Unit - III

Electromagnetic Waves in Conductor

Consider a homogeneous/ linear and isotropic Conducting media having conductivity σ.

let permitivity - ϵ

permeability - μ

and charge density ρ

Then maxwell's eqn are

$$
\left.\begin{aligned}
\nabla\cdot\vec{D} &= \rho \\
\nabla\cdot\vec{B} &= 0 \\
\nabla\times\vec{H} &= \vec{J}+\frac{\partial \vec{D}}{\partial t} \\
\nabla\times\vec{E} &= -\frac{\partial \vec{B}}{\partial t}
\end{aligned}\right\} \quad \text{---(1)}
\qquad \text{and} \quad
\begin{aligned}
\vec{J} &= \sigma\vec{E} \\
\vec{B} &= \mu\vec{H} \\
\vec{D} &= \epsilon\vec{E}
\end{aligned}
$$

taking curl of eqn IV of (1)

$$\nabla\times(\nabla\times\vec{E}) + \left(\nabla\times\frac{\partial\vec{B}}{\partial t}\right) = 0$$

Using vector identity, we get

$$\nabla(\nabla\cdot\vec{E}) - \nabla^2\vec{E} + \frac{\partial}{\partial t}(\nabla\times\vec{B}) = 0$$

$$\nabla(\rho/\epsilon) - \nabla^2\vec{E} + \frac{\partial}{\partial t}(\nabla\times\mu\vec{H}) = 0$$

$$\nabla(\rho/\epsilon) - \nabla^2\vec{E} + \frac{\partial}{\partial t}\mu\left(\sigma\vec{E}+\epsilon\frac{\partial\vec{E}}{\partial t}\right) = 0$$

$$\nabla(\rho/\epsilon) - \nabla^2\vec{E} + \mu\sigma\frac{\partial\vec{E}}{\partial t} + \epsilon\mu\frac{\partial^2\vec{E}}{\partial t^2} = 0$$

$$\nabla^2 \vec{E} = \mu\epsilon \frac{\partial^2 \vec{E}}{\partial t^2} + \sigma\mu \frac{\partial E}{\partial t} + \nabla(\rho/\epsilon) \quad — (2)$$

Similarly taking curl of eq^n III of (1), we get-

$$\nabla^2 \vec{H} = \mu\epsilon \frac{\partial^2 \vec{H}}{\partial t} + \sigma\mu \frac{\partial H}{\partial t} \quad — (3)$$

(i) Modified wave eq^n

Let us consider the plane polarized e.m. wave propagating in z direction with $\vec{E}$ in x-direction, so that

$$\left.\begin{aligned} \frac{\partial}{\partial x} &= \frac{\partial}{\partial y} = 0 \\ \text{and} \quad \vec{E} &= \hat{i} E_x \\ \vec{H} &= \hat{j} H_y \end{aligned}\right\} \quad — (4)$$

Then eq^n (2) & (3) becomes

$$\left.\begin{aligned} \frac{\partial^2 E_x}{\partial z^2} &= \mu\epsilon \frac{\partial^2 E_x}{\partial t^2} + \sigma\mu \frac{\partial E_x}{\partial t} \\ \text{and} \quad \frac{\partial^2 H_y}{\partial z^2} &= \mu\epsilon \frac{\partial^2 H_y}{\partial t^2} + \sigma\mu \frac{\partial H_y}{\partial t} \end{aligned}\right| \quad — (5)$$

{ ρ is constant throughout the medium]

Assuming harmonic variation of E_x with respect to t then

$$E_x = E_0 e^{+j\omega t}$$

$$\frac{\partial E_x}{\partial t} = j\omega E_x$$

$$\frac{\partial^2 E_x}{\partial t^2} = -\omega^2 E_x \quad — (6)$$

using (6), eq^n (5) becomes

$$\frac{\partial^2 E_x}{\partial z^2} + \omega^2 \mu\epsilon E_x - j\omega\sigma\mu E_x = 0$$

$$\frac{\partial^2 E_x}{\partial z^2} - (j\omega\sigma\mu - \omega^2\mu\epsilon) E_x = 0 \quad \text{—(7)}$$

