

MOHANLAL SUKHADIA UNIVERSITY, UDAIPUR
SECOND YEAR B. Sc. MATHEMATICS 2016-17

PAPER – III
MECHANICS

Duration: 3 Hours

Max. Marks: 75

UNIT – I

Equilibrium of bodies under three or more forces, Friction, common category.

UNIT – II

Virtual work, Projectile on inclined plane and Impact.

UNIT – III

Velocity and Accelerations (Tangential, normal, radial, transversal), Rectilinear motion, Hooke's law and motion of horizontal and vertical strings.

UNIT – IV

Constrained motion (circular and cycloidal), motion under resisting medium (resistance varies as velocity and square of velocity).

UNIT – V

Fluid pressure and thrust on immersed plane surfaces. Center of pressure.

References:

- | | |
|-----------------------|---|
| 1. S. L. Loney | : Statics, Macmillan and Company, London. |
| 2. R.S. Verma | : A Text book of Statics (Pothishala) |
| 3. Ray & Sharma | : A Text book of Hydrostatics |
| 4. N.Sharma | : A Text book of Dynamics. |
| 5. M Ray | : A Text book of Dynamics. |
| 6. Bhargava & Agrawal | : Gati Vigyan |
| 7. Gokhroo, Saini | : Uchch Gati Vigyan |
| 8. Gokhroo & Others | : Hydrostatics(Hindi Ed.) |
| 9. Gokhroo & Others | : Statics (Hindi Ed.) |
| 10. Bhargava & Others | : Hydrostatics (Hindi Ed.) |
| 11. Bhargava & Others | : Statics (Hindi Ed.) |

Unit - I Equilibrium

Q.1. Find the resultant of the forces 5 and 9 kg wt. acting at an angle 120° .

Sol: Given that $P = 5 \text{ kg}$, $Q = 9 \text{ kg}$, $\alpha = 120^\circ$
for Resultant

$$R^2 = P^2 + Q^2 + 2PQ \cos \alpha$$

$$= 25 + 81 + 2 \times 5 \times 9 \times \cos(120^\circ)$$

$$= 106 - 90 \times \frac{1}{2} \quad \because \cos(90^\circ + \theta) = -\sin \theta$$

$$= 61$$

$$R = \sqrt{61} \text{ kg}$$

for Direction $\theta = \tan^{-1} \left[\frac{Q \sin \alpha}{P + Q \cos \alpha} \right]$

$$= \tan^{-1} \left[\frac{9 \times \sin(120^\circ)}{5 + 9 \cos(120^\circ)} \right]$$

$$= \tan^{-1} \left[\frac{9\sqrt{3}/2}{5 - 9 \times \frac{1}{2}} \right]$$

$$\theta = \tan^{-1}(9\sqrt{3})$$

Q.2. Find the magnitude of two forces such that, if they act at right angle, their resultant is $\sqrt{10} \text{ kg wt}$ while when they act at an angle of 60° , their ~~resultant~~ resultant is $\sqrt{13} \text{ kg wt}$.

Sol: ~~Given that~~

Let forces are P & Q

If they act at right angle i.e. $\alpha = 90^\circ$

their resultant is $\sqrt{10} \text{ kg}$ i.e. $R = \sqrt{10}$

$$\text{by } R^2 = P^2 + Q^2 + 2PQ \cos \alpha \quad (1)$$

$$10 = P^2 + Q^2 \quad (2) \quad \because \cos 90^\circ = 0$$

If they act at an angle 60° i.e. $\alpha = 60^\circ$
resultant i.e. $R = \sqrt{13}$

by (1)

$$13 = P^2 + Q^2 + PQ \quad (3)$$

$$\because \cos 60^\circ = \frac{1}{2}$$

by (2) & (3)

$$PQ = 3 \quad (4)$$

by (2) & (4)

$$\boxed{P = 1 \text{ kg} \text{ \& } Q = 3 \text{ kg}}$$

$$P^2 + \frac{9}{P^2} = 10$$

$$\Rightarrow P^4 + 9 = 10P^2$$

$$\Rightarrow P^4 - 10P^2 + 9 = 0$$

$$\Rightarrow P^4 - P^2 - 9P^2 + 9 = 0$$

$$\Rightarrow P^2(P^2 - 1) - 9(P^2 - 1) = 0$$

$$\Rightarrow (P^2 - 9)(P^2 - 1) = 0$$

$$P = 1 \text{ or } 3$$

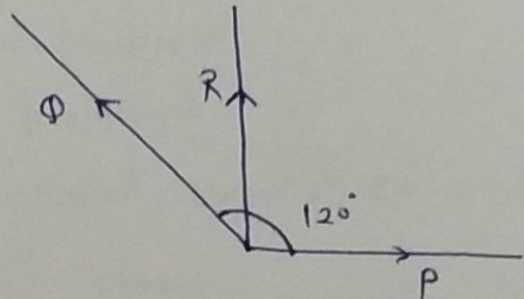
$$\text{Similarly } Q = 3 \text{ or } 1$$

$$\text{Hence } P = 1, Q = 3$$

Q.3 The angle between two forces is 120° , Greater force is of 80 kg wt. and their resultant is perpendicular to the smaller force, find the smaller force.

