Structural Analysis of Transport Network

Dr. Sabiha Khan

- Graph theory is a branch of combinational topology which provides us with proper language suitable for measurement and analysis of the structure of transportation networks (Kansky, 1963).
- Topological and graph theoretic indices are arrived at by simplifying the networks to their basic components, that is the number of points in the network and the lines connecting those points.
- Though different terminology has been used, points or nodes are now commonly referred to as vertices, and lines or routes as edges and the 'areas' are mentioned as 'regions' (chorley and Hagget 1969, Taaffe and Gauthier 1973).
- Network analysis has been done by the actual distance between the nodes or vertices, capacity of these linkages and angle between the linkages.

The three basic parameters, on which the indices based are

  - (i) sub-graphs
  - (ii) edges and
  - iii) vertices.

The graph theoretic measures applied in the study explains:

(1) The layout and geometry of the transportation networks through an analysis of individual elements and characteristics and

(2) Efficiency of the network structure and its comparability to measure the economic development.

Network analysis gives measures of accessibility and connectivity. And allows comparisons between networks.

The measures of network structure derived from graph theoretical concepts may be broadly classified into two groups:

- Non-ratio measures
- Ratio measures

Non – Ratio measure: A single number which describes the aggregate pattern of the network as a whole.

Cyclomatic number and diameter of network.

Ratio measures: a vector of numbers which measure the relationship between individual elements (Vertices or edges) and the whole network (R. Busachrr and T Saaty, 1965).

The first geographical application was made by Kansky (1963) who used various graph theoretic measures such as Alpha, Beta, Gamma, Cyclomatic number, Eta, Theta and pie index.

Applications of these measures of transport networks are following:

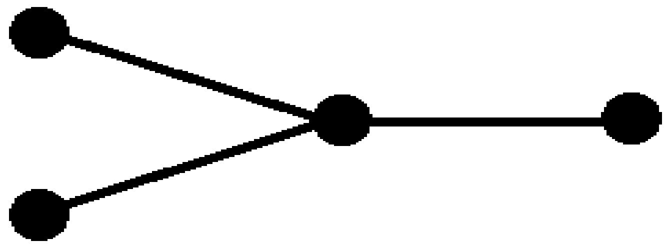
- The connectivity of networks
- The centrality within a network
- The spread and diameter of a network
- Detours

What is connectivity ?

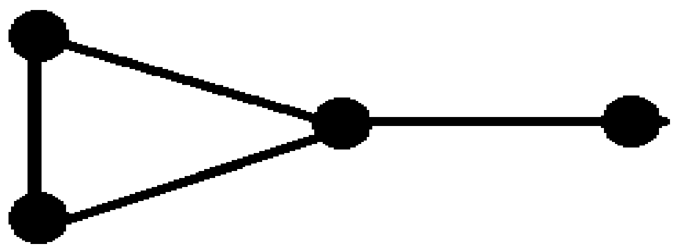
- “The connectivity of a network may be defined as the degree of completeness of the links between nodes” (Robinson and Bamford, 1978).
- “The degree of connection between all vertices is defined as the connectivity of the networks” (Taaffe & Gauthier).
  - The greater the degree of connectivity within a transportation network, the more efficient with that system be.
- Kansky (1963) developed several descriptive indices for measuring the connectivity of networks, i.e., beta, gamma, alpha indices and cyclomatic number.

Beta Index **(β)**

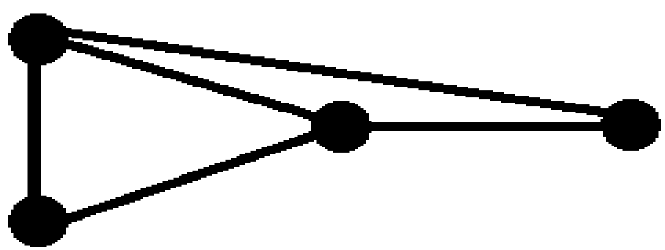
- The beta index is a very simple measure of connectivity, which can be found by dividing the total number of arcs in a network by the total number of nodes.
- β = arcs / nodes
- The beta index ranges from 0.0 (only nodes , no arcs) to 1.0 for a well connected network.
- Some characteristics of β index are:
  - β value for tree type and disconnected network would always be less than 1. it would be 0, when there is no edge in a network.
  - β value for any circuit network structure would always be equal to 1.
  - β value exceeds 1 for a complicated network structure having more than one circuit.
  - Beta value for planner graph ranges between 0 to 3.
  - Beta value for non- planner graph ranges between 0 to ∞.



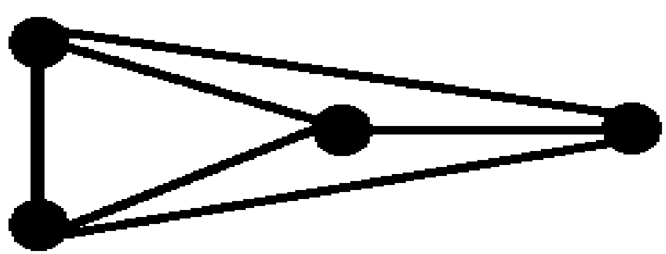
$v = 4, e = 3$
$\beta = e / v$
$= 3/4 = 0.75$



$v = 4, e = 4$
$\beta = e/v$
$= 4/4 = 1.0$



$v = 4, e = 5$
$\beta = e/v$
$= 5/4 = 1.25$



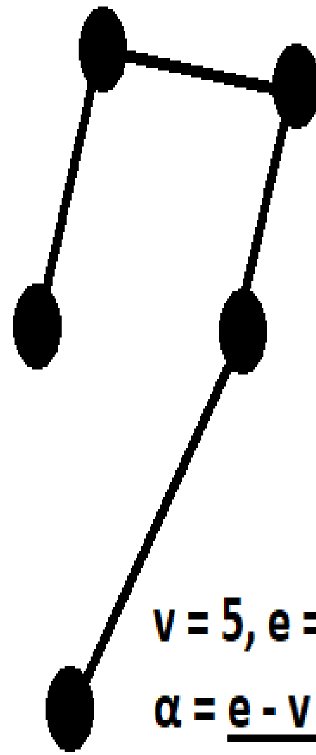
$v = 4, e = 6$
$\beta = e/v$
$= 6/4 = 1.50$

Alpha Index (α)

- Most useful measure of the connectivity of a network.
- It is the ratio between the actual number of circuits and maximum possible circuits within a network.
- α = actual circuits / maximum possible circuits.
- $$\boldsymbol{\alpha = \frac{e - v + g}{2v - 5}}$$
- The alpha index ranges from 0 to 1, that is 0 to 100 per cent.
- The higher the index value, the greater is the degree of connectivity in the network.

- Alpha index would be zero for non - connected graph or branched network (circuit absent).
- As the number of alternative linkage or route increased in minimum connectivity, circuit formed.
- Number of alternative routes = **e - v + g**
- Maximum number of connections = **2v – 5**
- $$\alpha = \frac{e - v + g}{2v - 5}$$

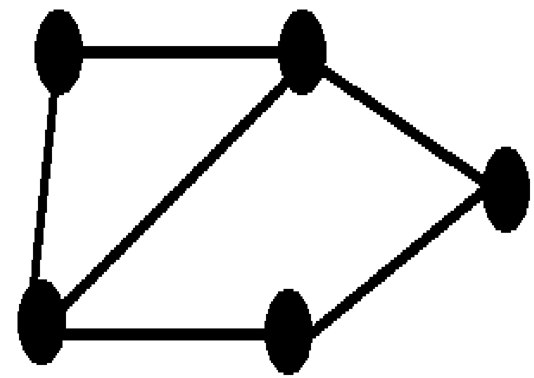
- These networks having the same number of vertices (v = 5), but different structures.
- The number of actual circuits is zero in first network and hence it takes the minimum value.
- There are two circuits formed in second network and hence the value is 0.40.



