MOMENTS

- Moments in mathematical statistics involve a basic calculation. These calculations can be used to find a probability distribution's mean, variance, and skewness.
- *Moments in statistics are popularly used to describe the characteristic of a distribution.*
- **1 Moment:** Measure of central location
- **2 Moment:** Measure of dispersion
- **3 Moment:** Measure of asymmetry
- **4 Moment:** Measure of peakedness

- ***First moment- Mean***
- Measure the location of the central point.

$$\overline{x} = \frac{\sum_{i=1}^{N} x_i}{N}$$

- ***Second moment- Standard Deviation (SD, σ(Sigma)):***
- Measure the spread of values in the distribution OR how far from the normal.

$$\sigma^2 = \frac{\sum_{i=1}^{N}(X_i - \bar{X})^2}{N}$$

σ = (Variance)^.5

- **Small SD** : Numbers are close to mean
  High SD : Numbers are spread out**For normal distribution:**
  Within 1 SD: 68.27% values lie
  Within 2 SD: 95.45% values lie
  Within 3 SD: 99.73% values lie
- **Advantages over Mean Absolute Deviation(MAD):**
  1. Mathematical properties- Continuous, differentiable.
  2. SD of a sample is more consistent estimate for a population- When drawing repeated samples from a normally distributed population, the standard deviations of samples are less spread out as compare to mean absolute deviations.

- ***Third moment- Skewness***
- Measure the symmetry in the distribution.

$$Skew = \frac{1}{N}\sum_{i=1}^{N}\left[\frac{(X_i - \bar{X})}{\sigma}\right]^3$$

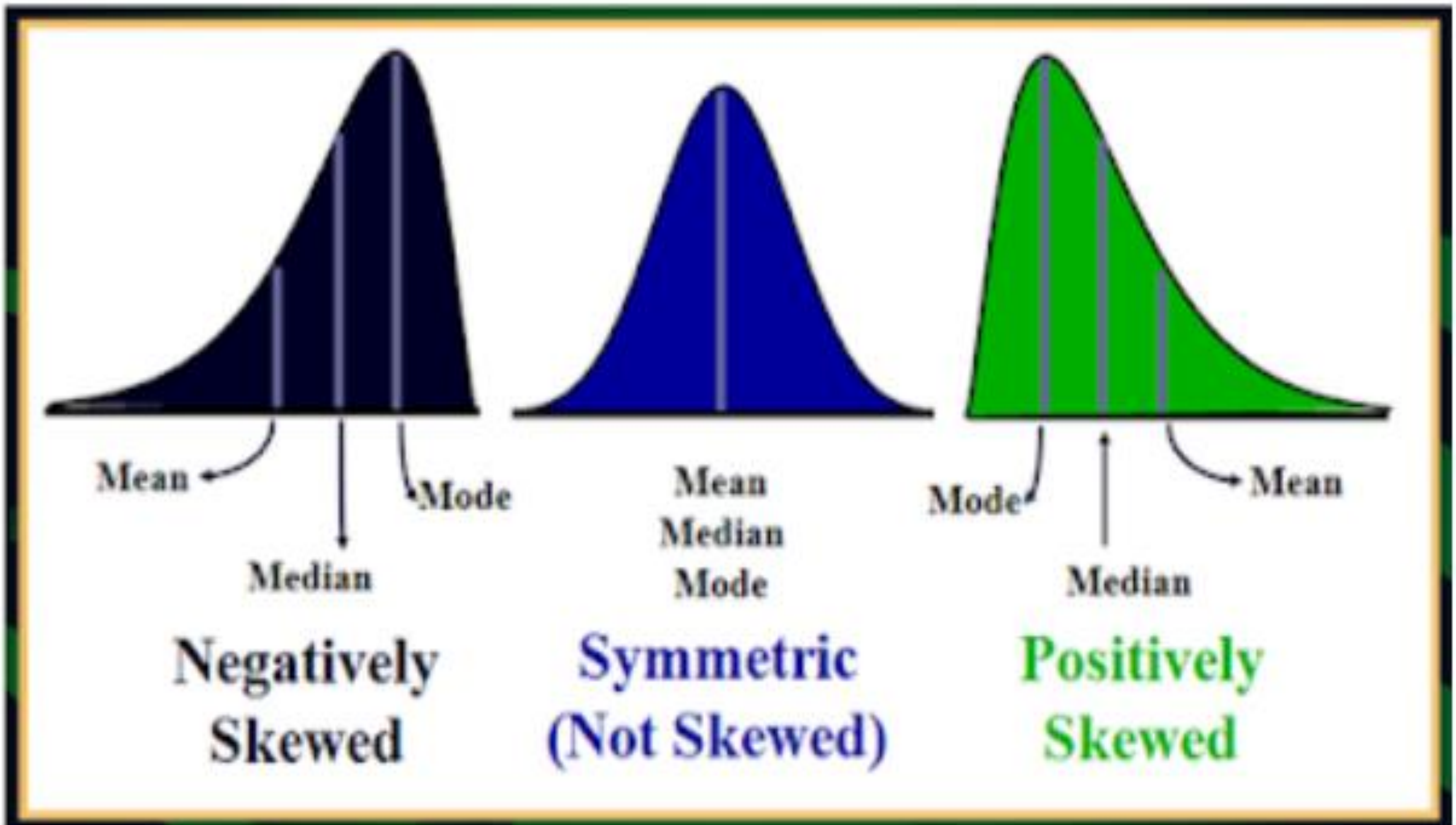
- Skewness=0 **[Normal Distribution, Symmetric]Other Formulas:**

