Measures of Nodal Accessibility

By Shortest Path Matrix & Associate Number

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Accessibility can be measured in two ways:

- **(1*) By shortest path matrix (Shimbel Index or D Matrix-)***– the number of arcs used in the shortest path between all possible pairs (as shown in Figure 4.8)
- **(2) *By the associated number*** – the number of arcs needed to connect a node to the most distant node from it; and

1) By shortest path matrix:

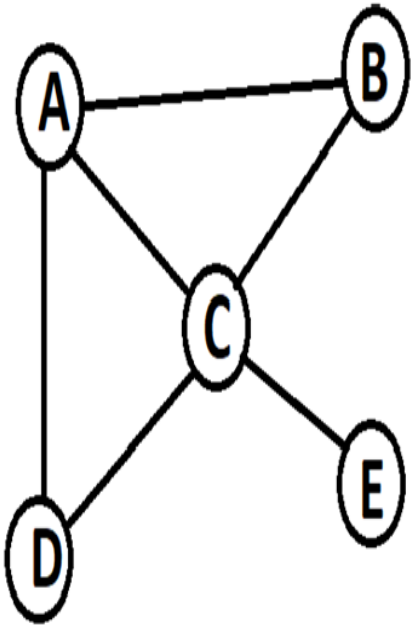
(Shimbel Index or D Matrix)

- In order to consider redundancies, Alfonso Shimbel developed a procedure through which it is possible to bring the length of the shortest route between any two places.
- In this approach, accessibility is computed via the distance between vertices (nodes).
- To accomplish this, Shimbel proposed the construction of a simple matrix (D) that would reveal the shortest path between all sets of nodes in a network.
- The Shimbel Index Equation: $\mathbf{Ai = \sum_{j=1} dij}$

- The Shimbel Distance Matrix (or D-Matrix) holds the shortest paths between the nodes of a network, which are always equal or lesser to the diameter.
- To construct this matrix, C matrices of Nth order are built until the diameter of the network is reached.
- Each C matrix is converted in a corresponding D matrix.
- In this case, two C matrices, C1(connectivity matrix) and C2 (two-linkages paths; C1*C1) are built since the diameter is 2.

The first order Shimbel Matrix (D1) is a simple adaptation of C1, where all the direct links are kept.

- A value of 0 is assigned for all the cii cells since the shortest path between a node and itself is always 0.
- Cells that have a value of 0 in the C1 matrix (outside cii cells) remain unfilled on the D1 matrix.



C1

	A	B	C	D	E
A	0	1	1	1	0
B	1	0	1	0	0
C	1	1	0	1	1
D	1	0	1	0	0
E	0	0	1	0	0

= 1

D1

	A	B	C	D	E
A	0	1	1	1	
B	1	0	1		
C	1	1	0	1	1
D	1		1	0	
E			1		0

The second order Shimbel Matrix (D2) is built from the first order matrix D1 but only from its unfilled cells.

- A value of 2 is assigned for each cells on the D2 matrix that have a value greater than 0 on the C2 matrix, but if a value of 1 already exists (D1 matrix), this value is kept.
- This means that on the D2 matrix, only the values of the yellow cells have been changed to 2. Since the diameter of this network is 2, the **D2 matrix is the Shimbel distance matrix**.

C2

	A	B	C	D	E
A	3	1	2	1	1
B	1	2	1	2	1
C	2	1	4	1	0
D	1	2	1	2	1
E	1	1	0	1	1

= 2

D2

	A	B	C	D	E
A	0	1	1	1	2
B	1	0	1	2	2
C	1	1	0	1	1
D	1	2	1	0	2
E	2	2	1	2	0

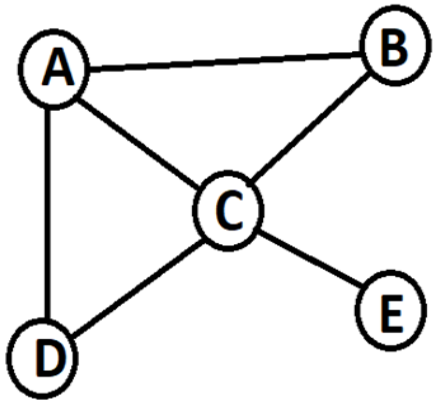
Nth order Shimbel Matrix (DN)