$v = 5, e = 4$

$$\alpha = \frac{e - v + g}{2v - 5}$$

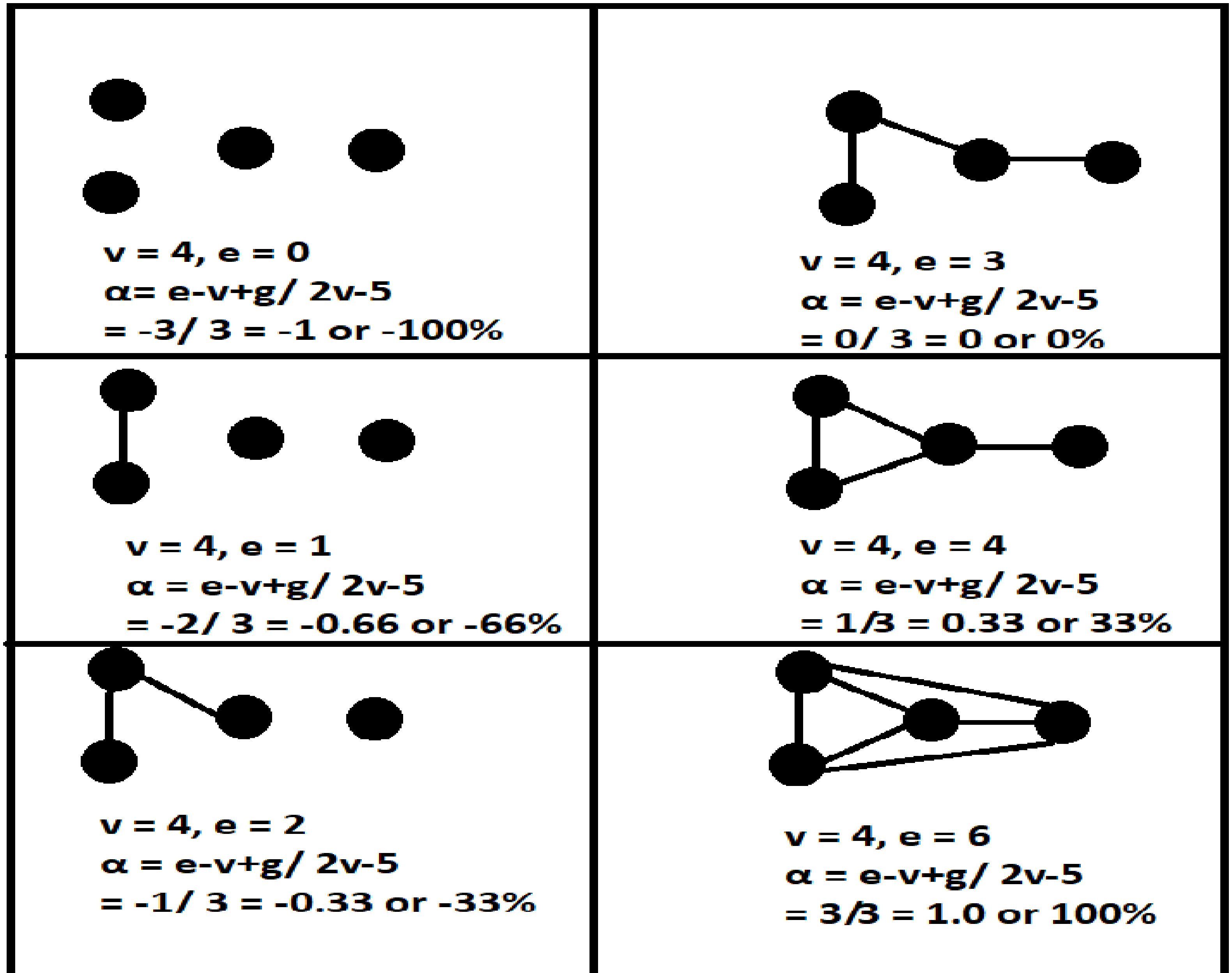
$\alpha = 0$



$v = 5, e = 6$

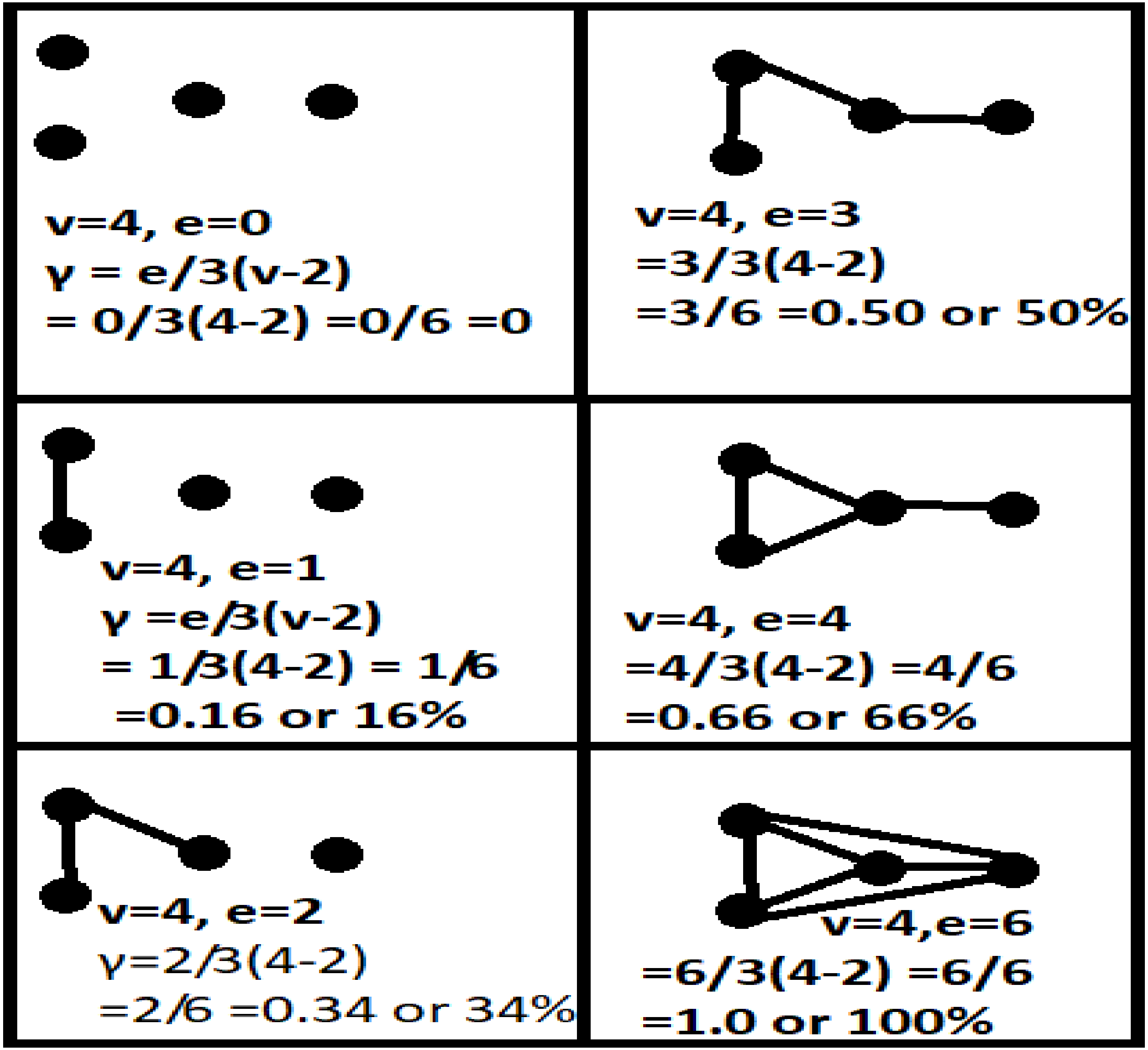
$$\alpha = \frac{e - v + g}{2v - 5}$$

$\alpha = 0.40$


v = 4, e = 0
α= e-v+g/ 2v-5
= -3/ 3 = -1 or -100%
v = 4, e = 3
α = e-v+g/ 2v-5
= 0/ 3 = 0 or 0%
v = 4, e = 1
α = e-v+g/ 2v-5
= -2/ 3 = -0.66 or -66%
v = 4, e = 4
α = e-v+g/ 2v-5
= 1/3 = 0.33 or 33%
v = 4, e = 2
α = e-v+g/ 2v-5
= -1/ 3 = -0.33 or -33%
v = 4, e = 6
α = e-v+g/ 2v-5
= 3/3 = 1.0 or 100%

Gamma Index (γ)

- It is a ratio between the observed number of edges and vertices of a given transport network.
- $\gamma = \dfrac{e}{3\,(v-2)}$ or e/e_{max}
- The numerical range for the gamma index is between 0 and 1 or 0 to 100 per cent.
- The connectivity varies from a set of nodes having no inter connection to the one in which every node has an edge connected to every other node in the graph.
- The inclusion of one additional node to a network of more than two nodes, increases the number of possible linkages by a value of three.