1. Skewness = (Mean-Mode)/SD
2. Skewness = 3*(Mean-Median)/SD

(Mode = 3*Median-2*Mean)

Transformations (to make the distribution normal)**:**

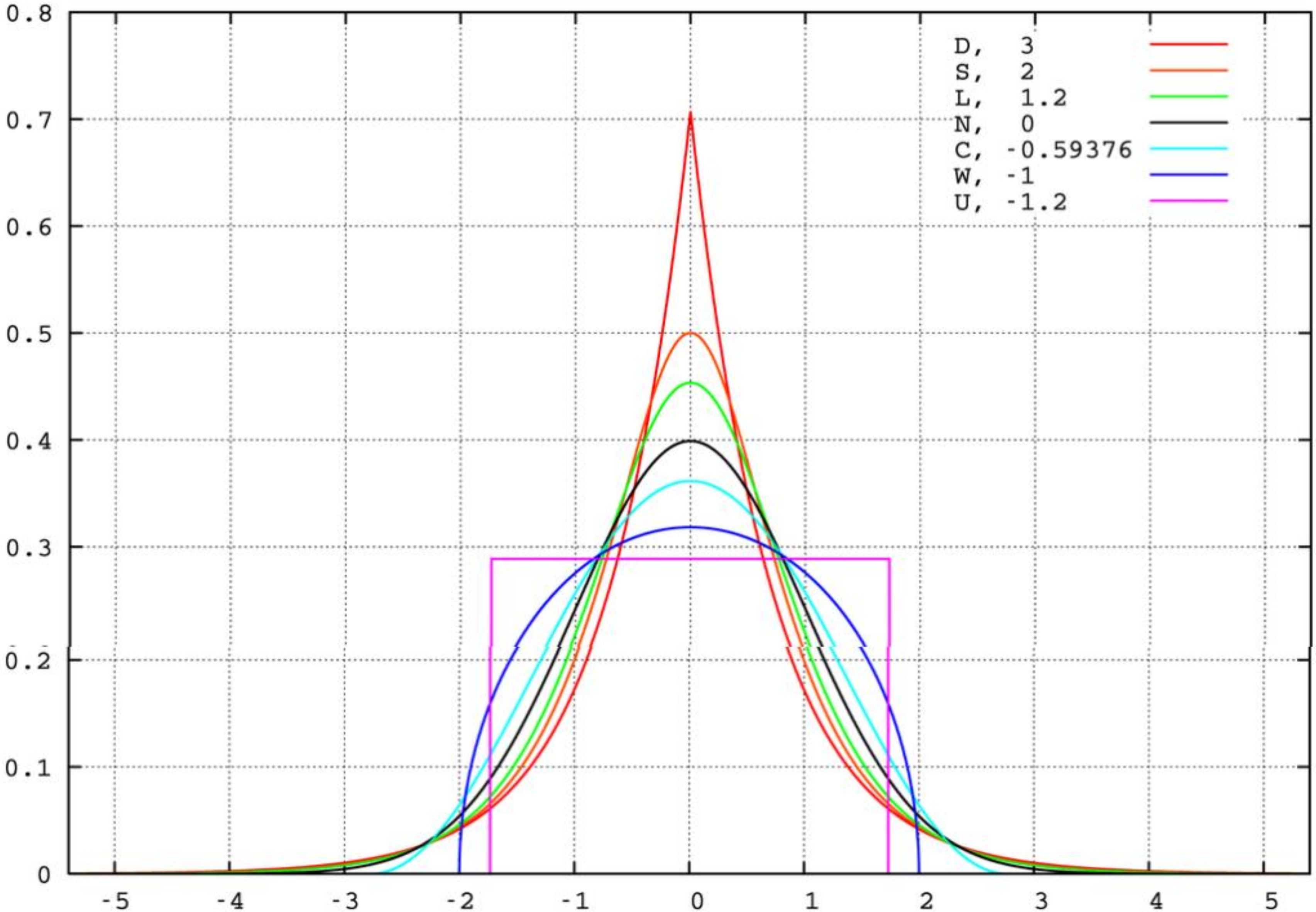
a. Positively skewed (right): Square root, log, inverse
b. Negatively skewed (left) : Reflect and square[sqrt(constant-x)],
reflect and log, reflect and inverse


Mean
Median
Mode
Negatively Skewed
Mean
Median
Mode
Symmetric (Not Skewed)
Mode
Median
Mean
Positively Skewed

- ***Fourth moment- Kurtosis:***
- Measure the amount in the tails.

$$Kurt = \frac{1}{N}\sum_{i=1}^{N}\left[\frac{(X_i - \bar{X})}{\sigma}\right]^4$$

Kurtosis=3 **[Normal Distribution]**
Kurtosis<3 [Lighter tails]
Kurtosis>3 [Heavier tails]**Other Formulas:**
*Excess Kurtosis = Kurtosis - 3***Understanding:**
Kurtosis is the average of the standardized data raised to fourth power. Any standardized values less than |1| (i.e. data within one standard deviation of the mean) will contribute petty to kurtosis.
The standardized values that will contribute immensely are the outliers.
High Kurtosis alerts about attendance of outliers.



Excess Kurtosis for Distributions [Laplace (D)ouble exponential; Hyperbolic (S)ecant; (L)ogistic; (N)ormal; ©osine; (W)igner semicircle; (U)niform]

THANK YOU

- Statistics — Moments of a distribution by Harsh Sinhal
- https://medium.com/analytics-vidhya/statistics-moments-of-a-distribution-1bcfc4cbbd48