Allocation Model

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- This model determines the best possible or optimal pattern of flow in the existing network.
- Such models typically allocate flows of goods between producing and consuming areas in a way that minimizes transport costs.
- It is normative type of model, which determine an optimal pattern or solution for the given problem.
- For instance gravity model and Shimbel Index are also normative models.

The Transportation Model:

- It is most simplest and elementary model for the transportation problem related with spatial pattern of flow.
- It is a static flow analysis intended to allocate flows of goods between different geographic locations in terms of some efficiency criteria.

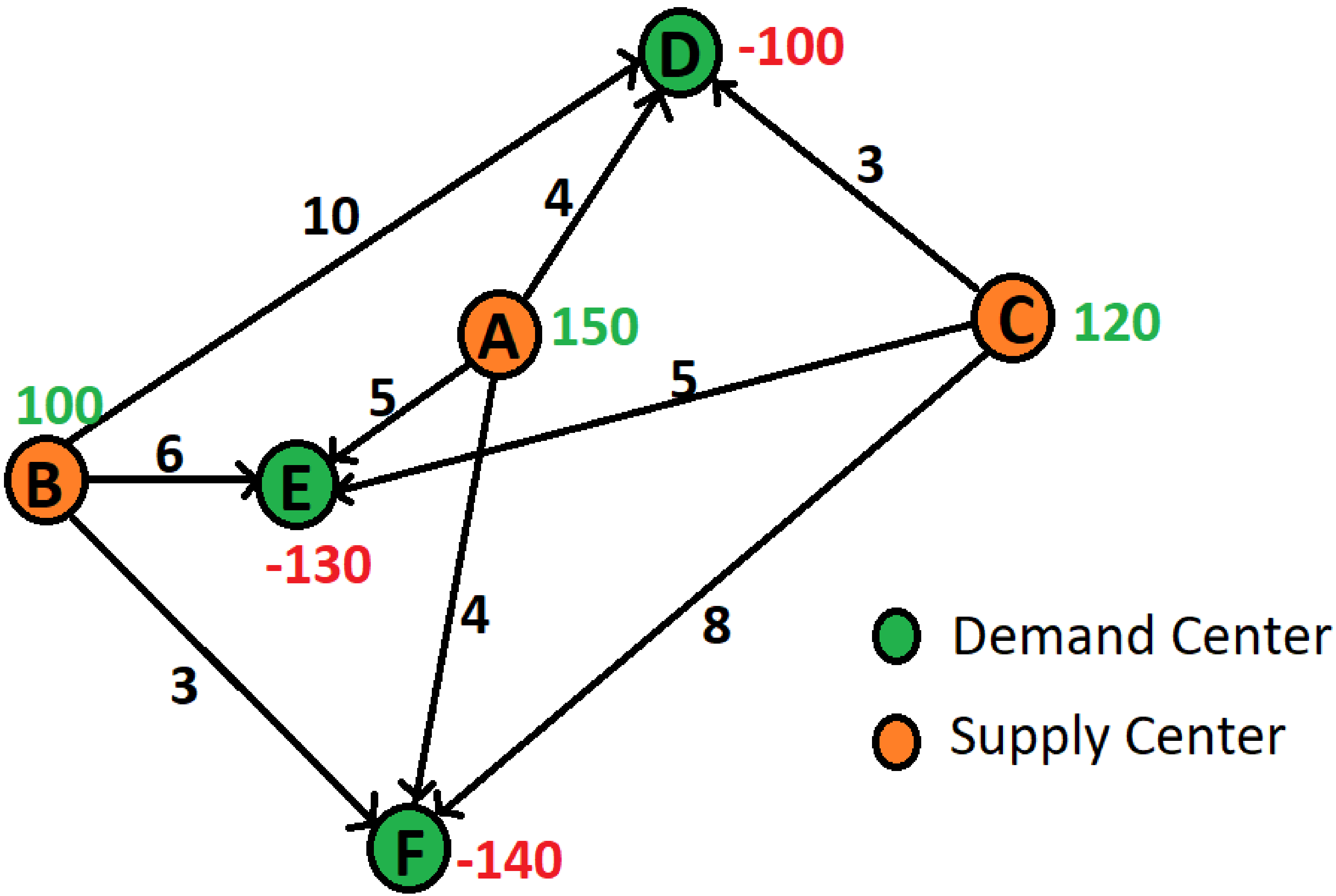
or

- Basically this model efforts to determine or allocate the spatial flow between the nodes /centers, according to their capacity (demand or supply) to minimize the cost of transportation.