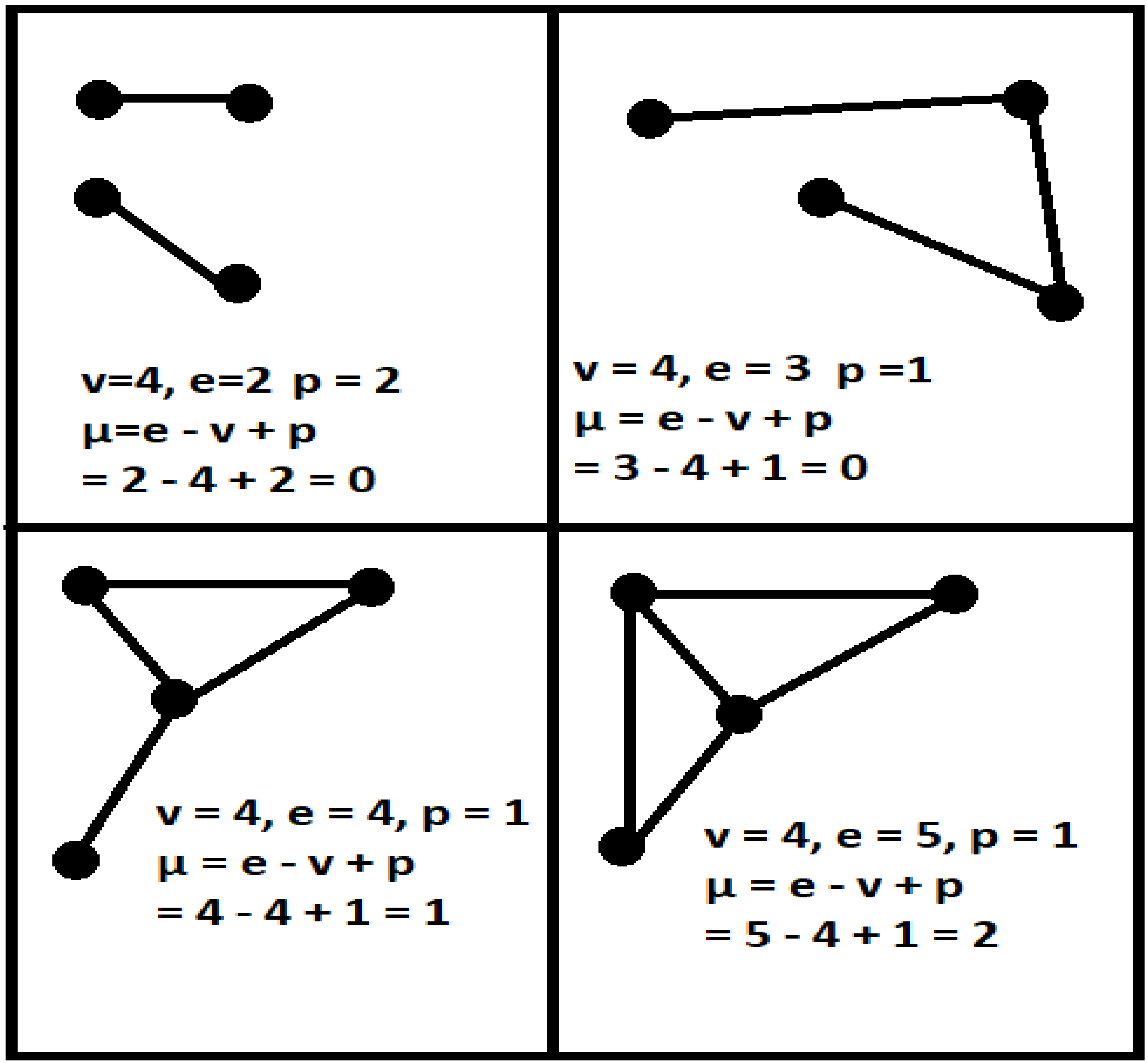


v=4, e=0
γ = e/3(v-2)
= 0/3(4-2) =0/6 =0
v=4, e=3
=3/3(4-2)
=3/6 =0.50 or 50%
v=4, e=1
γ =e/3(v-2)
= 1/3(4-2) = 1/6
=0.16 or 16%
v=4, e=4
=4/3(4-2) =4/6
=0.66 or 66%
v=4, e=2
γ=2/3(4-2)
=2/6 =0.34 or 34%
v=4,e=6
=6/3(4-2) =6/6
=1.0 or 100%

Cyclomatic Number (μ)

- This is different type of measuring index for connectivity.
- This is based upon the condition that as soon as a connected network has enough arcs or links to form a tree, then any additional arcs will result in the formation of circuits.
- Thus, the number of circuits in a connected network equals the total number of arcs minus the number of arcs required to form a tree, i.e., one less than the nodes or vertices. It may be written as:

- Cyclomatic number ($\boldsymbol{\mu}$) = a – (n – 1) or a – n + 1 or e – v + p
  - e = number of edges or arcs
  - v = number of vertices or nodes
  - p = number of non-connected sub graphs
- Where there happens to be two or more sub-graphs. Then, the formula for cyclomatic number is: ($\boldsymbol{\mu}$) = a – n + x
  - where x equals the number of subgraphs.


v=4, e=2 p = 2
μ=e - v + p
= 2 - 4 + 2 = 0
v = 4, e = 3 p =1
μ = e - v + p
= 3 - 4 + 1 = 0
v = 4, e = 4, p = 1
μ = e - v + p
= 4 - 4 + 1 = 1
v = 4, e = 5, p = 1
μ = e - v + p
= 5 - 4 + 1 = 2

- In a disconnected position or a tree type graph has zero cyclomatic number.
- As the graph move closer or closer to a connected state, then the cyclomatic number increases.
- The limitation of the cyclomatic number depends on the number of vertices and edges.

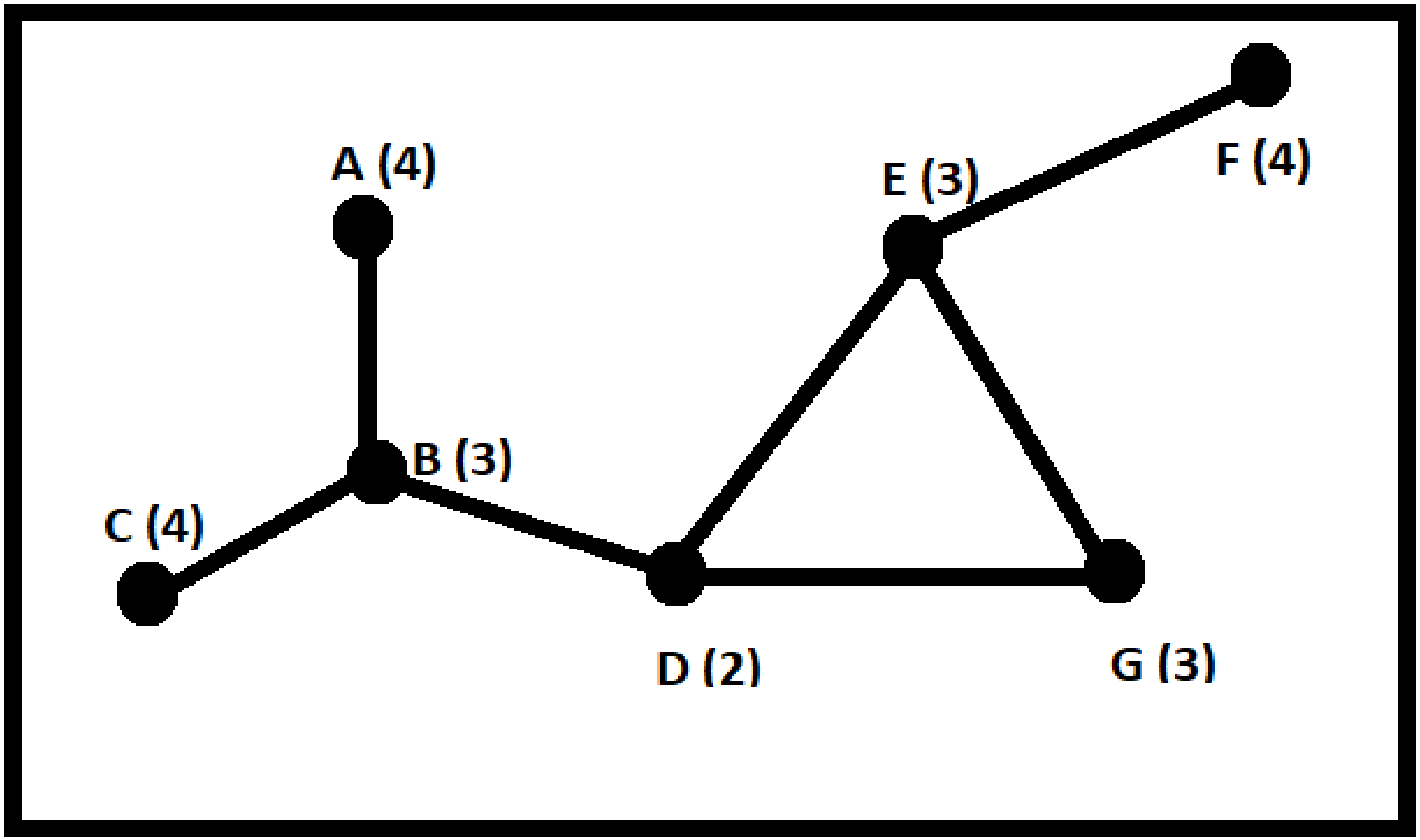
Centrality within a network

- D.Koning has developed an index known as koning index.
- This index describes the degree of centrality of any node on a network.
- Shortest path distance of a node with its most distanced node in a network.

or

- The koning index for each node is calculated by adding up the number of arcs from each other node using the shortest available path.