Let us put, γ propagation constant, as

$$\gamma^2 = j\omega\mu\sigma - \omega^2\mu\epsilon$$

then eqn (7) becomes

$$\frac{\partial^2 E_x}{\partial z^2} - \gamma^2 E_x = 0 \quad \text{—(8)}$$

Similarly eqn II of (5), we get

$$\frac{\partial^2 H_y}{\partial z^2} - \gamma^2 H_y = 0 \quad \text{—(9)}$$

eqn (8) and (9) are the wave eqn's for $\vec{E}$ and $\vec{H}$

(ii) phase velocity and Refractive index

The solution of eqn (8) for a wave travelling in z-direction

$$E_x = E_0 e^{-\gamma z} \quad \text{—(10)}$$

where $\gamma = \sqrt{j\omega\mu\sigma - \omega^2\mu\epsilon}$ —(10)

$$\gamma = \sqrt{j\omega\mu\sigma\left(1 + j\frac{\omega\epsilon}{\sigma}\right)}$$

for good conductor

$$\frac{\sigma}{\omega\mu} >> 1$$

{ Conductivity is very high

Then $\gamma = \sqrt{j\omega\mu\sigma}$

$$\gamma = (1+j)\sqrt{\frac{\omega\mu\sigma}{2}} \quad \text{— (11)}$$

γ has a real and imaginary parts so we write

$$\gamma^2 = (1+j)^2 \left(\frac{\omega\mu\sigma}{2}\right)$$
$$= (1+j^2+2j)\left(\frac{\omega\mu\sigma}{2}\right)$$
$$= (1-1+2j)\left(\frac{\omega\mu\sigma}{2}\right)$$
$$\gamma^2 = 2j\left(\frac{\omega\mu\sigma}{2}\right) = j\omega\mu\sigma$$

$$\gamma = \alpha + j\beta \quad \text{— (12)}$$

α → real part of γ ⇒ called attenuation constant

β — imaginary part of γ ⇒ called phase constant

from eq (11)

$$\alpha = \beta = \sqrt{\frac{\omega\mu\sigma}{2}} \quad \text{— (13)}$$

so the real and imaginary part of wave no. γ are equal

Using eqn (11) in eqn (10), we get

$$E_x = E_0 \exp\left\{-(1+j)\sqrt{\frac{\omega\mu\sigma}{2}} \cdot z\right\}$$
$$= E_0 \exp\left\{-\sqrt{\left(\frac{\omega\mu\sigma}{2}\right)} \cdot z\right\} \exp\left\{-j\sqrt{\left(\frac{\omega\mu\sigma}{2}\right)} \cdot z\right\} \quad \text{— (14)}$$

in eqn (14), the attenuation factor is

$$\exp\left\{-\sqrt{\frac{\omega\mu\sigma}{2}} \cdot z\right\}$$

and phase factor is

$$\exp\left\{-j\sqrt{\frac{\omega\mu\sigma}{2}} \cdot z\right\}$$

Eqn (14) indicates that field attenuates exponentially and retards linearly in phase with increasing z, since conductivity for good conductor is very large (Cu, $\sigma \approx 10^7$ S/m) due to which α and β are very large. (3)

Phase velocity of wave in good conductor

$$v = \frac{\omega}{\beta} = \sqrt{\frac{2\omega}{\mu\sigma}} \quad \text{—— (15)}$$

and Refractive index of conducting medium is

$$\eta = \frac{c}{v} = c\sqrt{\frac{\mu\sigma}{2\omega}} \quad \text{—— (16)}$$

(iii) Depth of penetration (skin depth)

When wave propagating in conductor, suffers attenuation. At radio freq. in conductor the factor $\omega\sigma$ will be very large, then α will be quite high and wave may penetrate only a very short distance in conductor. An existing wave in a conducting medium is rapidly attenuated. The penetration of wave into the conducting medium is called depth of penetration or skin depth.

The depth of penetration (δ) ⇒ depth in which the wave attenuates to $1/e$ times of its strength that is before penetration.

