GROUND CONTROL POINTS
(Control points for image to map transformation)

A **ground control point (GCP) is a location on** the surface of the Earth (e.g., a road intersection) that can be identified on the imagery and located accurately on a map.

There are two distinct sets of coordinates associated with each GCP:

- source or image coordinates specified in i rows and j columns, and
- Reference or map coordinates (e.g., x, y measured in degrees of latitude and longitude or meters in a Universal Transverse Mercator projection).

Ground Control Points (GCPs) cont..

- The paired coordinates (i, j and x, y) from many GCPs can be modeled to derive geometric transformation coefficients.

- These coefficients may be used to geometrically rectify the remote sensor data to a standard datum and map projection

Ground Control Points (GCPs) cont..

- Accurate GCPs are essential for accurate rectification
- Sufficiently large number of GCPs should be selected
- Well dispersed GCPs result in more reliable rectification
- GCPs for Large Scale Imagery

  – Road intersections, airport runways, towers, buildings etc.
- for small scale imagery
- – larger features like Urban area or Geological features can be used
- For topomap or scanned map
- Tic points or points of known real world coordinates.
- NOTE : landmarks that can vary (like lakes, other water bodies, vegetation etc) should not be used.
- GCPs should be spread across image
- Requires a minimum number depending on the type of transformation

COORDINATE TRANSFORMATION

- Polynomial equations are used to convert the source file coordinates to rectified map coordinates.
- Depending upon the distortions in the imagery, the number of GCPs used, their location relative to one other, complex polynomial equations are used.
- The degree of complexity of the polynomial is expressed as ORDER of the polynomial.
- The order is simply the highest exponent used in the polynomial

First-Order Polynomial Transformation – Affine

General rule suggests use of affine transformation for map to map or image to map transformation

Mathematically expressed as pair of first order polynomial equations

$$X = Ax + By + C$$

$$Y = Dx + Ey + F$$

x and y are input coordinates

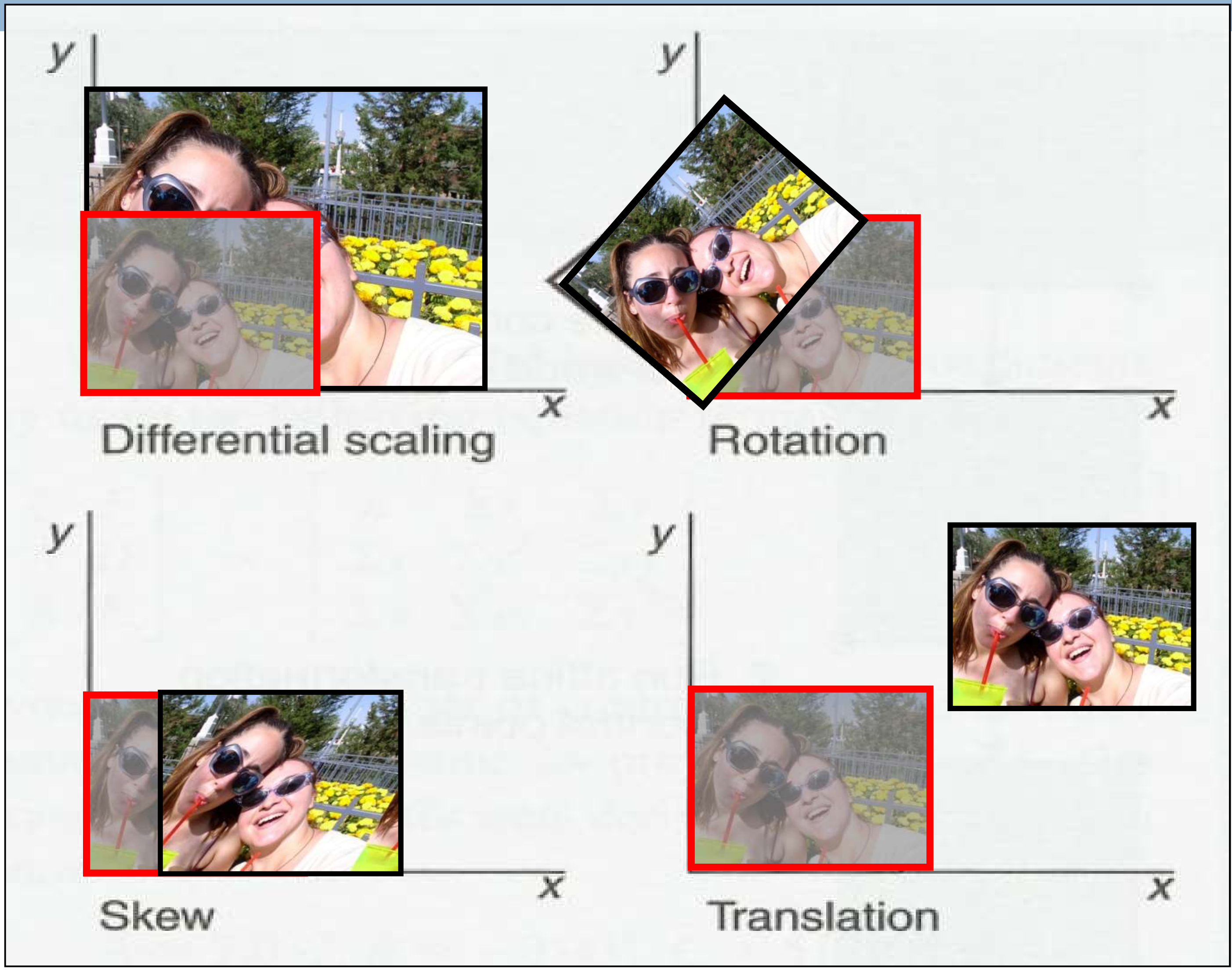
X and Y output coordinates - to be determined