Koning Index (Associate Number)



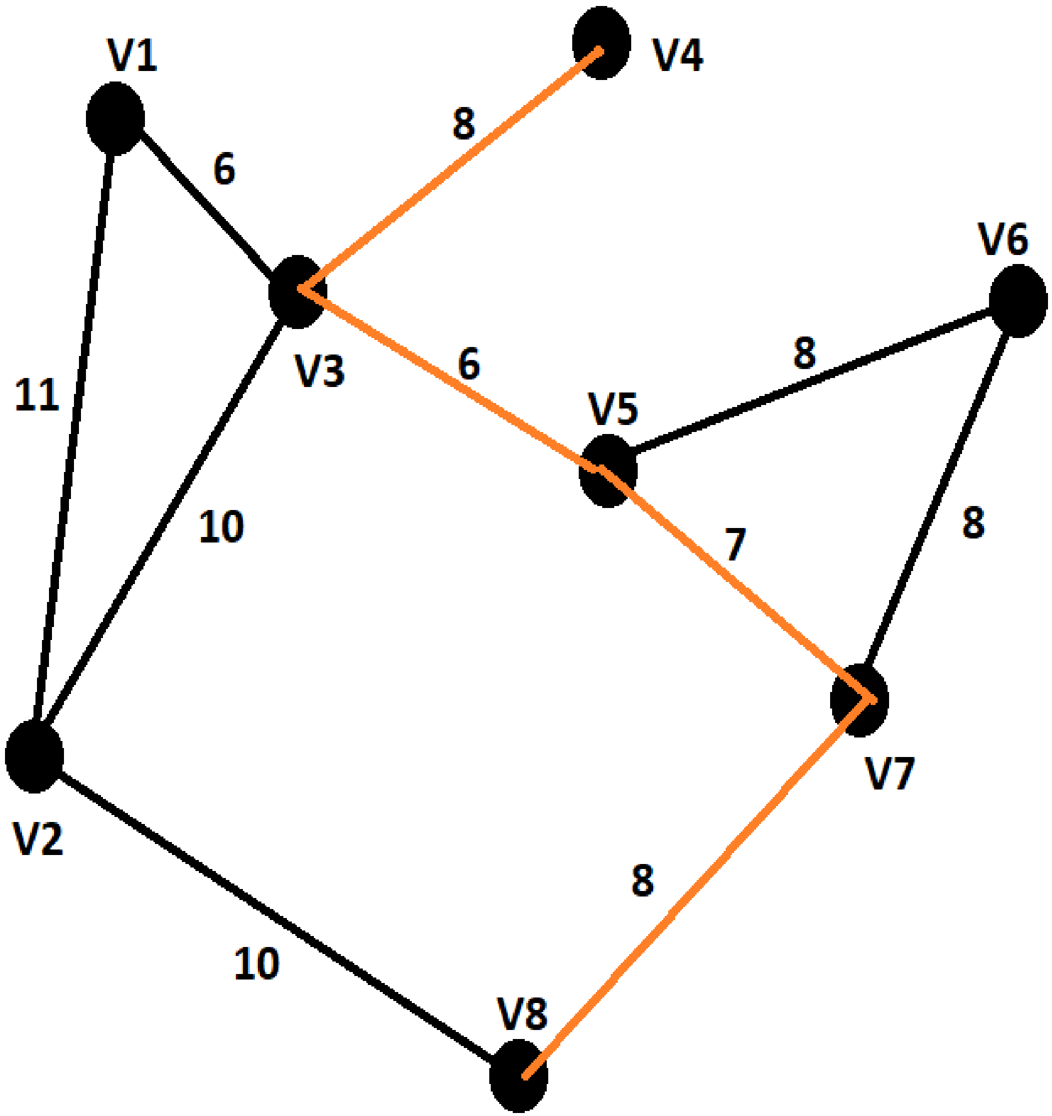
Spread and Diameter of Networks:

- There are two indices to measure diameter and spread of the network.
    - Pi index and
- Eta index.
- These indices were developed by Kansky.

Pi index (π):

- The relationship between the total length of the graph (C) and the distance along its diameter (d). A high index shows a developed network. It is a measure of distance per units of diameter and the indicator of the shape of the network.
- It is developed to investigate the relationship between the whole transportation network and its diameter.
- It is the ratio between the total length of the network and the length of the diameter.
- $\pi = \dfrac{\text{total distance of network}}{\text{distance of diameter}}$ or $\dfrac{c}{d}$

- Higher π value will be ascribed to more complicated networks and it would reflects the higher degree of development.



