More Valid Rules of Inference for Complex problems

Rules of Inference	Related Logical Implication	Name of Rule
1. $\begin{array}{l} p \\ p \to q \\ \hline \therefore q \end{array}$	$[p \wedge (p \to q)] \to q$	Rule of Detachment
2. $\begin{array}{l} p \to q \\ q \to r \\ \hline \therefore p \to r \end{array}$	$[(p \to q) \wedge (q \to r)] \Rightarrow (p \to r)$	Transitive Rule
3. $\begin{array}{l} p \to q \\ \sim q \\ \hline \therefore \sim p \end{array}$	$[(p \to q) \wedge \sim q] \to \sim p$	Modus Tollens
4. $\begin{array}{l} p \\ q \\ \hline \therefore p \wedge q \end{array}$	—	Rule of Conjunction
5. $\begin{array}{l} p \vee q \\ \sim p \\ \hline \therefore q \end{array}$	$[(p \vee q) \wedge \sim p] \to q$	Rule of Disjunctive Syllogism
6. $\begin{array}{l} \sim p \to F_0 \\ \hline \therefore p \end{array}$	$(\sim p \to F_0) \Rightarrow p$	Rule of Contradiction
7. $\begin{array}{l} p \wedge q \\ \hline \therefore p \end{array}$	$(p \wedge q) \to p$	Rule of Conjunctive Simplification
8. $\begin{array}{l} p \\ \hline \therefore p \vee q \end{array}$	$p \to (p \vee q)$	Rule of Disjunctive Amplification

Ex. Prove that the following argument is valid.

If the band could not play rock music or the snacks were not delivered on time, then the New Year's party would have been cancelled & Reina would have been angry. If the party were cancelled then refunds would have had to be made. No refunds were made. Therefore, the band could play rock music.

Sol. Let p : the band could play rock music

q : snacks were delivered on time

r : New Year's party was cancelled

s : Reina was angry

t : Refunds had to be made.

$\therefore$ Argument is

$$(\sim p \vee \sim q) \rightarrow (r \wedge s)$$

$$r \rightarrow t$$

$$\sim t$$

$$\therefore p$$

Inference (9)

Steps	Reasons
1. $r \to t$	premise
2. $\sim t$	premise
3. $\sim r$	Steps 1, 2 & Modus Tollens
4. $\sim r \vee \sim s$	Step 3 & Rule of Disjunctive Amplification
5. $\sim (r \wedge s)$	Step 4 & De Morgan's Laws
6. $(\sim p \vee \sim q) \to (r \wedge s)$	Premise
7. $\sim (\sim p \vee \sim q)$	Step 6 & 5 & Modus Tollens
8. $p \wedge q$	Step 7, De Morgan's Laws & Law of Double Negation $(\sim\sim p \equiv p)$
9. $\therefore p$	Step 8 & Rule of Conjunctive Simplification.

Conclusion could be derived. ∴ Argument is Valid. HP.

Predicate - This an open proposition

A predicate is a declarative sentence which contains ≥ 1 variable and is not a proposition.

ex. x is a rational no., universe OR Universe of discourse is of all Real nos.

Predicate of this ex. will be:

$R(x)$: x is a rational no., universe is of all Real nos

~~x~~ is a member from this universe.

if $x = 3/2$ then $R(3/2)$ is True $\because$ $3/2$ is a Rational no from the Universe of Real nos

$R(\sqrt{2})$ is False $\because$ $\sqrt{2}$ is irrational.

ex. A set of Teachers can be denoted as

$T(x)$: x is a Teacher.

if Nikhil is a Teacher then T(Nikhil) is True.

ex. Sum of 2 integers is 4.

$S(x, y)$: $x+y=4$, $S(2,2)$ is T & $S(3,2)$ is F.

Note: Value of predicates depends on Universe & Often universe is not mentioned when obvious

Predicates can be combined with logical connectives like propositions.

ex. x is a rational no or z is >3.

Let $R(x)$: x is a rational no.

$Z(x)$: x is >3

~~$R(x) \vee G$~~ $R(x) \vee Z(x)$

Quantifiers – These answer the questions 'How many?' (Quantity without using no.)

Universal Quantifier – 'All'

ex. All triangles have 3 sides

This quantifier is denoted by $\forall x$ & is known as also 'For every x', 'For all x', 'For each x'.

Existential Quantifier '$\exists x$'. This denotes 'For some x', 'There is ~~so~~ an x' etc.

$\forall x, F(x) \Rightarrow$ For each x in the universe, $F(x)$ is True.

$\exists x, F(x) \Rightarrow$ There is at least one x in the universe s.t. $F(x)$ is True.

Home Work (HW) Exercise

Establish Validity of the given argument

$p \to r$

$\sim p \to q$

$q \to s$

$\therefore \sim r \to s$

Note: Note: ~~Following~~ Properties – given on Page (4) of Logic also may be used in this kind of problems.

~~Properties (Equivalence of Propositions)~~ –

Fallacies (Invalid Arguments) –

(1.) Fallacy of affirming the consequent

~~2. Falla~~

$p \to q$

q

$\therefore p$

Ex. If the price of gold is rising, then inflation is surely coming. Inflation is surely coming. Therefore, price of gold is rising.

Sol. p: Price of gold is rising
q: Inflation is surely coming

Argument takes the form of (1)

$p \to q$

q

$\therefore q$ $\therefore$ Fallacy

(2) Fallacy of assuming the opposite

$p \to q$

$\sim p$

$\therefore \sim q$

(3) non sequitur fallacy $\frac{p}{\therefore q}$ ("it does not follow")

ex. p: A triangle (Δ) has 3 sides
q: Therefore, a Δ is a square

It is of form (3) fallacy $\frac{p}{\therefore q}$