- For a network having a diameter of 3, a D3 matrix would have to be built from a C3 matrix (C1*C2) because at least 1 cell would have remained empty in the D2 matrix.
- Repeat the construction of Nth order Shimbel matrices until the diameter is reached.

The Shimbel Matrix (D)

- The order of the Shimbel distance matrix that corresponds to the diameter is the D matrix.
- The summation of rows or columns represents the Shimbel distance for each node.
- In the D matrix of the above example, node C is having the least summation of shortest paths (4) and is thus the most accessible, followed by node A (5), nodes B and D (6) and node E (7).
- The total summation of minimal paths is 28.

	A	B	C	D	E	Σ
A	0	1	1	1	2	5
B	1	0	1	2	2	6
C	1	1	0	1	1	4
D	1	2	1	0	2	6
E	2	2	1	2	0	7
Σ	5	6	4	6	7	28



C1

	A	B	C	D	E
A	0	1	1	1	0
B	1	0	1	0	0
C	1	1	0	1	1
D	1	0	1	0	0
E	0	0	1	0	0

= 1

D1

	A	B	C	D	E
A	0	1	1	1	
B	1	0	1		
C	1	1	0	1	1
D	1		1	0	
E			1		0

C2

	A	B	C	D	E
A	3	1	2	1	1
B	1	2	1	2	1
C	2	1	4	1	0
D	1	2	1	2	1
E	1	1	0	1	1

= 2

D2

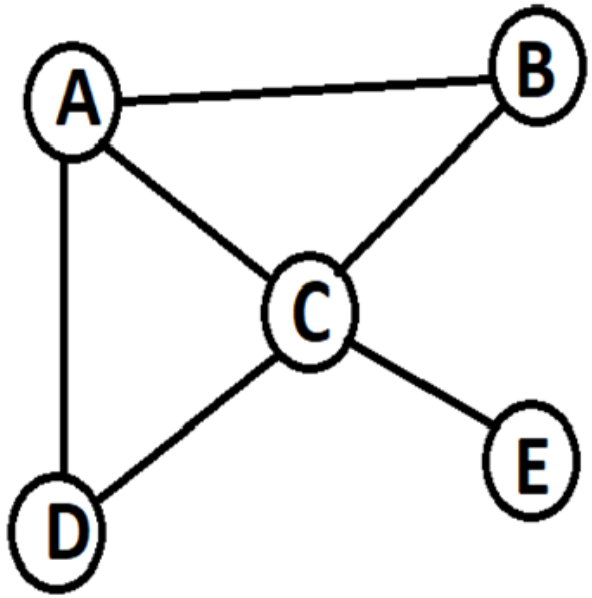
	A	B	C	D	E
A	0	1	1	1	2
B	1	0	1	2	2
C	1	1	0	1	1
D	1	2	1	0	2
E	2	2	1	2	0

Shortest Path Matrix (D Matrix)

	A	B	C	D	E	∑
A	0	1	1	1	2	5
B	1	0	1	2	2	6
C	1	1	0	1	1	4
D	1	2	1	0	2	6
E	2	2	1	2	0	7
∑	5	6	4	6	7	28

2) *By the associated number:*

The number of arcs needed to connect a node to the most distant node from it.



	A	B	C	D	E	Associate Number
A	0	1	1	1	2	2
B	1	0	1	2	2	2
C	1	1	0	1	1	1
D	1	2	1	0	2	2
E	2	2	1	2	0	2

THANK YOU