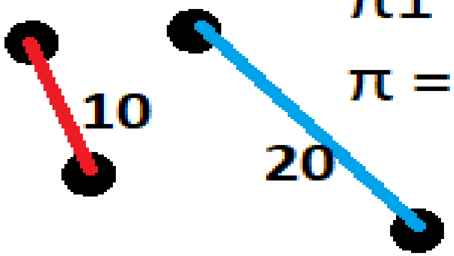
$$\pi = c/d$$

$$\pi = \frac{\underline{11+10+10+6+8+8+6+8+7+8}}{8+6+7+8}$$

$$\pi = 82/29 = 2.83$$

$c = 10$ km, $d = 10$ km

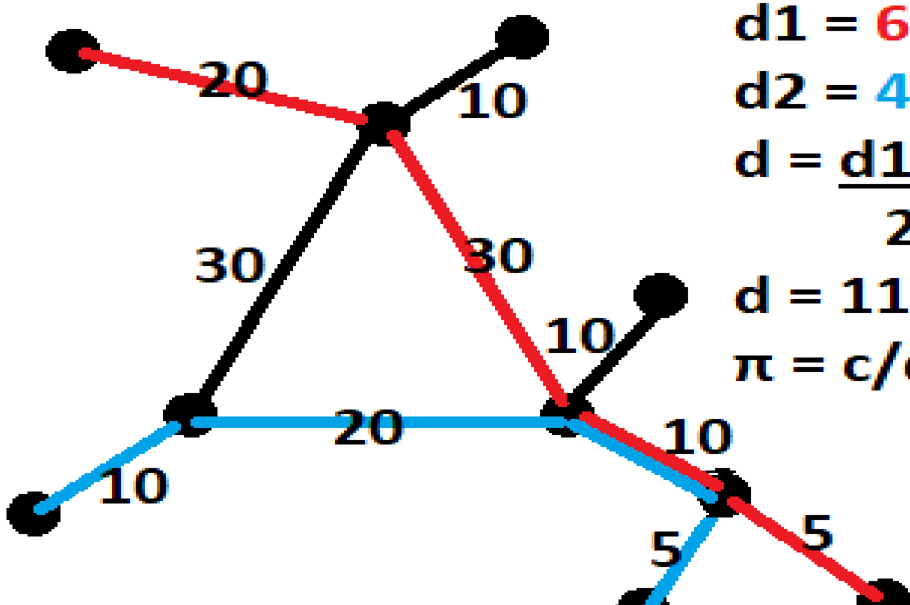
$\pi = c/d = 10/10 = 1.0$



$\pi 1 = 10/10 = 1.0$, $\pi 2 = 20/20 = 1.0$

$$\pi = \frac{\pi 1 + \pi 2}{\text{No. of subgraphs}} = \frac{1.0 + 1.0}{2}$$

$\pi = 1.0$



$c = 150$ kms

$d1 = 65$ kms

$d2 = 45$ kms

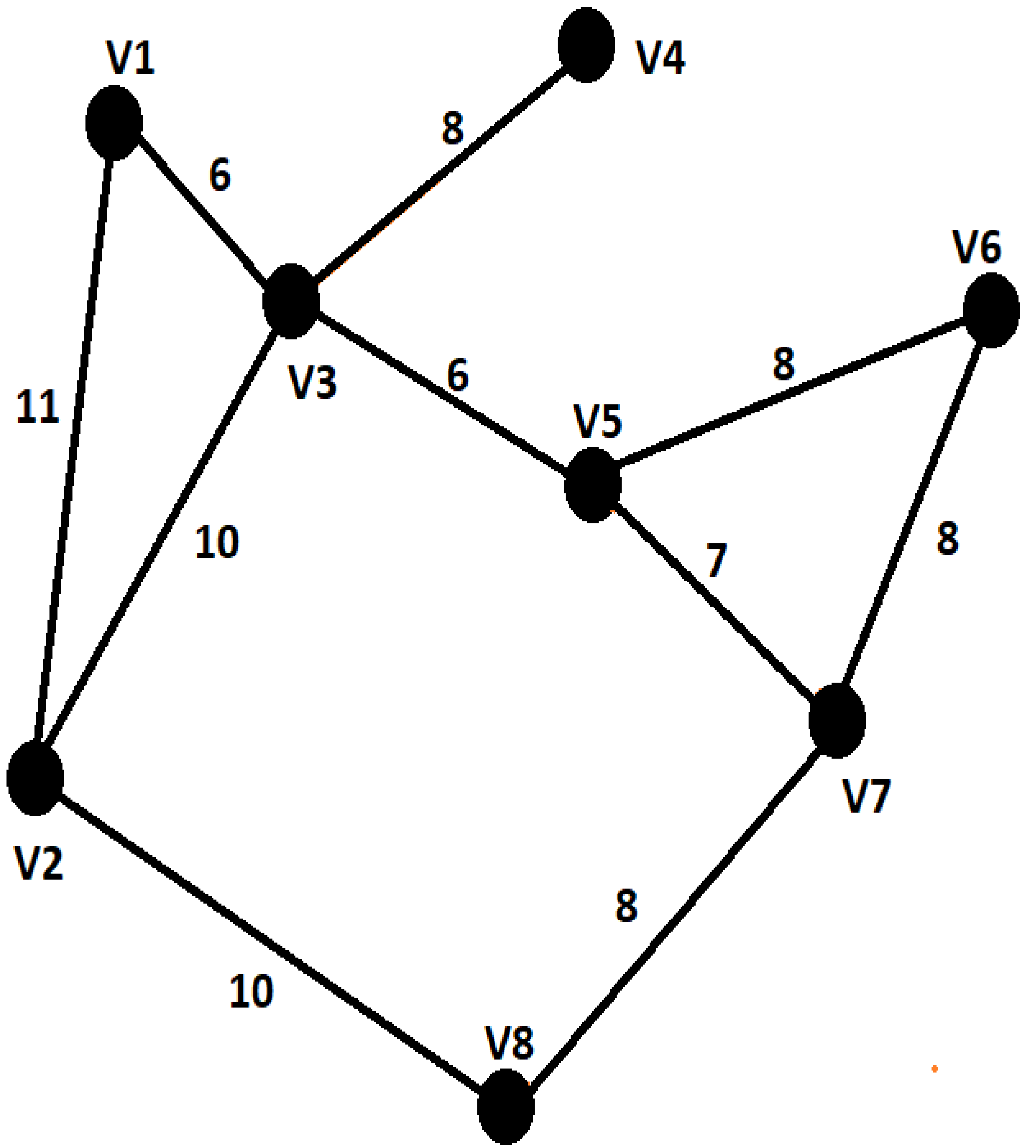
$$d = \frac{d1 + d2}{2} = \frac{65 + 45}{2}$$

$d = 110/2 = 55$

$\pi = c/d = 150/55 = 2.73$

Eta Index (η)

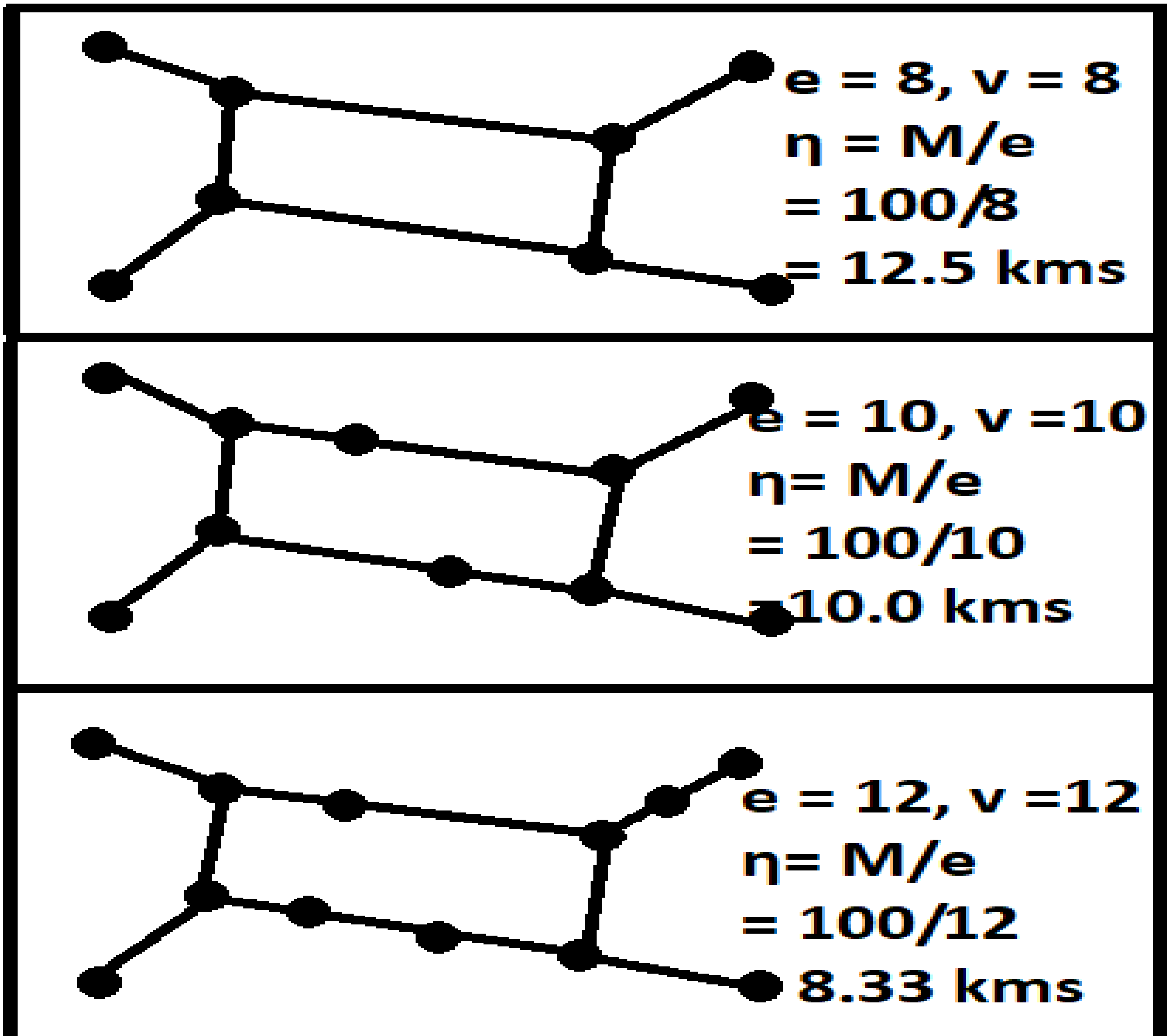
- The average length per link. This index used to find out the spread of the network.
- $\eta = \dfrac{\text{total distance of the network}}{\text{number of arcs}}$
- or M/E
- Eta is measure of average length of the linkages or edges in the given network.
- Adding new nodes will cause a decrease in eta as the average length per linkage declines.
- As the number of nodes or vertices add in the network, the value of eta decreases. Because increase in number of nodes increases the number of edges.



$$\eta = M/e$$

$$\eta = \frac{11+10+6+10+8+8+6+8+7+8}{10}$$

$$n = 82/10 = 8.2$$



e = 8, v = 8
η = M/e
= 100/8
= 12.5 kms
e = 10, v =10
η= M/e
= 100/10
10.0 kms
e = 12, v =12
η= M/e
= 100/12
8.33 kms