A,B,C,D,E,F are transformation coefficients.

•Allows Differential Scaling , skews, rotates, and translates the layer coordinates.

•The affine transformation requires a **minimum of three control points.**

First-Order Transformation - Affine



First-Order Polynomial Transformation – Affine

- If the coefficients A ,B,C, D, E and F are known then, the polynomial can be used to relate a point on map to its corresponding point on image and vice versa. Hence six coefficients are required for this transformation (three for X and three for Y).
- Six coefficients provide for **scale changes in X and Y direction, Skew in degrees, rotation angle, Translation in X and Y direction.**
- Requires Minimum THREE GCP's for solving the above equation. Four or more GCPs are used to reduce measurement errors and for a least squares solution.
- However before applying rectification to the entire set of the data, it is important to determine how well the six coefficients derived from the least square regression of the initial GCPs account for the geometric distortion in the input image

Accuracy of transformation

- In this method, we check how good do selected points fit between the map and the Image?
- To solve linear polynomials we first take three GCP's to compute the six coefficients. Its source coordinates in the **original input image** are say **x_o and y_o.**
- The position of the same points **in reference map** in degrees, feet or meters are say **x, y.**

Accuracy of transformation

- Now, if we input the map x,y values for the first GCP back into the linear polynomial equation with all the coefficients in the place, we would get the computed or retransformed x_r and y_r values , which are supposed to be location of this point in input image.
- Ideally measured and computed values should be equal.
- **In reality this does not happen.**
- There is **discrepancy between the measured and computed coordinates** i.e we have to check the accuracy of transformation

RMS Root Mean Square error

Deviation between the estimated x, y values and the actual x, y values

- RMS for a tic = sqrt($((x_{act} - x_{est})^2 + (y_{act} - y_{est})^2)$)

- Average RMS = sqrt($((\Sigma(x_{act,i} - x_{ext,i})^2 + \Sigma(y_{act,i} - y_{est,i})^2 / n)$)