solⁿ Given $\alpha = 120^\circ$, $Q = 80$, $\theta = 90^\circ$, $P = ?$

$$\text{by } \theta = \tan^{-1} \left[\frac{Q \sin \alpha}{P + Q \cos \alpha} \right]$$



$$\Rightarrow \tan(90^\circ) = \frac{80 \sin(120^\circ)}{P + 80 \cos(120^\circ)}$$

$$\Rightarrow \infty = \frac{80 \times \frac{\sqrt{3}}{2}}{P - 80 \times \frac{1}{2}}$$

$$\Rightarrow \frac{1}{0} = \frac{40\sqrt{3}}{P - 40}$$

$$\Rightarrow P - 40 = 0 \quad (\text{by cross multiplication})$$

$$\Rightarrow P = 40 \text{ kg wt.}$$

Q.4 The resultant of two forces P and Q is of the magnitude P. show that if the force P be doubled, Q remaining unaltered, the new resultant will be at right angles to Q and its magnitude will be $\sqrt{4P^2 - Q^2}$.

solⁿ Given $R = P$

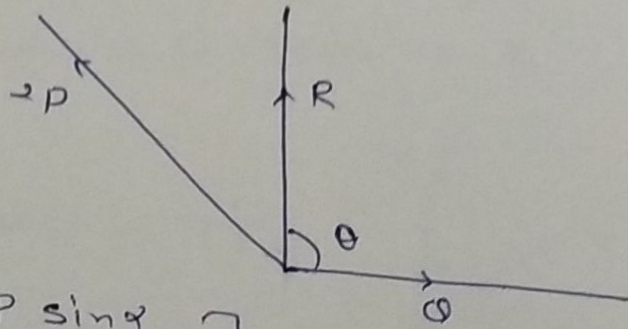
$$\text{then by } R^2 = P^2 + Q^2 + 2PQ \cos \alpha \quad \text{--- (1)}$$

$$\Rightarrow P^2 = P^2 + Q^2 + 2PQ \cos \alpha$$

$$\Rightarrow Q^2 + 2PQ \cos \alpha = 0 \Rightarrow Q + 2P \cos \alpha = 0 \quad \text{--- (2)}$$

If the force P be doubled then the resultant will ~~be~~ be at an angle θ with Q

ie



$$\text{by } \theta = \tan^{-1} \left[\frac{2P \sin \alpha}{Q + 2P \cos \alpha} \right]$$

$$\Rightarrow \tan \theta = \frac{2P \sin \alpha}{0} \quad \text{by (2)}$$

$$\Rightarrow \tan \theta = \infty$$

$$\Rightarrow \boxed{\theta = 90^\circ}$$

for Resultant by (1)

$$R^2 = 4P^2 + Q^2 + 2 \cdot 2P \cdot Q \cos \alpha$$

$$= 4P^2 + Q^2 + 4PQ \cdot \left(-\frac{Q}{2P}\right) \quad \text{by (2)}$$

$$= 4P^2 + Q^2 - 2Q^2$$

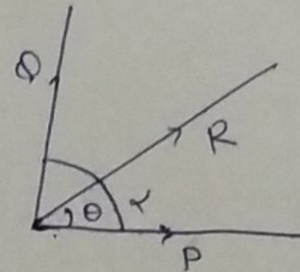
$$R = \sqrt{4P^2 - Q^2}$$

Q.5 Two forces P and Q act at a point and have resultant R . If Q be replaced by $(R^2 - P^2)/Q$ acting in the direction opposite to that of Q . Show that the resultant still remains R .