~~formly~~

from eq^n (10)

$$E_x = E_0 e^{-\alpha z} \quad \text{--- (17)}$$

if $\alpha z = 1$

$$\frac{E_x}{E_0} = \frac{1}{e} \quad \text{--- (18)}$$

i.e. on travelling distance

$$z = \frac{1}{\alpha} \quad \text{--- (19)}$$

the amplitude of wave falls to $1/e$ times its value at $z=0$

By definition, such distance is equal to the depth of penetration δ,

So that for good conductor

$$\delta = 1/\alpha$$

$$\delta = \sqrt{\frac{2}{\omega\mu\sigma}}$$

(iv) Impedance of the Conductivity medium

The solution of wave eqn for $\vec{E}$ of a linearly polarized plane wave propagating in z-direction in conductor with $\vec{E}$ in x-direction

$$E_x^* = E_0 e^{j(\omega t - \gamma z)} \quad \text{--- (20)}$$

$$\left\{ \begin{array}{l} \frac{\partial^2 E_x}{\partial z^2} = \mu\epsilon \frac{\partial^2 E_x}{\partial t^2} + \sigma\mu \frac{\partial E_x}{\partial t} \\ \frac{\partial^2 H_y}{\partial z^2} = \mu\epsilon \frac{\partial^2 H_y}{\partial t^2} + \sigma\mu \frac{\partial H_y}{\partial t} \end{array} \right.$$

and for magnetic field vector

$$H_y^* = H_0 e^{j((\omega t - \phi) - \gamma z)} \quad \text{--- (21)}$$

where ϕ is lag time phase of H_y^* with respect to E_x^*

The intrinsic impedance $\eta^* = \frac{E_x^*}{H_y^*}$

$$= \frac{E_0 e^{j(\omega t - \gamma z)}}{H_0 e^{j((\omega t - \phi) - \gamma z)}}$$

$$= \frac{E_0}{H_0 e^{-j\phi}}$$

$$\eta^* = \frac{E_0}{H_0} e^{j\phi} \quad \text{--- (22)}$$

$$\eta^* = \eta \angle \phi$$

where η is the magnitude of intrinsic impedance η^* and ϕ is its phase angle.

We can obtain η^* in term of ω, μ, σ as

The maxwell's eqn (III)

$$\nabla \times E^* = -\frac{\partial B^*}{\partial t}$$

for ~~a~~ wave propagating in z-direction

$$E_z^+ = H_y^+ = 0$$

and $$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

and E^+ in x and H_y^+ in y-direction

$$E_y^+ = 0$$

and $$H_x^+ = 0$$

then $\nabla \times E^+$ becomes

$$\frac{\partial E_x^+}{\partial z} = -\mu \frac{\partial H_y^+}{\partial t} \quad -(23)$$

$$\left\{ \nabla \times E^+ = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ E_x^+ & E_y^+ & E_z^+ \end{vmatrix} \right.$$

from eq^n (20)

$$\frac{\partial E_x^+}{\partial z} = -\gamma E_x^+ \quad -(24)$$

and from eq^n (21)

$$\frac{\partial H_y^+}{\partial t} = +j\omega H_y^+ \quad -(25)$$

using eq^n's (24) and (25) in (23), we get

$$\gamma E_x^+ = j\omega\mu H_y^+$$

$$\frac{E_x^+}{H_y^+} = \frac{j\omega\mu}{\gamma} \quad -(26)$$

for good conductor

$$\gamma \simeq 1+j\sqrt{\frac{\omega\mu\sigma}{2}}$$

then

$$\eta^* = \frac{j\omega\mu}{(1+j)\sqrt{\frac{\omega\mu\sigma}{2}}}$$

$$= \frac{(1-j)\,j}{(1+j)(1-j)}\sqrt{\frac{2\omega\mu}{\sigma}}$$

$$= \frac{(j+1)}{2}\sqrt{\frac{2\omega\mu}{\sigma}}$$

$$\eta^* = \frac{(1+j)}{\sqrt{2}}\sqrt{\frac{\omega\mu}{\sigma}} \quad \text{--- } (27)$$

$$\eta^* = \eta\cos\phi + j\eta\sin\phi \quad \text{--- } (28)$$

$\left\{ \begin{array}{l} \eta^* = \eta\angle\phi = \eta e^{j\phi} \\ = \eta(\cos\phi + j\sin\phi) \end{array} \right.$

From eqn (27) and (28)

$$\left.\begin{array}{l} \eta\cos\phi = \sqrt{\frac{\omega\mu}{2\sigma}} \\ \eta\sin\phi = \sqrt{\frac{\omega\mu}{2\sigma}} \end{array}\right\} \text{--- } (29)$$