ROOT MEAN SQUARE (RMS) ERROR

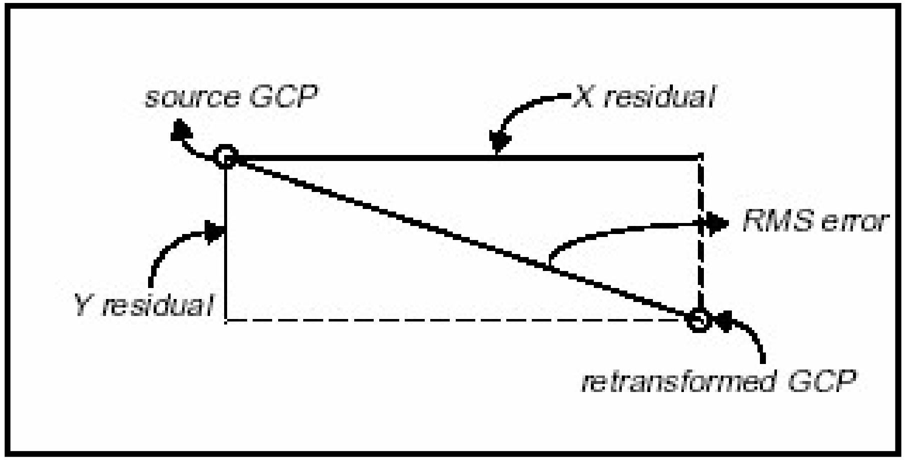
- The method used involves computing **Root Mean Square Error (RMS error) for each of the ground control point**
- **RMS error is the distance between the input (source or measured) location of a GCP and the retransformed (or computed) location for the same GCP.**
- RMS error is computed with a Euclidean Distance Equation.

Root Mean Square (RMS) Error….

- RMS error is expressed as distance in the source coordinate system.
- It is the distance in pixel widths
- An RMS error of 2 means that the reference pixel is 2 pixels away from the retransformed pixel.

Root Mean Square (RMS) Error

Deviation between the actual location and the estimated location of the control points.