THANK YOU

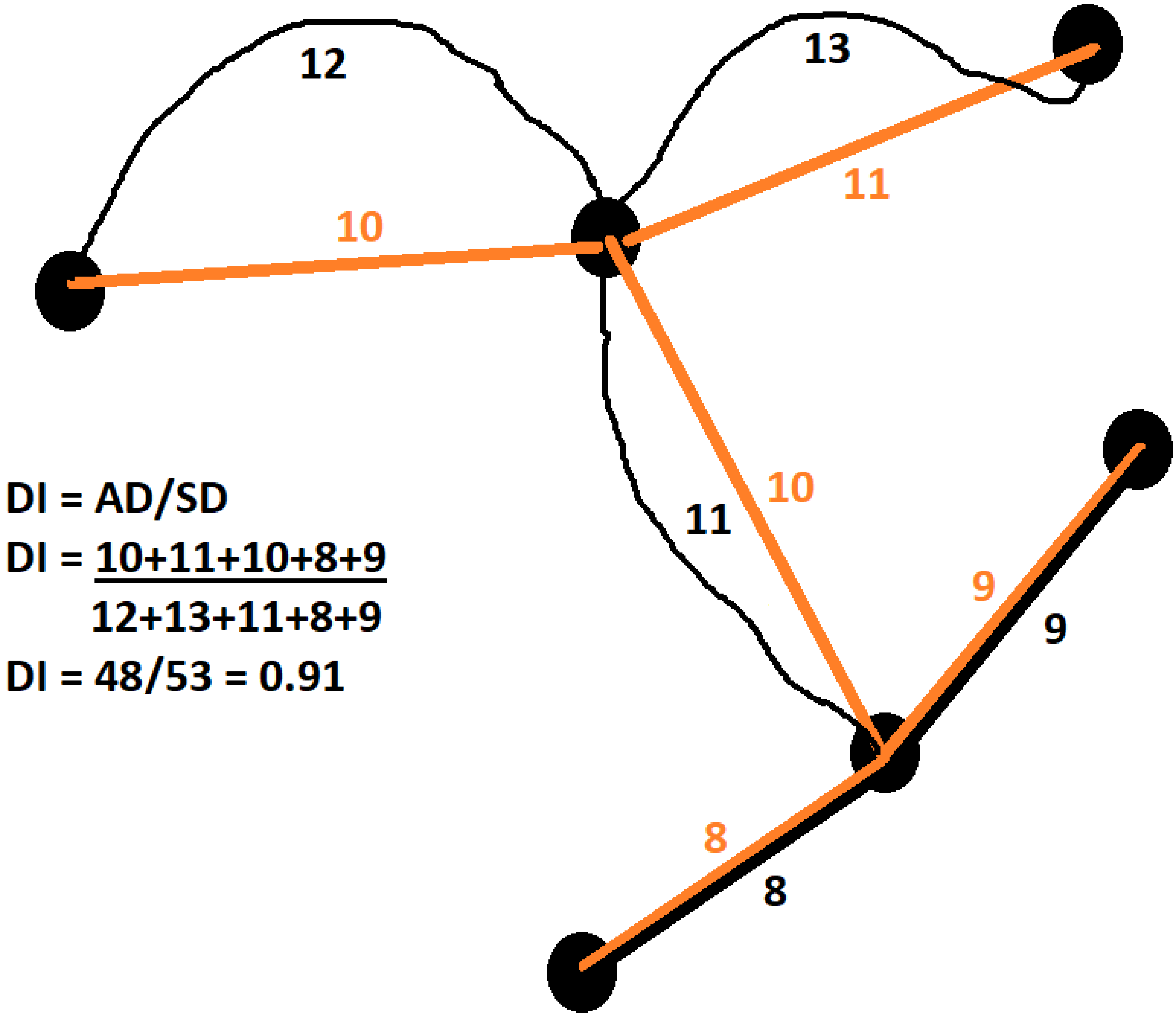
Detour Index:

- A measure of the efficiency of a network in terms of how well it overcomes distance or the friction of distance.
- The straight routes between two places two places or direct routes (also known as desire lines) are the routes, which travellers used to follow because of their shortest distance.
- But straight routes are rarely found in reality.
- Even the most direct route deviates from the straight line, because of physical or some other obstacle.
- The detour index is the actual journey distance calculated as a percentage of desire line distance.

- It is another measure to identify the efficiency of a transport network by reducing distance hurdles i.e. if an index scores 100, the network is considered more efficient. It is calculated as:

- Detour Index = $\frac{\text{actual route distance}}{\text{straight line distance}} \times 100$

- The detour index is the journey distance calculated as a percentage of desire line distance.
- The actual route distance is almost always greater than the desire line distance. In this case the value of detour index would be greater.
- The lower value of detour index shows the more direct is a given route.
- This index is used for assessing the effects, which the addition or abstraction of linkages produce in a given network.


12
13
10
11
10
11
9
9
8
8
DI = AD/SD
DI = 10+11+10+8+9 / 12+13+11+8+9
DI = 48/53 = 0.91

Circuitry

- Distance between measured route length and geometric distance between two places.
- Circuitry is a means of measuring both the routes and nodes on a network. Route circuitry is simply:
- $k_{ij} = \dfrac{l_{ij}-d_{ij}}{l_{ij}}$
- Where
  - k is the route circuitry,
  - l is the actual route length,
  - d is the straight line (geometric) distance, and
  - i and j are nodes.
- Basically this is a measure of the ratio difference between the actual route length and the geometric distance.

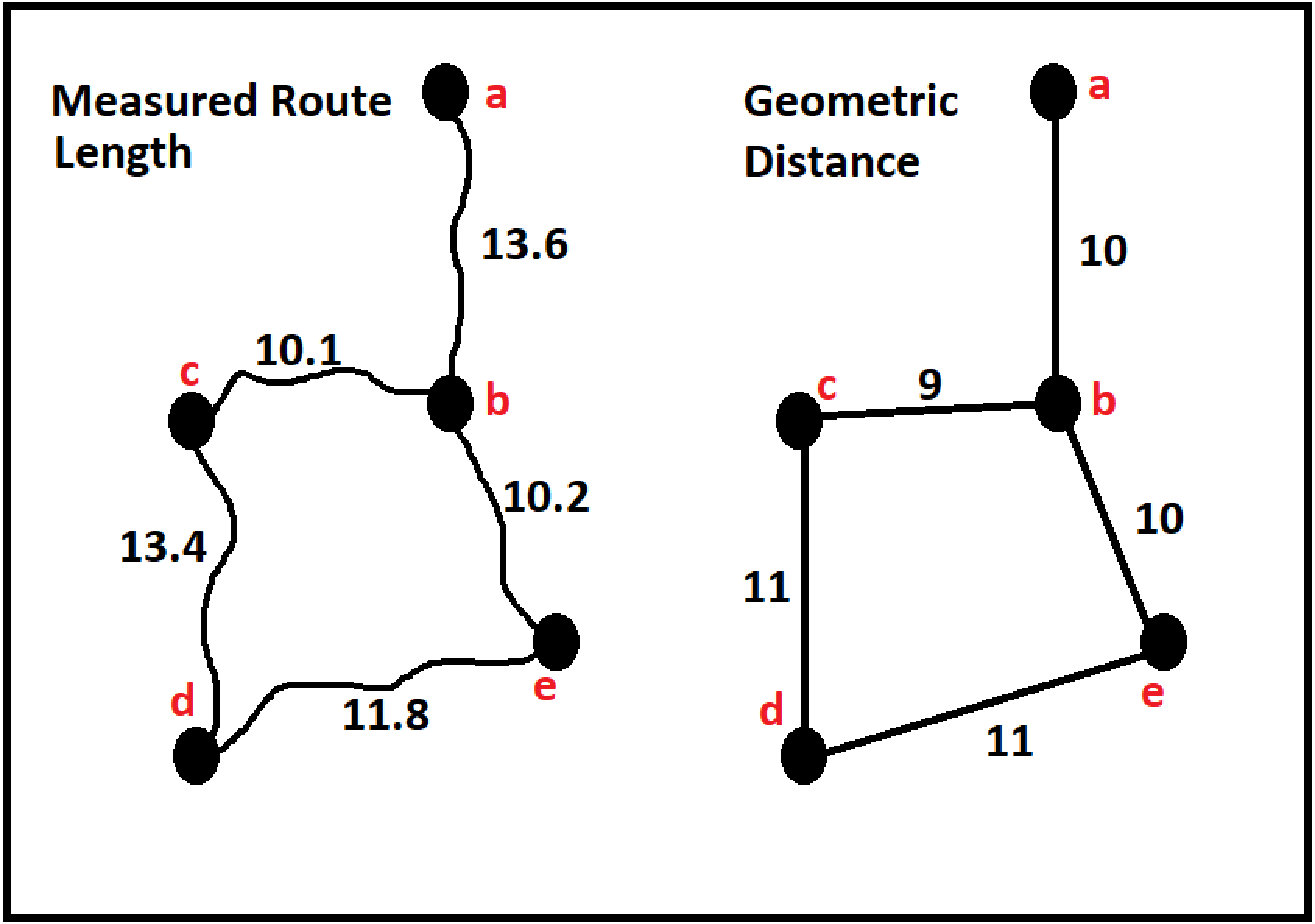
- There are many ways to measure node circuity, but the most common was developed by Kansky in 1963

$$\textit{Degree of Circuity}_{Node} = \frac{\sum_{j=1}^{n} (l_j - d_j)^2}{n^2 - 1}$$

- We can also measure the circuity for an entire network using the equation:

