at random

iii) The popⁿ units corresponding to the no. ~~selecting~~ selected in step II constitute the random sample.

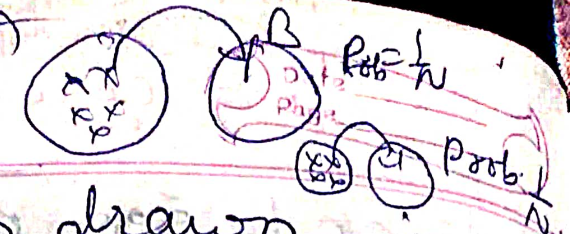
Unit - II

* **Simple random sampling :→**

It is a technique of drawing the a sample in such a way that each unit of the popⁿ has an equal and indep chance of being included in the sample

(i) **Simple random sampling (SRS) without replacement :→** (SRSWOR)

If in simple random sampling the selected

SRSWOR = 

unit at each draw is not replaced back into the popⁿ before making succeeding draws of the sample then procedure is called SRSWOR.

Let us suppose that a sample of size 'n' is drawn from a popⁿ of size N. Then we have $\boxed{\binom{N}{n} n!}$ total sample of size 'n'. (considering order of the units)

If order of units isn't considered then we shall have $\binom{N}{n}$ total samples of size n.

→ (i) In SRS WOR prob. of selectg. any unit at the first draw is equal to $\frac{1}{N}$

(ii) Prob. of selecting any unit out of remaining N-1 units in the second draw = $\frac{1}{N-1}$ and so on.

Rmk:- Prob. that i^{th} unit ~~not~~ is selected at r^{th} draw given that i^{th} unit is not selected

6—2 $\binom{6}{2}2!$ order 1 2 3 4 5 6

(1,2) (1,6)
(1,3) (2,1)
(1,4) ⋮

Prob. $\frac{1}{N}$

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at previous $r-1$ draws is

~~$P(y_r = Y_i \mid y_{r'} \neq Y_i$~~

$$P(y_r = Y_i \mid y_{r'} \neq Y_i;\ r' = 1, 2 \ldots r-1)$$
$$= \frac{1}{N-(r-1)} = \frac{1}{N-r+1} \quad r = 1, 2 \ldots n$$

The required prob. of obtaining ~~of sampling~~ of sample of size n (in any order) is

$$p = \frac{1}{\binom{N}{n}}$$

Imp. * Result ⟶ The prob. of selecting a specified unit of the popⁿ at any given draw is equal to the prob. of its being selected at the first draw

06

⟶ Let E_r be the event that any specified units is selected at the r^{th} draw. then prob. of E_r $\{P(E_r)\}$

$P(E_r)$ = Prob. { that the specified unit isn't selected in any one of the previous $r-1$ draws & then selected at the r^{th} draw

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$$\therefore P(E_r) = \prod_{i=1}^{r-1} P[\text{it is not selected at the } i^{th} \text{ draw}]$$

$\times P$[it is selected at the r^{th} draw given that it is not selected at the previous $r-1$ draws { by compound prob. thm. }]

$$= \left[\prod_{i=1}^{r-1}\left\{1 - \frac{1}{N-i+1}\right\}\right] \times \frac{1}{N-r+1}$$

$$= \left[\prod_{i=1}^{r-1}\left\{\frac{N-i}{N-i+1}\right\}\right] \times \frac{1}{N-r+1}$$

$$= \frac{N-1}{N} \cdot \frac{N-2}{N-1} \cdot \frac{N-3}{N-2} \cdots \frac{N-r+1}{N-r+2} \times \frac{1}{N-r+1}$$

$$= \frac{1}{N}$$

$$\boxed{P(E_r) = \frac{1}{N} = P(E_1)}$$ Hence prove.