Error for a control point is

$$\sqrt{(x_{act} - x_{est})^2 + (y_{act} - y_{est})^2}$$

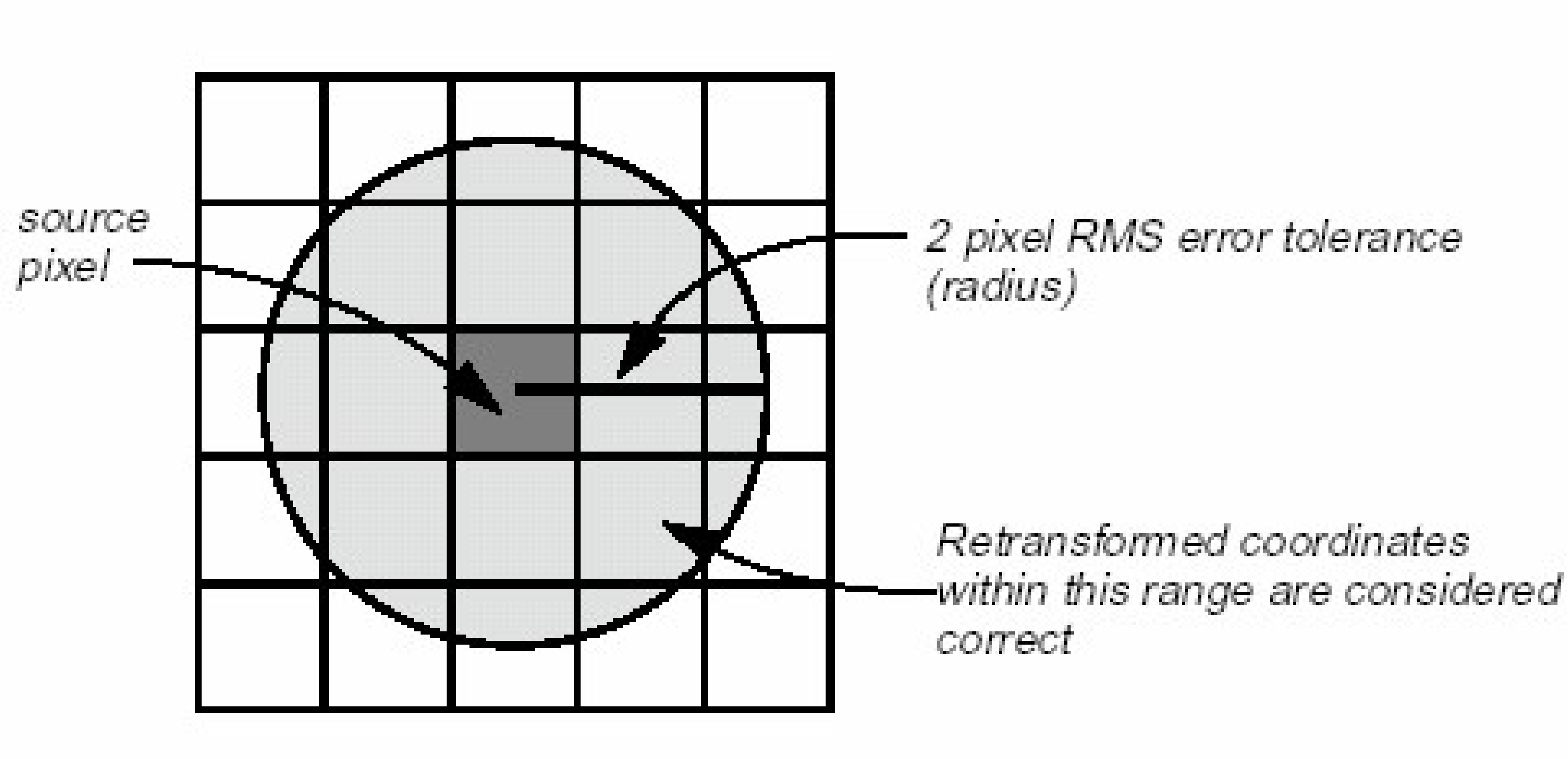
Average RMS is

$$\sqrt{\left(\sum_{i=1}^{n}(x_{act,\,i} - x_{est,\,i})^2 + \sum_{i=1}^{n}(y_{act,\,i} - y_{est,\,i})^2\right)\Big/ n}$$

Acceptable RMS error or Tolerance of the error

- Normally, a user specifies a certain amount (a threshold) of acceptable total RMS error.
- The amount of RMS error that is tolerated can be thought of as a window around each source coordinate, inside which a retransformed coordinate is considered to be correct.
- For example, if the threshold is 2, then the retransformed pixel can be **2 pixels away from the source pixel and still be considered accurate.**

Acceptable RMS error or Tolerance of the error



Acceptable RMS error….

- Acceptable RMS error depends upon the

– End use of the data

– The type of data being used, and

– The accuracy of the GCP and the ancillary data.

- **Normally an RMS error of less than 1 per GCP and a total RMS error of less than half a pixel (0.5) is acceptable**

Acceptable RMS error….

- After each computation of a transformation, the total RMS error reveals that a given set of control points exceeds the threshold then
  - Delete the GCP from the analysis that has greatest amount of individual error
  - Recompute the coefficients and the RMS error for all points

These are the first order transformation equations:

1. $x' = a + bx + cy$
2. $y' = d + ex + fy$

Now, we have the following RMS table :

T A B L E

Name	On	Edit	Cell X	Cell Y	Northing	Easting	Height	RMS
1	On	Edit	453.98	405.95	30.38N	77.73E	0.00	0.03
2	On	Edit	279.03	245.14	30.44N	77.68E	0.00	0.58
3	On	Edit	377.85	190.91	30.45N	77.72E	0.00	0.53
4	On	Edit	281.32	245.00	30.44N	77.68E	0.00	0.01

Co-ordiantes of the pixel in the satellite image (x,y)

Corresponding latitude-longitude values in the toposheet (x',y')

Error

Now, we have got three GCP's each, in satellite image as well as in toposheet, and thus six co-ordinates, and hence twelve values in all.

Out of which, three values each refer to x, y, x', y'. We will use these values to solve for the values of constants... a,b,c,d,e,f...and finally get the transformation equations...

...contd

Actual to image co-ordinates:

$$x' = 29.4266 + 0.0325488x - 0.0061536y$$
$$y' = 77.8772 - 0.006085x - 0.0323303y$$

Image to actual co-ordinates:

$$x = -250 + 29.6671x' - 5.64673y'$$
$$y = 2950 - 5.584x' - 29.8676y'$$

The last coloumn refers to the RMS error in the selection of GCP's. For better and high accuracy the RMS error should be less than unity.