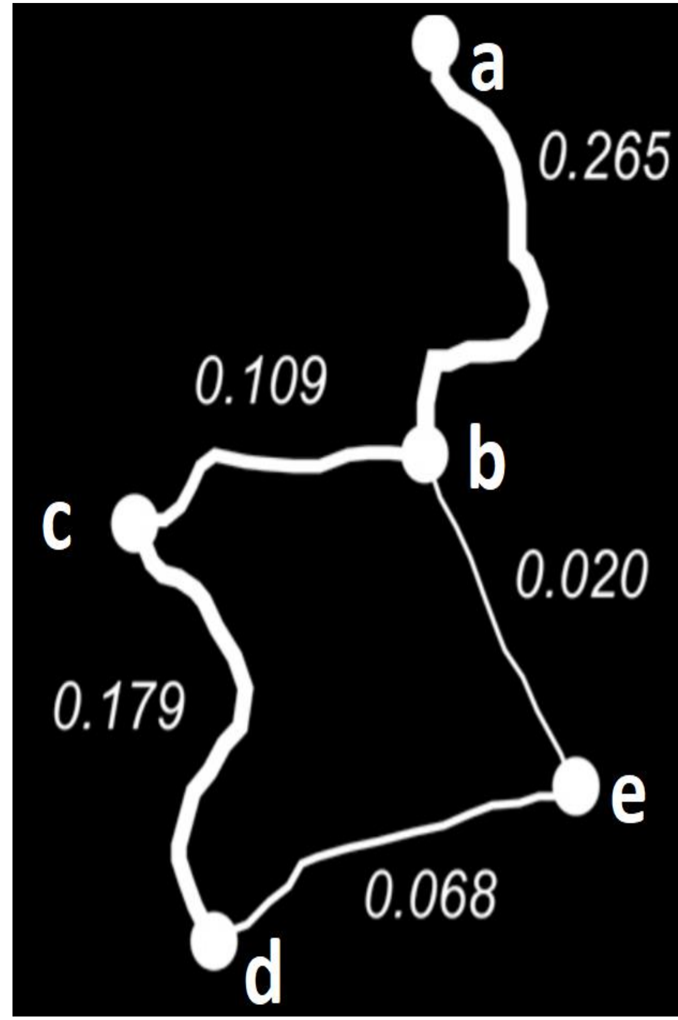
$$\textit{Degree of Circuity}_{Network} = \frac{\sum_{i=1}^{n} \sum_{j=1}^{n} (l_{ij} - d_{ij})^2}{n^2 - n}$$

- Circuitry is a measure of route efficiency, in that straighter routes are more efficient.
- Circuitry ranges between 0 to 1.
- Value approaches at 1 means, the route is very circuitous.
- But value never reaches to 1, since the potential difference between actual distance and geometric distance is infinite.
-


Measured Route Length
a
13.6
c
10.1
b
10.2
13.4
e
d
11.8
Geometric Distance
a
10
c
9
b
10
11
e
d
11

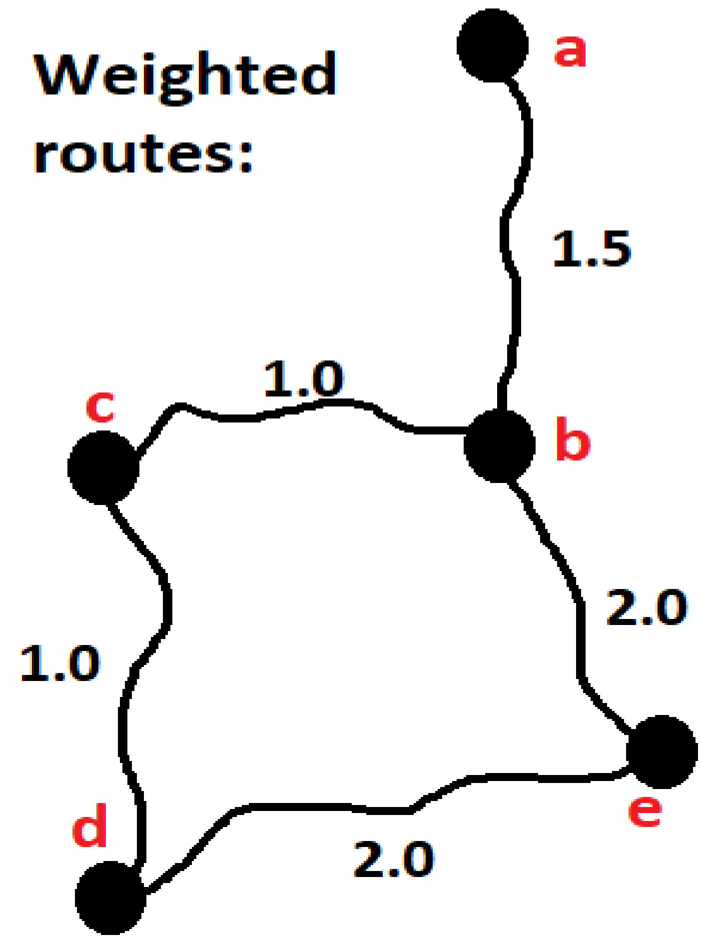
Non-Weighted Circuitry Index:

- $K_{ab} = \dfrac{l_{ab} - d_{ab}}{l_{ab}} = \dfrac{13.6 - 10}{13.6} = 0.265$
- $K_{bc} = \dfrac{l_{bc} - d_{bc}}{l_{bc}} = \dfrac{10.1 - 9}{10.1} = 0.265$
- $K_{be} = \dfrac{l_{be} - d_{be}}{l_{be}} = \dfrac{10.2 - 10}{10.2} = 0.020$
- $K_{cd} = \dfrac{l_{cd} - d_{cd}}{l_{cd}} = \dfrac{13.4 - 11}{13.4} = 0.179$
- $K_{de} = \dfrac{l_{de} - d_{de}}{l_{de}} = \dfrac{11.8 - 11}{11.8} = 0.068$



- Thus far we have treated all routes as being equal.
- This is often not the case.
- There are many factors which influence routes:
  – 1. Presence of street lights, stop signs, etc.
  – 2. Number of lanes.
  – 3. Speed limit.

Therefore it may be preferable to weight the routes based on some factor measurement.



Weighted Circuitry:

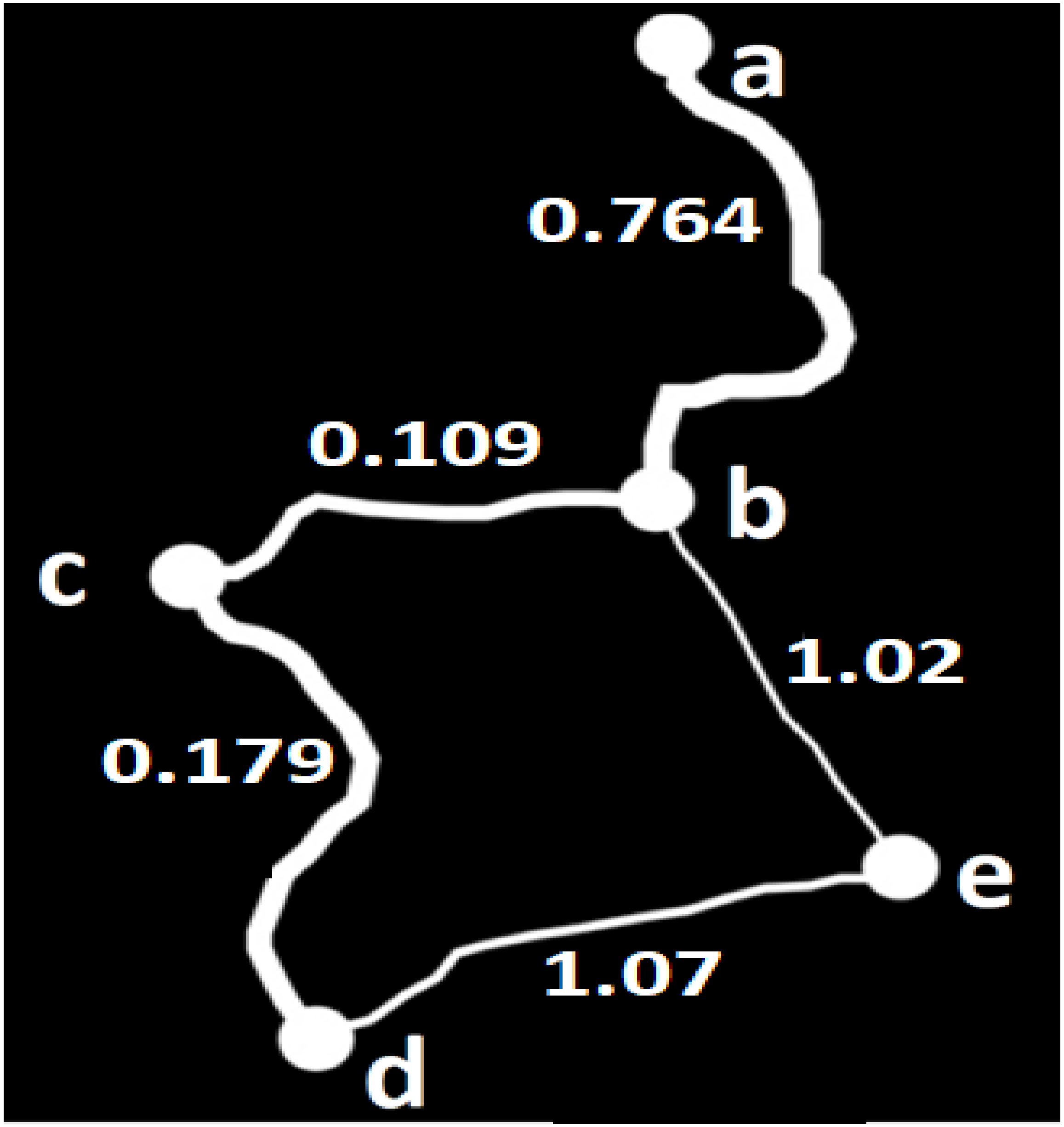
$$K_{ij} = \frac{((l_{ij} \times w_{ij}) - d_{ij})}{l_{ij}}$$

Weighted circuitry index ranges 0 to ∞.