From eqn (29) $\frac{\eta\sin\phi}{\eta\cos\phi} = \frac{\sqrt{\frac{\omega\mu}{2\sigma}}}{\sqrt{\omega\mu/2\sigma}} = 1$

$$\tan\phi = 1$$

$$\phi = \tan^{-1}(1) = 45$$

$$\phi = 45 \quad \text{--- } (30)$$

Squaring eqn (29) and adding we get

$$\eta^2(\cos^2\phi + \sin^2\phi) = \frac{\omega\mu}{2\sigma} + \frac{\omega\mu}{2\sigma}$$

$$\eta^2 = \frac{\omega\mu}{\sigma}$$

$$\eta = \sqrt{\frac{\omega\mu}{\sigma}} \quad \text{--- } (31)$$

Using eqn (30) and (31) in eqn (22), we get

$$\eta^* = \eta < \phi$$

$$\eta^* = \sqrt{\frac{\omega\mu}{\sigma}} < 45 \quad — (32)$$

This is intrinsic impedance of a good conductor

Since σ is quite large so η^* is very small and has reactive component.

The phase angle of this impedance is always $45°$ for good conductor.

$$\eta^* = \frac{E_x^*}{H_y^*} = \sqrt{\frac{\omega\mu}{\sigma}} < 45°$$

The $\vec{E}_x^*$ and H_y^* are out of phase in conductor. The phase difference b/w $\vec{E}$ and $\vec{H}$ is $\pi/4$.

$\vec{E}$ leads $\vec{H}$ by $\pi/4$ radian in conductor

The current which produces ~~to~~ $\vec{H}$ in conductor is conductor current and not the displacement current as in non-conductor.

The relative amplitude of $\vec{E}$ and $\vec{H}$ depends on freq.

but / value η $\vec{E}$ is small compared to $\vec{H}$

Example at freq 1 Mc/sec, $E/H \approx 10^{-3}$ for copper

≈ 377 for air

The real part eqn (37)

$$\left.\begin{aligned} E &= E_0 e^{-\beta z} \cos(\omega t - \alpha z) \\ H &= H_0 e^{-\beta z} \cos(\omega t - \alpha z - \phi) \end{aligned}\right\} \quad \text{— (36)}$$

then

$$|\vec{S}| = EH = E_0 H_0 e^{-2\beta z} \cos(\omega t - \alpha z) \cos(\omega t - \alpha z - \phi)$$

$$= E_0 H_0 e^{-2\beta z} \left[\cos^2(\omega t - \alpha z) \cos\phi + \cos(\omega t - \alpha z) \sin(\omega t - \alpha z) \sin\phi\right]$$

The average of this

$$S_{av} = |\vec{S}|_{av} = E_0 H_0 e^{-2\beta z} \left(\tfrac{1}{2}\cos\phi - 0\right)$$

$$= \frac{1}{2} E_0 H_0 e^{-2\beta z} \cos\phi \quad \text{— (37)}$$

$S_{av} \propto e^{-2\beta z}$ (Square of amplitude of waves)

From eqn (37), we can write

$$S_{av} = \frac{1}{2}\left(E_0 e^{-\beta z} \cdot e^{-j(\omega t + \alpha z)}\right)\left(H_0 e^{-\beta z} \cdot e^{j(\omega t - \alpha z)}\right) \cos\phi$$

$$= \frac{1}{2} \mathrm{Re}\left\{\left(i E_0 e^{-\beta z} e^{-j(\omega t - \alpha z)}\right) \times \left(j H_0 e^{-\beta z} e^{j(\omega t - \alpha z - \phi)}\right)\right.$$

$$= \frac{1}{2} \mathrm{Re}\left(\vec{E} \times \vec{H}^*\right)$$

where $\vec{H}^* = j H_0 e^{-\beta z} e^{j(\omega t - \alpha z - \phi)}$ is complex conjugate of $\vec{H}$

Conclusions -

(i) $\vec{E}$ and $\vec{H}$ are mutually perpendicular and also perpendicular to direction of propagation of e.m. wave.

(ii) $\vec{E}$ and $\vec{H}$ are attenuating exponentially ($E_x = E_0 e^{-\alpha z}$) as wave penetrates in conductor

(iii) $\vec{E}$ and $\vec{H}$ are not in phase in conductor. phase difference is $\pi/4$

(iv) The energy flow is along the direction of propagation of e.m. wave, but is damped off exponentially.