Sol.]

$$R^2 = P^2 + Q^2 + 2PQ \cos \alpha \quad \text{--- (1)}$$

$$\tan \theta = \frac{Q \sin \alpha}{P + Q \cos \alpha} \quad \text{--- (2)}$$



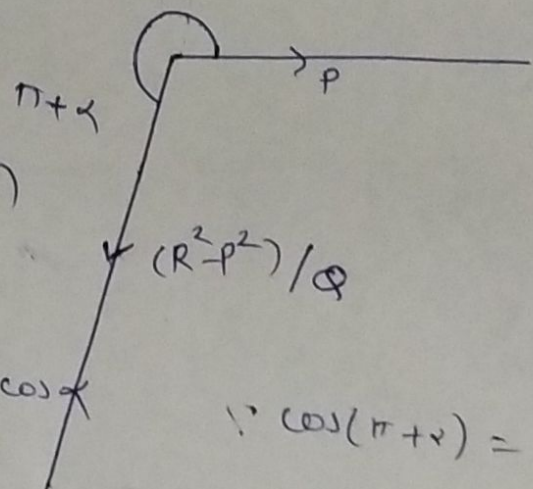
If Q be replaced by $(R^2 - P^2)/Q$ acting in the direction opposite to that of Q i.e. $\pi + \alpha$

New Resultant is R_1

$$R_1^2 = P^2 + \left(\frac{R^2 - P^2}{Q} \right)^2$$

$$+ 2 \cdot P \cdot \left(\frac{R^2 - P^2}{Q} \right) \cos(\pi + \alpha)$$

$$R_1^2 = P^2 + \frac{(R^2 - P^2)^2}{Q^2} - \frac{2P}{Q} (R^2 - P^2) \cos \alpha$$



$$\because \cos(\pi + \alpha) = -\cos \alpha$$

$$R_1^2 = P^2 + \frac{(Q^2 + 2PQ \cos \alpha)^2}{Q^2} - \frac{2P}{Q} (Q^2 + 2PQ \cos \alpha) \cdot \cos \alpha \quad \text{by (1)}$$

$$= P^2 + (Q + 2P \cos \alpha)^2 - 2P(Q + 2P \cos \alpha) \cos \alpha$$

$$= P^2 + Q^2 + 4P^2 \cos^2 \alpha + 4PQ \cos \alpha - 2PQ \cos \alpha - 4P^2 \cos^2 \alpha$$

$$= P^2 + Q^2 + 2PQ \cos \alpha$$

$$= R^2$$

$\Rightarrow R_1 = R$ Hence resultant still remains R .

Resolution of forces : To find the component of given force in ~~concrete~~ ^{given} direction, called Resolution of forces.

If components of force are perpendicular to each other then Resolution is called rectangular, otherwise its called oblique resolution.

Q:1. Find the components of a given force in two given directions.

So! Let given force is F acting at O in direction OC and OA, OB are two given directions which make an angles α & β respectively with OC .

Let components of force in direction $OA \perp OB$ are P & Q resp.

then by sine formulae

$$\frac{OM}{\sin \beta} = \frac{ON}{\sin \alpha} = \frac{OC}{\sin[\pi - (\alpha + \beta)]}$$

$$\Rightarrow \frac{P}{\sin \beta} = \frac{Q}{\sin \alpha} = \frac{F}{\sin(\alpha + \beta)}$$

$$\because \sin(\pi - \theta) = \sin \theta$$

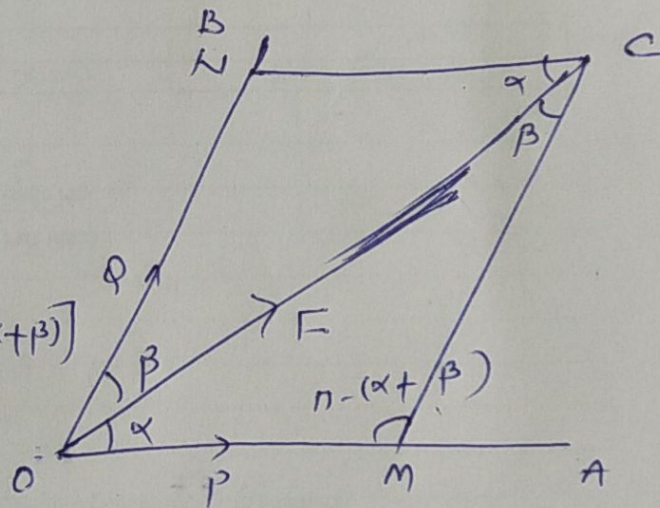
$$\Rightarrow P = \frac{F \sin \beta}{\sin(\alpha + \beta)}$$

$$Q = \frac{F \sin \alpha}{\sin(\alpha + \beta)}$$

If $P \perp Q$ then $\alpha + \beta = \pi/2$

$$\therefore P = F \sin \beta = F \sin(\frac{\pi}{2} - \alpha) = F \cos \alpha$$

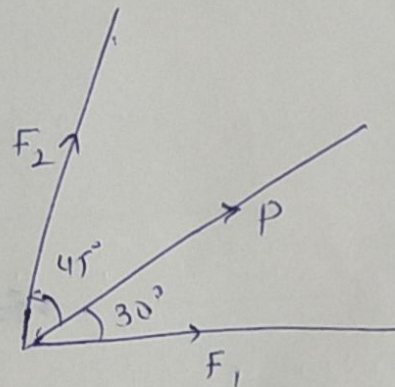
$$Q = F \sin \alpha$$



Q. 2 Find the components of the force P making angles 30° and 45° with it in opposite directions.

Sol. Given that force P making an angles 30° and 45° with it in opposite direction
i.e $\alpha = 30^\circ$, $\beta = 45^\circ$