Higher Order Transformation

Complex distortions can be corrected

Requires more links and control points

Second- Order

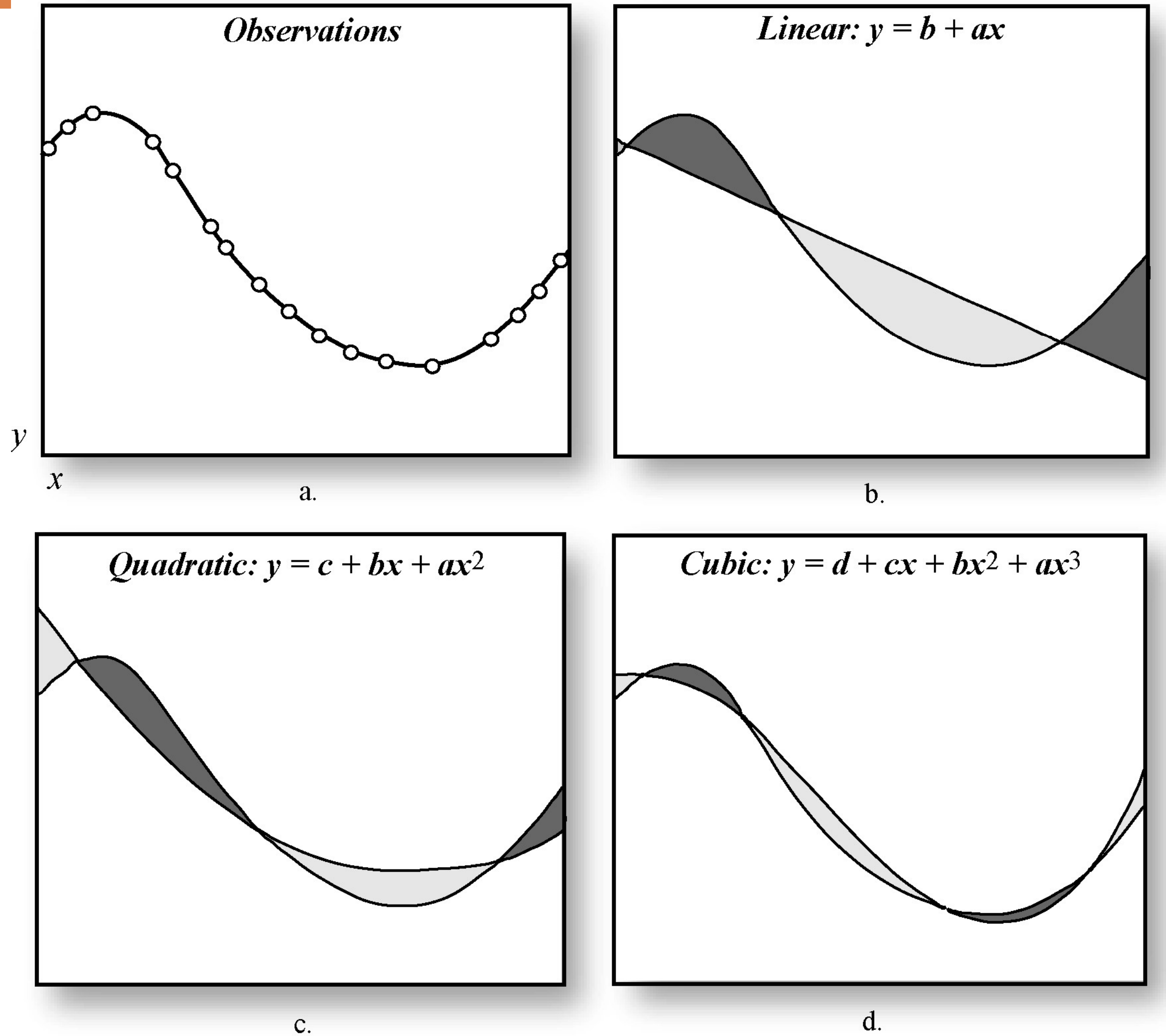
- Minimum of 6 control points

Third - Order

- Minimum of 10 control points

First – Order is suitable for most purposes

How different-order transformations fit a hypothetical surface



Concept of illustrated in cross-section.

a) Original observations.

b) First-order linear transformation fits a plane to the data.

c) Second-order quadratic fit.

d) Third-order cubic fit.

Order of Equation / No. of GCP

ORDER OF EQUATION	NO. OF COEFFICIENTS	MINIMUM NUMBER OF GCPs REQUIRED
First	6	3
Second	12	6
Third	20	10
n	(n+1)*(n+2)	[(n+1)*(n+2)]/ 2

- ArcMap supports first to third order transformation.
- Command line supports first to twelfth order transformation.