking a cake.

If Nita is baking a cake then she is not practising her flute.

If Nita is not practising her flute then her father will not buy her a car.

Therefore, Nita's father will not buy her a car.

Sol. Let p: Nita is baking, q: Nita is practising her flute

r: Her father will buy her a car. Argument is:

$$\begin{array}{l} p \\ p \to \sim q \\ \sim q \to \sim r \\ \hline \therefore \sim r \end{array}$$

Steps	Reasons
1. $p \to \sim q$	premise
2. $\sim q \to \sim r$	premise
3. $p \to \sim r$	follows from steps 1. & 2. & Transitive Law
4. p	premise
5. $\therefore \sim r$	follows from steps 3. & 4. & Rule of Detachment

We could derive the conclusion using the ~~R~~ Valid Rules of Inference so the Argument is Valid.

<u>Note</u>: All the premises must be considered & they can be in any order. If we can derive the conclusion then the argument is Valid. The same problem can be done differently as follows:

Steps	Reasons
1. p	premise
2. $p \to \sim q$	premise
3. $\sim q$	Steps 1. & 2. & Rule of Detachment
4. $\sim q \to \sim r$	premise
5. $\therefore \sim r$	Steps 3. & 4. & Rule of Detachment.

$\therefore$ Argument is Valid.