(v) Ratio of $\vec{E}$ and $\vec{H}$ energy density $= \dfrac{\frac{1}{2}\epsilon E_0^2}{\frac{1}{2}\mu H_0^2} = \dfrac{\omega\epsilon}{\sigma} \leq \dfrac{1}{50}$

(v) Poynting vector in conductor

Let wave propagating in z-direction

$\vec{E}$ is in x-direction

$\vec{H}$ is in y-direction

The solution of wave eqn {eqn (20) and (21)}

$$E = \hat{i} E_0 e^{-j(\omega t - \gamma z)}$$

$$\vec{E} = \hat{i} E_0 e^{-\beta z} e^{-j(\omega t - \alpha z)}$$

and

$$\vec{H} = \hat{j} H_0 e^{-\beta z} e^{-j(\omega t - \alpha z + \phi)}$$

$\gamma = \alpha + i\beta$ — (32)

From eqn

$$\nabla\cdot(\vec{E}\times\vec{H}) = \vec{H}\cdot(\nabla\times\vec{E}) - \vec{E}(\nabla\times\vec{H})$$

$$= -\vec{H}\cdot\frac{\partial \vec{B}}{\partial t} - \vec{E}\cdot\left(\frac{\partial \vec{D}}{\partial t} + \vec{J}\right)$$

$$= -\frac{\partial}{\partial t}\left(\frac{1}{2}\mu H^2 + \frac{1}{2}\epsilon E^2\right) - \vec{E}\cdot\vec{J}$$

— (33)

Integrating over a volume V bounded by surface S

then

$$\int_V \nabla\cdot(\vec{E}\times\vec{H})\,dV = -\frac{\partial}{\partial t}\int_V\left(\frac{1}{2}\epsilon E^2 + \frac{1}{2}\mu H^2\right)dv - \int_V \vec{E}\cdot\vec{J}\,dv$$

Using Gauss divergence theorem

$$\int_S (\vec{E}\times\vec{H})\,ds = -\frac{\partial}{\partial t}\int_V\left(\frac{1}{2}\epsilon E^2 + \frac{1}{2}\mu H^2\right)dv - \int_V \vec{E}\cdot\vec{J}\,dv$$

— (34)

we know the poynting vector

$$\vec{S} = \vec{E}\times\vec{H} \quad W/m^2$$

$$|\vec{S}| = |\vec{E}\times\vec{H}| = EH$$

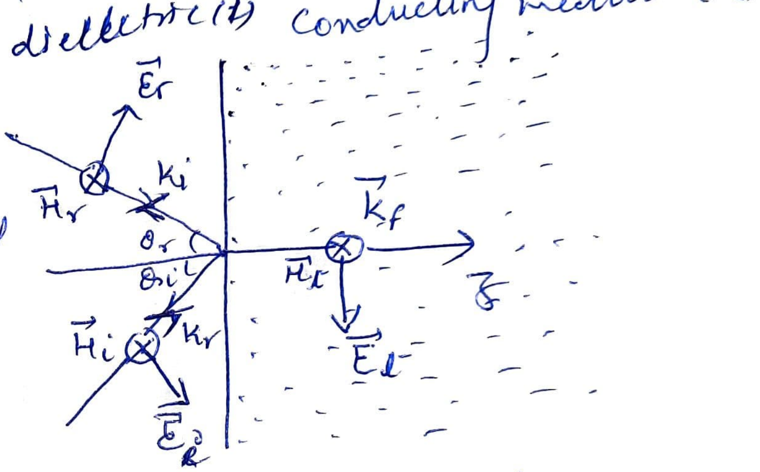
— (35)

Reflection and transmission at a conducting surface

(Reflection and transmission at the boundary of a non-conducting and a conducting media)

Consider the boundary surface separates a dielectric from a conducting medium

Let incident is normal



The boundary conditions for continuity of tangential components of $\vec{E}$ and $\vec{H}$

$$\left.\begin{aligned} E_i - E_r &= E_t \\ H_i + H_r &= H_t \end{aligned}\right\} \quad (1)$$

here $\theta_i = 0°$, $\theta_r = 0$

We know $J = \sigma \vec{E}$

we know that $\vec{E}$ vanishingly small inside conductor

$J \simeq 0$ unless $\sigma = \infty$

so the boundary condition

$$\left.\begin{aligned} \hat{n} \times \vec{H} &= J_\epsilon \\ \text{reduced to} \quad \hat{n} \times H &= 0 \end{aligned}\right\} \quad (2)$$