by resolution of force



$$F_1 = P \frac{\sin 45^\circ}{\sin(30+45^\circ)}$$

$$= P \frac{\frac{1}{\sqrt{2}}}{\frac{\sqrt{3}+1}{2\sqrt{2}}}$$

$$= \frac{2P}{(\sqrt{3}+1)}$$

$$= \frac{2(\sqrt{3}-1)}{(\sqrt{3}+1)(\sqrt{3}-1)} P$$

$$= (\sqrt{3}-1)P$$

$$F_2 = P \frac{\sin 30^\circ}{\sin(30+45^\circ)}$$

$$P = P \cdot \frac{\frac{1}{2}}{\frac{\sqrt{3}+1}{2\sqrt{2}}}$$

$$= \frac{\sqrt{2}P}{\sqrt{3}+1}$$

$$= \frac{\sqrt{2}(\sqrt{3}-1)P}{(\sqrt{3}+1)(\sqrt{3}-1)}$$

$$= \frac{1}{2}(\sqrt{6}-\sqrt{2})P$$

$$\begin{aligned} & \sin(30+45^\circ) \\ &= \sin 30^\circ \cos 45^\circ \\ & \quad + \cos 30^\circ \sin 45^\circ \\ &= \frac{1}{2} \cdot \frac{1}{\sqrt{2}} + \frac{\sqrt{3}}{2} \cdot \frac{1}{\sqrt{2}} \\ &= \frac{\sqrt{3}+1}{2\sqrt{2}} \end{aligned}$$

Q.3 If the resultant of two forces acting on a particle be at right angles to one of them and its magnitude be one third of the other, show that the ratio of the larger force to the smaller force is $3:2\sqrt{2}$.

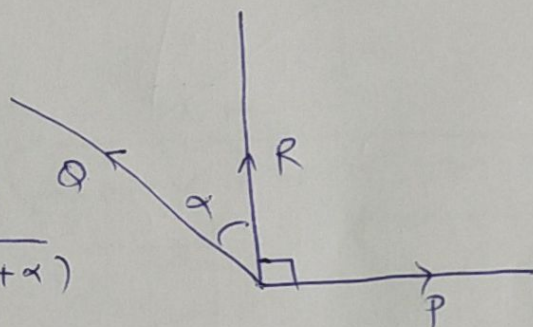
Sol: Let R is the resultant of forces P & Q .

R makes ~~at~~ ~~or~~ a right angle with P and α with Q

also ~~and~~ $R = \frac{1}{3}Q$

by Resolution of force

$$P = \frac{R \sin \alpha}{\sin(90^\circ + \alpha)}, \quad Q = \frac{R \sin 90^\circ}{\sin(90^\circ + \alpha)}$$



$$P = \frac{R \sin \alpha}{\cos \alpha} \quad (1), \quad Q = \frac{R}{\cos \alpha}$$

$$\frac{P}{Q} = \sin \alpha, \quad Q = \frac{1}{3}Q$$

$$\Rightarrow \cos \alpha = \frac{1}{3}, \quad \sin \alpha = \sqrt{1 - \cos^2 \alpha} = \frac{2\sqrt{2}}{3}$$

by (1)

$$P = \frac{1}{3}Q \cdot \frac{2\sqrt{2}}{3}$$

$$\frac{P}{Q} = \frac{2\sqrt{2}}{3}$$

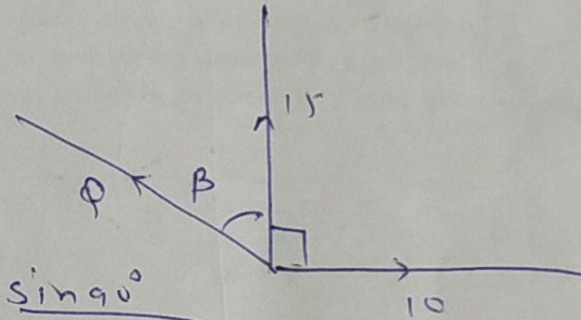
Hence $Q:P = 3:2\sqrt{2}$

Q.4 A force of 15 kg wt. acting vertically upwards has been resolved into two components. If its component in the horizontal direction is of 10 kg wt then find the magnitude and direction of the component.