There are few conditions for the implementation of the model:

- Competitive economy
- Uniformity of goods
- No effect of transportation on per unit of production of goods or products.
- Total demand should be equal to total supply.

A simple hypothetical example:



- We have to allocate optimal spatial flow between these supply and demand centers to minimize the transport cost.
- There are few constraints to schedule the shipments:
  - Warehouse must receive the required number of units, which can sell per day.
  - A factory cannot ship more units that it can produce daily.
- Several feasible shipping schedules can be derived that will satisfy these constraints.
- The objective of the problem to minimize the transportation cost.

In seeking a solution of the hypothetical transport problem, there are certain known factors:

1. The location of the supply centers (Factories) and demand centers (warehouses).

2. The per day production at the production/ supply center or factory.

3. The amount to be supplied to each warehouse/ demand center.

4. The cost of shipping/transportation per unit – weight of product from supply center (factory) to demand center (warehouse).

- All above information presented in the form of matrix.
- Rows present the supply centers (factory) and the columns present the demand centers (warehouse).
- Row total presents total production (surplus) and column total presents total demand(deficit).
- Known information (such as transport cost per km.)given on the margins of the matrix.
- There are 09 routes (X_{ij}) between these supply and demand centers.

- The allocation made from a given factory to the 3 warehouses cannot exceed this capacity.
- For example production capacity of **A factory** is 150 units; the allocation made from it to the D,E & F warehouses (flows x11,x12 & x13) must not exceeds 150 units.
- A column entry indicates the total demand of the individual warehouse.
- The allocation made from the three factories to a given warehouse must equal to this demand.
- i.e. the demand of **D warehouse**= 100 units. Therefore, allocation made from A,B &C factory to this D warehouse(flows x11,x21 & x31)must equal to 100 units.
- The sum of the supply capacity of factories is equal to the demand of warehouse.

		Warehouses			
	To / From	D	E	F	Surplus
Factory	A	4 x11	5 x12	4 x13	150
	B	10 x21	6 x22	3 x23	100
	C	3 x31	5 x32	8 x33	120
	Deficit	100	130	140	370

- After knowing the cost of shipping per unit weight of product from factory to warehouse.
- It is now necessary to determine the actual flow from the factory to warehouses.
- These flows are subject to certain constraints:

1. The shipments planned from each factory must not exceed the productivity capacity of that factory.
2. The shipments planned for each warehouse must equal its requirements.

- Many possible routings can meet these constraints.
- Our problem is to find the routing that accomplishes this objective with the least possible total transport cost.

A Feasible Solution:

- Let us begin by allocating production of the first factory/plant to the individual warehouses as cheaply as possible.
- Then allocate the surpluses of the other two plants to the warehouses in a manner that satisfies the constraint to balance production and demand.
- The matrix shows that the production of the **A plant** can be allocated either to the **D warehouse** or to the **F warehouse**- the per unit transport costs are equal…….120 units to **E warehouse**.(our constraint now force to use this route, even though it is not the lowest-cost route from C plant.)

		Warehouses			
	To / From	D	E	F	Surplus
Factory	A	4 100	5 0	4 50	150
	B	10 0	6 10	3 90	100
	C	3 0	5 120	8 0	120
	Deficit	100	130	140	370

- The above allocation is a feasible solution of the problem.
- Here, no plant/factory is supplying more than it can produce.
- The demand at each warehouse is being totally satisfied.
- It is feasible solution, but not be the optimal one.
- An alternative shipping schedule may exist, that will also satisfy the supply and demand requirements at an even lower transportation cost.
- The total transport cost in this initial allocation is determined by multiplying the amount shipped from a factory to a warehouse to a destination (warehouse) site by the per unit cost of the shipment.
- The total transportation cost = Rs. 1,530.

An Optimal solution:

- Primal transportation problem, allocation of shipments from supply areas to demand areas to minimize the transport costs.
- Dual transportation problem, having a maximum objectives. In this case maximum objective is the economic return to be realised from allocating the small consumer products from the supply areas to the demand areas.
- This dual component has important economic implications because it represents imputed values. Thereby using these values for the allocation of the shipments.
- Here, this dual component maximizes the difference in factory prices and warehouse price referred to **as Shadow prices.**
- If the flow is optimal on the basis of lowest transport cost, this dual component provides a set of equilibrium prices, which means that the difference between the price at the destination (warehouse) will be differ from the price at the origin (factory) by the amount of transportation between them.
- Trade in a particular commodity take place between two places (i & j), the transportation (cij) must be less than or equal to the price differential between the two places or **cij ≤pj-pi**. Now the whole system is in balance.