~~The prop.~~

* **Prob. of selecting any specified unit in the sample** :→

Since a specified unit can be included in the sample of size 'n' in

$P(A \cup B) = P(A$

In 'n' mutually exclusive ways i.e. It can be selected in the sample at the r^{th} draw $\{r = 1, 2 \ldots n\}$ and since prob. of $P(E_r) = \frac{1}{N}$; $r = 1, 2 \ldots n$

By the addit^n thm. of prob. we get;

The prob. that a specified unit is included in the sample $= \sum_{r=1}^{n} \left(\frac{1}{N}\right) = \frac{n}{N}$

(परिणाम), शक्यता

* Notation and terminology of SRSWOR :→

Let us consider a finite pop^n of N units and let Y be character under study var. (for eg. height) Let Y_i $(i = 1, 2, \ldots N)$ be the value of the character for the i^{th} unit in the pop^n and the corresponding small letters y_i $(i = 1, 2 \ldots n)$ denote the value of the character for the i^{th} unit selected in the sample then we define :

Pop^n Mean $\bar{Y}_N = \frac{\sum_{i=1}^{N} Y_i}{N}$ ——— ①

Sample mean $\bar{y}_n = \frac{\sum_{i=1}^{n} y_i}{n}$ ——— ②

Sample mean $\bar{y}_n$ may also be written alternatively as :→

$$\boxed{\bar{y}_n = \frac{1}{n}\sum_{i=1}^{N} a_i y_i} \longrightarrow (3)$$

Where $a_i = \begin{cases} 1 & \text{if } i^{th} \text{ unit is } \cancel{\text{selected}} \text{ included in the sample} \\ 0, & \text{if } i^{th} \text{ unit is not included in the Sample} \end{cases}$

Pop.ⁿ Mean sq.

S^2 = Mean square of popⁿ

$$S^2 = \frac{1}{N-1}\sum_{i=1}^{N}(Y_i - \bar{Y}_N)^2$$

OR

$$S^2 = \frac{1}{N-1}\left[\sum_{i=1}^{N} Y_i^2 - N\bar{Y}_N^2\right]$$

$$\left[\text{as } \frac{1}{N-1}\sum_{i=1}^{N}(Y_i^2 + \bar{Y}_N^2 - 2Y_i\bar{Y}_N)\right]$$

$$\left[\sum_{i=1}^{N} Y_i^2 - 2\bar{Y}_N\sum_{i=1}^{N} Y_i + \bar{Y}_N^2\sum_{i=1}^{N} 1\right.$$

$$= -2N\bar{Y}_N^2 + N\bar{Y}_N^2$$

Sample Mean Square

s^2 = Mean sq. for the Sample

$$s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(y_i - \bar{y}_n)^2$$

$$= \frac{1}{n-1}\left[\sum_{i=1}^{n} y_i^2 - n\bar{y}_n^2\right]$$

$E(x) = \sum x P(x)$

05, 07, 08, 09, 11, 13 $E(a_i) = \sum p(a_i)$

Th. In SRSWOR the sample mean is an unbiased estimate of the popn mean.

$t = \frac{1}{n}\sum_{i=1}^{n} a_i y_i$

Proof :→ We have

$$\bar{y}_n = \frac{1}{n}\sum_{i=1}^{N} a_i y_i$$

$$\therefore E(\bar{y}_n) = E\left[\frac{1}{n}\sum_{i=1}^{N} a_i y_i\right]$$

$$= \frac{1}{n}\sum_{i=1}^{N} E(a_i) y_i$$

Since a_i takes only two values 1 and 0

$E(a_i) = 1 \cdot P(a_i = 1) + 0 \cdot P(a_i = 0)$

$= 1 \cdot P(i^{th}$ unit is included in the sample of size $n)$

$+ 0 \cdot P(i^{th}$ unit isn't included in the sample of size $n)$

$\frac{1}{N} = \frac{n}{N}$

$$E(a_i) = 1 \cdot \frac{n}{N} + 0 \cdot \left(1 - \frac{n}{N}\right)$$

$$= \frac{n}{N}$$

Hence

$$E(\bar{y}_n) = \frac{1}{n}\sum_{i=1}^{N} \frac{n}{N} y_i$$

$$= \frac{1}{N}\sum_{i=1}^{N} y_i = \bar{Y}_N$$

$$\boxed{E(\bar{y}_n) = \bar{Y}_N}$$

we always don't take expectation of pop^n var.

$$E(\Sigma a_i)^2 = \Sigma a_i^2 + \sum_{i \neq j} a_i a_j]$$

Hence sample mean is an unbiased estimate of the pop^n mean

Thm:- In SRSWOR, the sample mean sq. is an unbiased estimate of the pop^n mean sq.

Proof: (We'll take s^2 and then its estimation if it comes to S^2 then its unbiased estimate of S^2.)