sol. as direction in question

$$F = 15, \quad \alpha = 90^\circ, \quad P = 10, \quad \beta = ?, \quad \phi = ?$$

by resolution of force



$$10 = \frac{15 \cdot \sin \beta}{\sin(90^\circ + \beta)}, \quad \phi = \frac{15 \cdot \sin 90^\circ}{\sin(90^\circ + \beta)}$$

$$\frac{2}{3} = \tan \beta$$

$$\boxed{\beta = \tan^{-1}\left(\frac{2}{3}\right)}$$

$$\begin{aligned} \cos \beta &= \frac{1}{\sin \beta} \\ &= \frac{1}{\sqrt{1 + \tan^2 \beta}} \end{aligned}$$

$$= \frac{3}{\sqrt{13}}$$

$$\phi = \frac{15}{\cos \beta}$$

$$\phi = \frac{15}{\frac{3}{\sqrt{13}}}$$

$$\boxed{\phi = 5\sqrt{13} \text{ kg wt.}}$$

Hence other component of force is $5\sqrt{13}$ kg wt acting in direction $\tan^{-1}\left(\frac{2}{3}\right)$ with vertical.

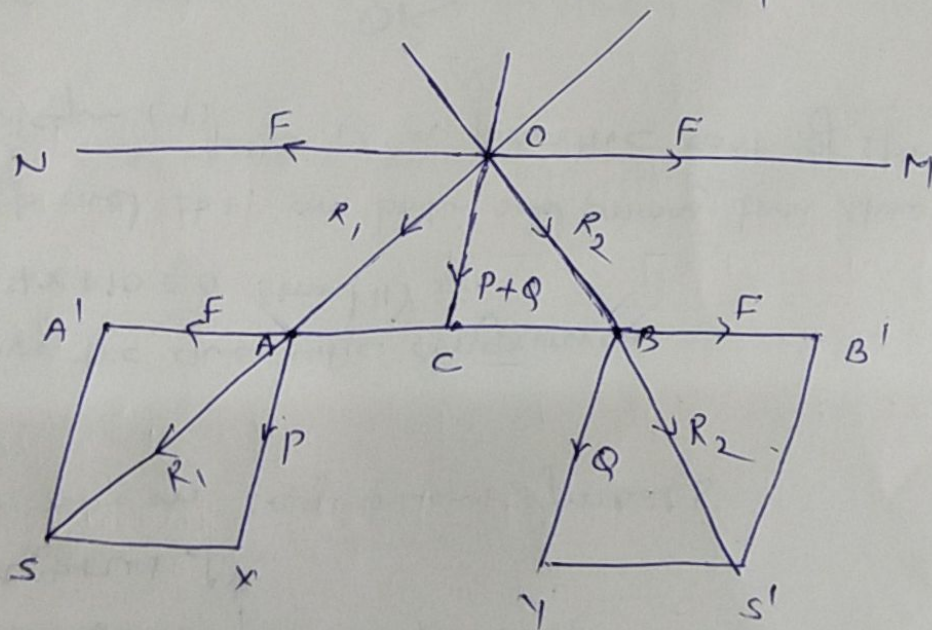
Parallel Forces: Forces whose lines of action are parallel are called parallel forces, they are two types.

(i) Like parallel forces: If their line of action are in same direction,

(ii) Unlike parallel forces: If their line of action are in opposite direction,

Q. Find the resultant of two unequal like parallel forces acting on a rigid body,

Sol: Let ~~two~~ P and Q are two unequal like parallel forces acting from the points A & B ^{of any rigid body} ~~respectively~~ in the direction AX & BY respectively.



for equilibrium force F applied in direction AA' & BB'

Now Draw two parallelogram $AA'XS$ & $BB'S'Y$ and the resultant of forces P & F is R_1 and Q & F is R_2 (say).

Let line of actions of R_1 & R_2 are AS & BS' and their opposite sides meet at O .

Now Draw a parallel line ϕ to AX & BY from O meet at C on AB say OC .

at O . Resolution of Resultant force R_1 in two components one F acting in OC parallel to AA' and other P along OC parallel to AX .

Similar for R_2 , F along OH & Q along OC .

at O ^{acting} four forces P, Q, F & F

where one F is drawn OG & other in direction OH both are same in magnitude but in opposite direction therefore they are in equilibrium.