An Optimal solution:

- **Logically, there should be an allocation from the factory to the warehouse over any route for which the price differential is greater than the transport cost.**
- Here we used 5 of the 9 possible shipping routes. (3+3-1=5)
- Now determine the **opportunity costs** or the costs of not allocating shipments over the **unused routes (4)**.
- If the difference between factory price and delivered price at warehouse is less than the transportation cost between factory and warehouse, we would have incurred losses by using these unused routes, so the opportunity cost is negative. (TC > DP = -OP) or $C_{ij} > P_j - P_i$
- However, if the price differential between the factory and warehouse is higher than the transportation costs over an unused route, then we could have shipped over that route and made a profit; therefore positive opportunity cost exists in the system. (TC < DP = +OP) or $C_{ij} < P_j - P_i$
- An optimal solution is one in which all opportunity costs are negative. (on the unused routes)

- To determine these price differentials, arbitrarily assign the zero value to factory **A.**
- **S.P. at D=** T.C. between A & D + S.P at A
  - T.C. between A to D = Rs. 4/- unit
  - S.P. at A = 0
  - S.P. at D = 4 + 0 = 4

		Warehouses			
	v To / u From	4 D	7 E	4 F	Surplus
Factory	0 A	4 0	5 +2	4 0	150
	1 B	10 -7	6 0	3 0	100
	2 C	3 -1	5 0	8 -6	120
	Deficit	100	130	140	370

- **T.C.= Transportation Cost**
- **S.P.= Shadow Price**

- **S.P. at F** = T.C. between A & F + S.P at A
    - T.C. between A factory to F Warehouse = Rs. 4/- unit,
    - S.P at A = 0
    - S.P. at F = 4 + 0 = 4
- **S.P. at B** = S.P at F - T.C. between B & F
    - F warehouse get supply from B factory also so,
    - – T.C. between B & F = Rs. 3/- unit
    - – S.P. at F = 4
    - – S.P. at B = 4 – 3 = 1
- **S.P. at E** = T.C. between B & E + S.P at B
    - T.C. between B factory to E Warehouse = Rs. 6/- unit,
    - S.P at B = 1
    - S.P. at E = 6 + 1 = 7
- **S.P. at C** = S.P at E - T.C. between C & E
    - – T.C. between C & E = Rs. 5/-- unit
    - S.P at E = 7
    - – S.P. at B = 7 -5 = 2

- We used these calculated shadow prices to determine the opportunity costs for unused routes.
- **Calculate the opportunity cost between factory B and warehouse D.**
  - Shadow Price at B = 1
  - Shadow Price at D = 4
  - Difference between Shadow Prices = 4 – 1 = 3
  - Transport Cost between B and D = Rs. 10/- unit
  - 3 – 10 = -7
  - Here, T.C. > D.P. = Negative opportunity cost
  - No shipping allocated

		Warehouses			
	To / From	4 D	7 E	4 F	Surplus
Factory	0 A	4 0	5 +2	4 0	150
	1 B	10 -7	6 0	3 0	100
	2 C	3 -1	5 0	8 -6	120
	Deficit	100	130	140	370

Used Routes

- **Calculate the opportunity cost between factory C and warehouse D**.
  - Shadow Price at C = 2
  - Shadow Price at D = 4
  - Difference between Shadow Prices = 4 – 2 = 2
  - Transport Cost between C and D = Rs. 3/-unit
  - 2 – 3 = -1
  - Here, T.C. > D.P. = Negative opportunity cost
  - No shipping allocated
- **Calculate the opportunity cost between factory A and warehouse E.**
  - Shadow Price at A = 0
  - Shadow Price at E = 7
  - Difference between Shadow Prices = 7 – 0 = 7
  - Transport Cost between A and E = Rs. 5/- unit
  - 7 – 5 = +2
  - Here, T.C. < D.P. = Positive opportunity cost
  - So, shipping allocation on this unused route (between A and E) will be beneficial.

- Calculate the opportunity cost between factory C and warehouse F.
  - Shadow Price at C = 2
  - Shadow Price at F = 4
  - Difference between Shadow Prices = 4 – 2 = 2
  - Transport Cost between C and F = Rs. 8/-unit
  - 2 – 8 = -6
  - Here, T.C. > D.P. = Negative opportunity cost
  - No shipping allocated
- After the evaluation of the opportunity cost, we can reallocate the shipping routes and minimize the transportation cost.

THANK YOU

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