Weighted Circuitry Index:

- $K_{ab} = \frac{((l_{ab} \times w_{ab}) - d_{ab})}{l_{ab}} = \frac{13.6 \times 1.5 - 10}{13.6} = 0.764$
- $K_{bc} = \frac{((l_{bc} \times w_{bc}) - d_{bc})}{l_{bc}} = \frac{10.1 \times 1 - 9}{10.1} = 0.109$
- $K_{be} = \frac{((l_{be} \times w_{be}) - d_{be})}{l_{be}} = \frac{10.2 \times 2.0 - 10}{10.2} = 1.02$
- $K_{cd} = \frac{((l_{cd} \times w_{cd}) - d_{cd})}{l_{cd}} = \frac{13.4 \times 1 - 11}{13.4} = 0.179$
- $K_{de} = \frac{((l_{de} \times w_{de}) - d_{de})}{l_{de}} = \frac{11.8 \times 2 - 11}{11.8} = 1.07$

Weighted Circuitry Index:



Degree of Circuitry:

- A node based measurement of the actual versus geometric distance summed from one node to all other nodes along the shortest route.

$$DC = \frac{\sum_{j=1}^{n} (E_j - d_j)^2}{n^2 - 1}$$

- Where E is the measured route length, d is the geometric distance of route j, and n is the number of vertices.

Degree of Circuitry:

$$DC\ c = \frac{(10.1-9)^2_{cb} + (13.4-11)^2_{cd} + (23.7-19)^2_{ca} + (20.3-19)^2_{ce}}{5^2 - 1}$$

DCc = 30.75/24 = 1.28

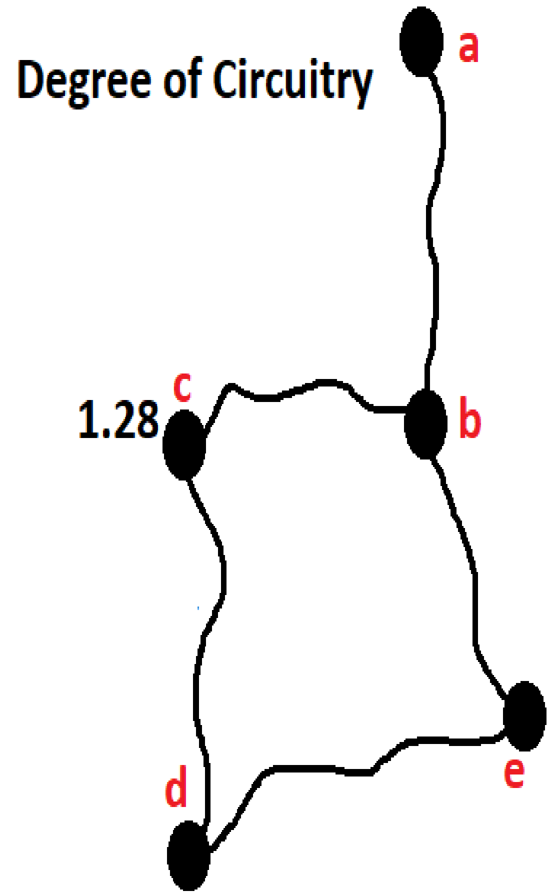
- **Calculate circuity of a node**:
- i.e. for node c
- From node c to b = $(10.1-9)^2 = (1.1)^2 = 1.21$
- From node c to d = $(13.4-11)^2 = (2.4)^2 = 5.76$
- From node c to a = $(23.7-19)^2 = (4.7)^2 = 22.09$
- From node c to e = $(20.3-19)^2 = (1.3)^2 = 1.69$

Sum:

=1.21 + 5.76 + 22.09 + 1.69 = 30.75

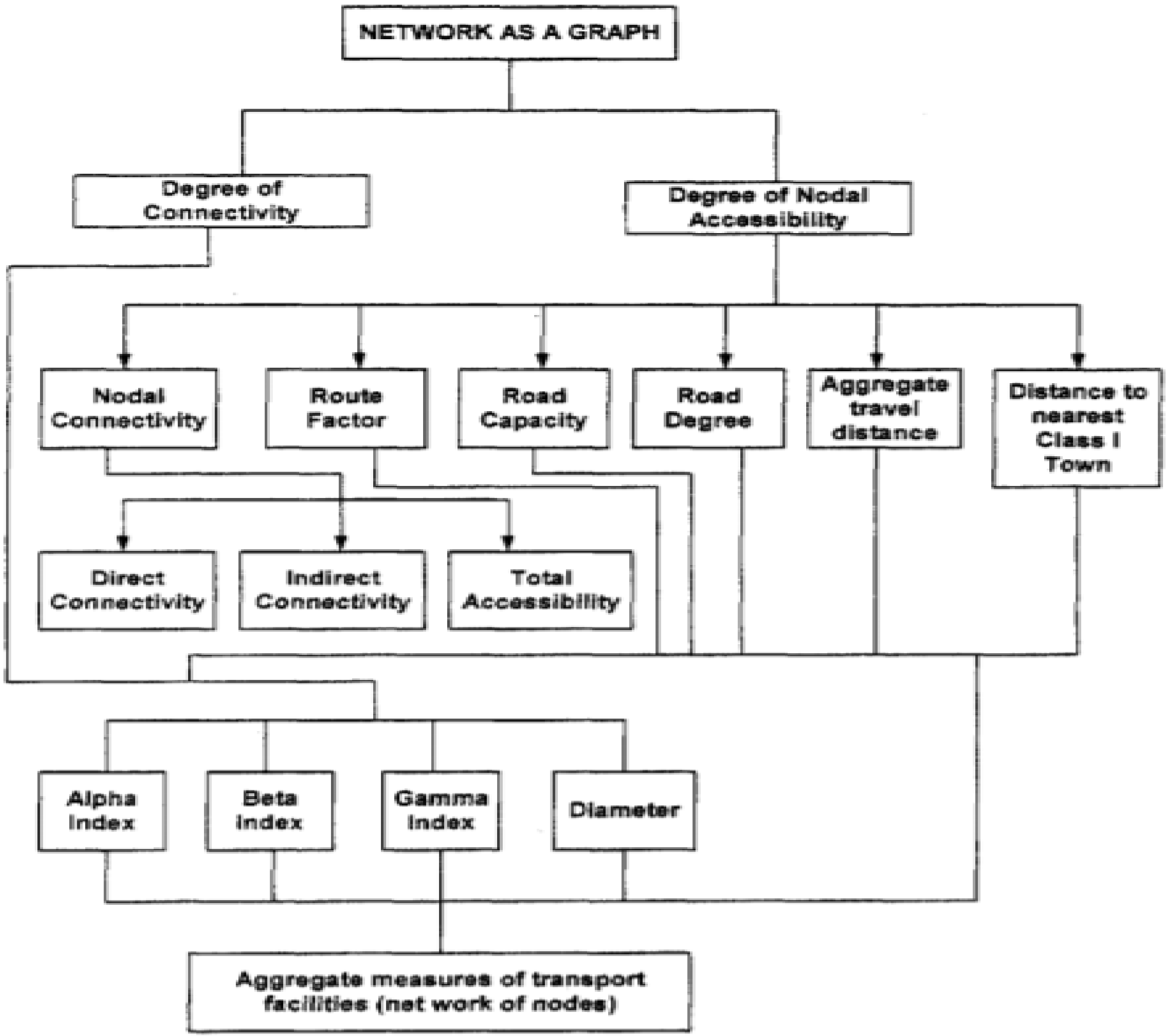
= 30.75/ 24 = 1.28

So, 1.28 is circuitry index of node c



Circuitry Matrix:

	a	b	c	d	e
a	-		22.09		
b		-	1.21		
c	22.09	1.21	-	5.76	1.69
d			5.76	-	
e			1.69		-
Column Total			30.75		



Source : Sukla Bhaduri, 1992

THANK YOU

Disclaimer: The content displayed in the PPT has been taken from variety of different websites and book sources. This study material has been created for the academic benefits of the students alone and I do not seek any personal advantage out of it.