Hence the Resultant of forces along OC is $(P+Q)$.

position of C $\because \triangle OCA$ & $\triangle AA'S$ are similar

$$\therefore \frac{OC}{AC} = \frac{A'S}{AA'}$$

$$\therefore \frac{OC}{AC} = \frac{P}{F}$$

$$\Rightarrow P \cdot AC = F \cdot OC \quad \text{--- (1)}$$

again $\triangle OCB$ & $\triangle BB'S'$ are similar

$$\frac{OC}{BC} = \frac{B'S'}{BB'} = \frac{Q}{F}$$

$$\Rightarrow Q \cdot BC = F \cdot OC \quad \text{--- (2)}$$

by (1) & (2) $P \cdot AC = Q \cdot BC$

$$\frac{AC}{BC} = \frac{Q}{P}$$

Q 2 Two men, one stronger than the other have to remove a block of stone weighing 300 kg with a light pole whose length is 3 metre.

If the weaker man can not carry more than 100 kg where must be stone be fastened to the pole so as just to allow him his full share of weight.

Sol: weight of stone 300 kg.

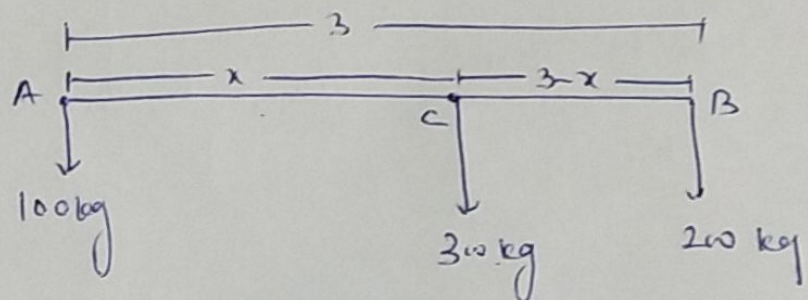
length of pole 3 meter.

weaker man carry maximum 100 kg wt.

so stronger man carry 200 kg wt

at stone fastened at x meter from weaker man

ie



as per the position of C ($P \cdot AC = Q \cdot BC$)