i.e. tangential component of $\vec{H}$ is continuous across the boundary we know that

$$|\vec{H}| = \frac{\eta}{Z_0}|\vec{E}| = \frac{\eta}{Z_0}|\vec{E}| \text{ as } \mu_r \simeq 1 \quad — (3)$$

from eqn (1) putting value of $\vec{H}$, we get

$$\eta_1(\vec{E}_i + E_r) = \eta_2 E_t \quad — (4)$$

from eqn (1) and (4), we get

$$E_i - E_r = \frac{\eta_1}{\eta_2}(E_i + E_r)$$

$$\frac{E_r}{E_i} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} \quad — (5)$$

from eqn (4) and (1)

$$E_i - \left(\frac{\eta_2}{\eta_1}E_t - E_i\right) = E_t$$

$$\frac{E_t}{E_i} = \frac{2\eta_1}{\eta_2 + \eta_1} \quad — (6)$$

we know that

$$k = \eta\frac{\omega}{c} \quad \text{i.e. } \eta = \frac{c}{\omega}k \quad — (7)$$

$$\left\{ \begin{array}{l} \lambda = \frac{2\pi}{k}\eta = \frac{c}{v}\eta = \frac{2\pi}{k}\eta \\ = c = \frac{\omega}{k}\eta \end{array} \right.$$

for good conductor

$$k \longrightarrow k^* = \alpha + i\beta \quad \text{with } \alpha \simeq \beta = \sqrt{\frac{\omega\mu_0\sigma}{2}}$$

so $\eta_2 \longrightarrow \eta^* = \frac{c}{\omega}(k^*) = \frac{c}{\omega}(\alpha + i\beta)$

i.e. $\eta_2 \longrightarrow \eta^* = \frac{c}{\omega}\alpha + i\left(\frac{c}{\omega}\beta\right)$

$$\eta_2 \longrightarrow \eta^* = n + ik \text{ with } n = k = \sqrt{\frac{\sigma}{2\omega\epsilon_0}} \quad \text{— (8)}$$

from eqn (5) and (6)

$$\frac{E_r}{E_i} = \frac{(n+ik) - n_1}{(n+ik)+n_1} = \frac{(n-n_1)+ik}{(n+n_1)+ik} \quad \text{— (9)}$$

$$\frac{E_t}{E_i} = \frac{2n_1}{(n+ik)+n_1} = \frac{2n_1}{(n+n_1)+ik} \quad \text{— (10)}$$

Now

$$(n - n_1) + ik = a e^{i\phi_1}$$

$$(n+n_1) + ik = b e^{i\phi_2}$$

with

$$a = \sqrt{(n-n_1)^2 + k^2}, \quad \phi_1 = \tan^{-1}\frac{k}{n-n_1}$$

$$b = \sqrt{(n+n_1)^2 + k^2}, \quad \phi_2 = \tan^{-1}\frac{k}{n+n_1} \quad \text{— (11)}$$

then eqn (9) and (10) becomes

$$\frac{E_r}{E_i} = \frac{a e^{i\phi_1}}{b e^{i\phi_2}} = \left[\frac{(n-n_1)^2 + k^2}{(n+n_1)+k^2}\right]^{1/2} e^{i(\phi_2 - \phi_1)} \quad \text{— (12)}$$

and

$$\frac{E_t}{E_i} = \frac{2n_1}{b e^{i\phi_2}} = \frac{2n_1}{[(n+n_1)^2 + k^2]^{1/2}} e^{-i\phi_2} \quad \text{— (13)}$$

Eqn (12) and (13) are final eq's representing the reflected and transmitted waves.

It is clear that both transmitted and reflected waves exist and they are not in phase with incident wave.

The amplitude of the transmitted wave is very small due to large values of n and k in the denominator.