We have

$$s^2 = \frac{1}{n-1} \sum_{i=1}^{n} (y_i - \bar{y}_n)^2$$

$$= \frac{1}{n-1} \left[\sum_{i=1}^{n} y_i^2 - n \bar{y}_n^2 \right]$$

$$= \frac{1}{n-1} \left[\sum_{i=1}^{n} y_i^2 - n \left(\frac{1}{n} \sum_{i=1}^{n} y_i \right)^2 \right]$$

$$= \frac{1}{n-1} \left[\sum_{i=1}^{n} y_i^2 - \frac{1}{n} \left(\sum_{i=1}^{n} y_i \right)^2 \right]$$

$$= \frac{1}{n-1} \left[\sum_{i=1}^{n} y_i^2 - \frac{1}{n} \left(\sum_{i=1}^{n} y_i^2 + \sum_{i \neq j=1}^{n} y_i y_j \right) \right]$$

$$= \frac{1}{n-1} \left[\left(1 - \frac{1}{n}\right) \sum_{i=1}^{n} y_i^2 - \frac{1}{n} \sum_{i \neq j=1}^{n} y_i y_j \right]$$

if ... indep.

$E(XY) = E(X)E(Y)$

but in dpt. we can't write as above

$$= \frac{1}{n}\sum_{i=1}^{n} y_i^2 - \frac{1}{n(n-1)}\sum_{i\neq j=1}^{n} y_i y_j \quad \text{——— (A)}$$

$$E(s^2) = \frac{1}{n} E\left[\sum_{i=1}^{n} y_i^2\right] - \frac{1}{n(n-1)} E\left[\sum_{i\neq j=1}^{n} y_i y_j\right]$$

we have

$$E\left[\sum_{i=1}^{n} y_i^2\right] = E\left[\sum_{i=1}^{N} a_i Y_i^2\right]$$

$$= \sum_{i=1}^{N} E(a_i) Y_i^2$$

$$= \sum_{i=1}^{N} \frac{n}{N} Y_i^2$$

$$= \frac{n}{N}\sum_{i=1}^{N} Y_i^2 \quad \text{——— (2)}$$

and

$$E\left[\sum_{i\neq j=1}^{n} y_i y_j\right] = E\left[\sum_{i\neq j=1}^{N} a_i a_j Y_i Y_j\right]$$

$$= \sum_{i\neq j=1}^{N} E(a_i a_j) Y_i Y_j$$

Now

$$E(a_i a_j) = 1 \cdot P(a_i a_j = 1) + 0 \cdot P(a_i a_j = 0)$$

$$= 1 \cdot P(a_i = 1 \cap a_j = 1)$$

$\because P(A \cap B) = P(A) \cdot P(B/A)$

$$= P(a_i = 1) \cdot P(a_j = 1 \mid a_i = 1)$$

$$= \frac{n}{N}\left(\frac{n-1}{N-1}\right)$$

Here prob. of a_i

$P(a_i = 1) = P$[i^{th} unit is included in the sample of size n]

$= \frac{n}{N}$

and $P[a_j = 1 \mid a_i = 1]$

$= P$[j^{th} unit is included in the sample given that i^{th} unit is already included in the sample.]

$= \frac{n-1}{(N-1)}$

$$E\left[\sum_{i \neq j=1}^{n} y_i y_j\right] = \sum_{i \neq j=1}^{N} \frac{n(n-1)}{N(N-1)} Y_i Y_j$$

$$\boxed{= \frac{n(n-1)}{N(N-1)} \sum_{i \neq j=1}^{N} Y_i Y_j} \quad (3)$$

Substituting the value from (2) and (3) in eq^n (1), we get

$$E(s^2) = \frac{1}{n} \cdot \frac{n}{N} \sum_{i=1}^{N} Y_i^2 - \frac{1}{n(n-1)} \frac{n(n-1)}{N(N-1)} \cdot \sum_{i \neq j=1}^{N} Y_i Y_j$$

from (A)

$$= \frac{1}{N} \sum_{i=1}^{N} Y_i^2 - \frac{1}{N(N-1)} \sum_{i \neq j=1}^{N} Y_i Y_j$$

$$= \frac{1}{N} \Sigma Y_i^2 - \frac{1}{N(N-1)} \left[(\Sigma Y_i)^2 - \Sigma Y_i^2 \right] \quad \text{H/W}$$