$$100 \cdot x = 200 \cdot (3-x)$$

$$100x = 600 - 200x$$

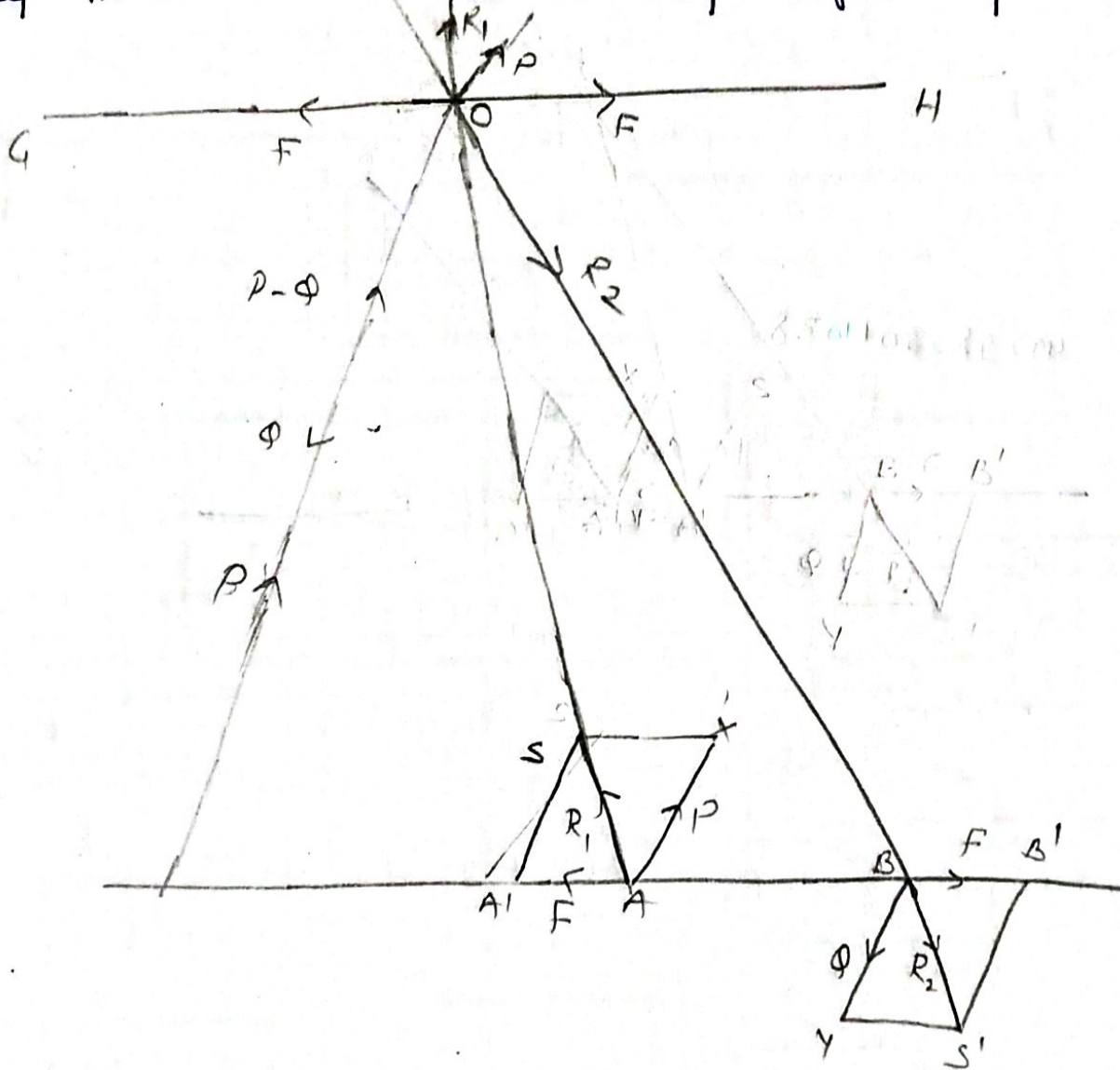
$$300x = 600$$

$$x = 2 \text{ meter.}$$

Hence stone must be fastened at 2 meter from weaker man so to allow him his full share of weight.

Q.3. Find the resultant of the unequal unlike parallel forces acting on a rigid body.

solⁿ Let P and Q (where $P > Q$) are two unequal unlike parallel forces acting from the points A & B of the rigid body in the direction Ax and By respectively.



Now apply the force F on A along AA' and also at B apply force F along BB'

∴ both F are same in magnitude and in opposite direction

$\therefore$ No effect on equilibrium.

Draw the parallelogram $AXSA'$ and $BYSB'$

let the resultant of forces P and F at A is R , (say)

similarly resultant of forces $\odot$ & F at B is R_2 (say).

the line of action of R_1 & R_2 are AS & BS' respectively.

~~Continue~~

Continue the lines AS & BS' (in opposite direction) meet at O .

Now Draw a parallel line OC to AX & BY

also Draw a parallel line GH to AB

at O , Resolution of force R_1 in two components
one F along OG parallel to AA' and other P along CO .

similarly for R_2 , F along OH parallel to BB' & other Q along OC

therefore at O , four forces P, Q, F & F acting,
both F are in same magnitude and in opposite
direction therefore they are in equilibrium.

Remaining two forces are P along CO & Q along OC .

$\therefore P > Q$

$\therefore$ resultant is $P - Q$, along CO

Position of C.

$\therefore \triangle ACO$ & $\triangle AA'S$ are similar

$$\therefore \frac{CO}{AC} = \frac{A'S}{AA'}$$

$$2) \frac{CO}{AC} = \frac{P}{F}$$

$$2) P \cdot AC = F \cdot CO$$

$$\Rightarrow \boxed{P \cdot CA = F \cdot OC} \quad \text{--- (1)}$$

Similarly $\triangle BCO$ & $\triangle BB'S'$ are similar

$$\therefore \frac{OC}{BC} = \frac{B'S'}{BB'}$$

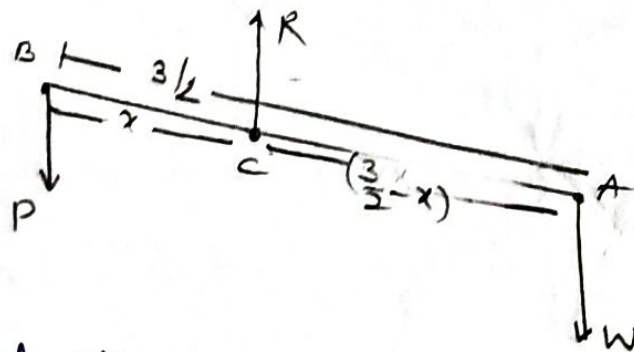
$$\Rightarrow \frac{OC}{BC} = \frac{Q}{F}$$

$$\Rightarrow Q \cdot BC = F \cdot OC \quad \text{--- (2)}$$

$$\text{by (1) \& (2)} \quad \boxed{P \cdot CA = Q \cdot BC} \Rightarrow \frac{CA}{CB} = \frac{Q}{P}$$

Q 4 A man carries a bundle at the end of a stick $\frac{3}{2}$ meters long which is placed on his shoulders. What should be the distance between his hand and shoulder, in order that the pressure on the shoulder may be three times the weight of the bundle.