The reflection coefficient is

$$R = \left|\frac{E_r}{E_i}\right|^2 = \frac{(n-n_1)^2+k^2}{(n+n_1)^2+k^2} \quad \text{as } \theta_r = \theta_i = 0$$

$$R = \frac{n^2+k^2-2nn_1+n_1^2}{n^2+k^2+2nn_1+n_1^2} \quad \text{—— (14)}$$

$$= \frac{n^2+k^2-2nn_1}{n^2+k^2+2nn_1} \quad \text{as } n \simeq k >> n_1$$

$$= \frac{2n^2-2nn_1}{2n^2+2nn_1} \quad \text{as } n \simeq k$$

$$= \frac{1-n_1/n}{1+n_1/n} = \left(1-\frac{n_1}{n}\right)\left(1+\frac{n_1}{n}\right)^{-1}$$

$$= \left(1-\frac{n_1}{n}\right)\left(1-\frac{n_1}{n} \cdots\right)$$

$$= 1 - 2\frac{n_1}{n} + \frac{n_1^2}{n^2} + \cdots$$

$$= 1 - 2\frac{n_1}{n} + 2\frac{n_1^2}{n^2}$$

$$\simeq 1 - 2\frac{n_1}{n} \simeq 1 - 2\frac{n_1}{k} \quad \text{—— (15)}$$

From eqn (15) for greater value of k, more nearly R is equal to unity.

The waves which are most strongly absorbed are very strongly reflected i.e. good absorbers are good reflectors

from eq^n (15)

$$R = 1 - \frac{2n_1}{k} = 1 - 2n_1\sqrt{\frac{2\omega\epsilon_0}{\sigma}} \qquad \text{as } n = k = \sqrt{\frac{\sigma}{2\omega\epsilon_0}} \qquad \text{— (16)}$$

At low freq. for substance of high conductivity, reflection coefficient will be close to unity

So all energy will be reflected and this is why metals are opaque to light

The little ~~heat~~ energy that flows into metal is rapidly dissipated by heat loss associated with the induced current.

As reflection coefficient (R) is almost unity for metallic surface, the magnitude of E_r and E_i are almost equal

then

$$\left.\begin{aligned} E_r &= -\hat{n}_x E_0 e^{-i(\omega t - k_i z)} \\ E_i &= n_x E_0 e^{-i(\omega t - k_i z)} \end{aligned}\right\} \qquad \text{— (17)}$$

total $\vec{E}$ is $\vec{E} = \vec{E_r} - \vec{E_i}$ — (18)

$$E = e^{-i\omega t}\left(e^{ik_i z} - e^{-ik_i z}\right) \qquad \text{— (19)}$$

taking real part of eq^n (19)

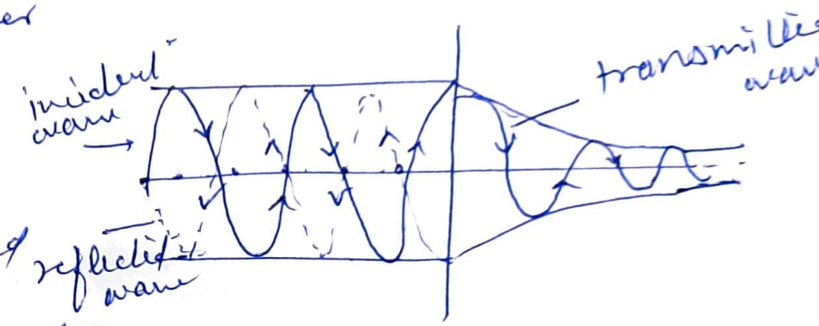
$$\vec{E} = 2\hat{n} E_0 \sin\omega t \sin k_1 z \quad \text{—} \quad (20)$$

Total $\vec{E}$ is represented by standing wave

and

$$k_1 z = n_1 \frac{\omega}{c} z = \frac{2\pi n_1}{\lambda} z \quad \text{—} \quad (21)$$

<u>phase of electric vector</u>



The phase change suffered by $\vec{E}$ due to reflection at normal incident from the surface of conducting medium

$$\phi = \phi_2 - \phi_1$$

$$\tan\phi = \tan(\phi_2 - \phi_1)$$

$$\tan\phi = \frac{\tan\phi_2 - \tan\phi_1}{1 + \tan\phi_1 \tan\phi_2}$$

we know $\tan\phi_1 = \frac{k}{n - n_1}$ and $\tan\phi_2 = \frac{k}{n + n_1}$

Then

$$\tan\phi = \frac{\frac{k}{n - n_1} - \frac{k}{n + n_1}}{1 + \left(\frac{k}{n - n_1}\right)\left(\frac{k}{n + n_1}\right)}$$

$$\tan\phi = \frac{2kn_1}{k^2 + n^2 - n_1^2} \quad \text{—} \quad (22)$$