Sol. Let A man carries a bundle of wt w kg at A of stick ~~at~~ AB of $\frac{3}{2}$ meter.



pressure of hand at other end of stick is P and the reaction of shoulder is R i.e. $R = 3w$
 $\therefore$ Wt w , P & R is ~~equilibria~~ ~~equilibri~~ equilibrium

$$\therefore R = w + P \implies P = 2w$$

for position of C

$$W \cdot AC = P \cdot BC$$

$$\Rightarrow w \cdot \left(\frac{3}{2} - x\right) = 2w \cdot x$$

$$\Rightarrow \frac{3}{2} - x = 2x$$

$$\Rightarrow 3x = \frac{3}{2}$$

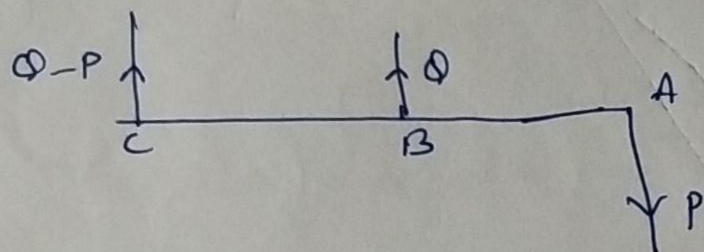
$$\Rightarrow x = \frac{1}{2} \text{ m.}$$

Hence the distance between hand and shoulder is $\frac{1}{2}$ meters

Q.5 Let P & Q are two unlike unequal parallel forces.

If P is doubled, it is found that the line of action of Q is mid way between the line of action of the new and the original resultant. Prove that $4P = 3Q$.

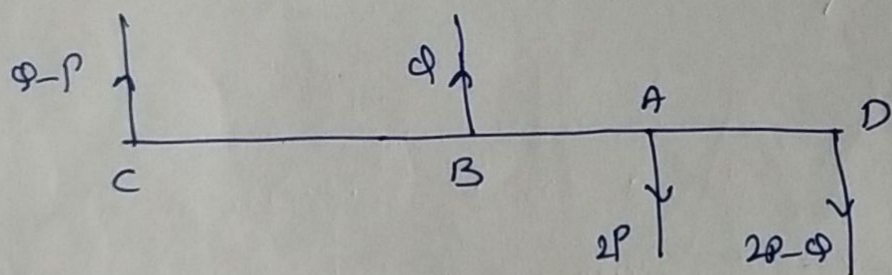
sol: let P & Q ($Q > P$) are acting at A & B resp.



$$Q \cdot BC = P \cdot AC$$

$$\frac{Q}{AC} = \frac{P}{BC} = \frac{Q-P}{AC-BC} = \frac{Q-P}{AB} \quad \text{--- (1)}$$

Now If P is doubled, then the line of action of Q is mid way between the line of action of new and original resultant i.e.



$$\text{where } BC = BD \quad \text{--- (2)}$$

$$\text{or } Q \cdot BD = 2P \cdot AD$$

$$\Rightarrow \frac{Q}{AD} = \frac{2P}{BD} = \frac{2P-Q}{BD-AD} = \frac{2P-Q}{AB} \quad \text{--- (2)}$$

by (1) & (2)

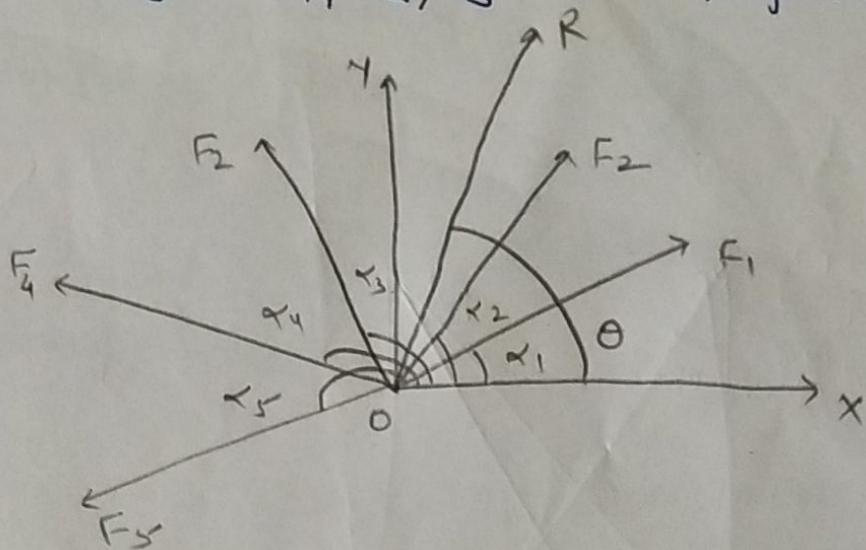
$$\left(\frac{P}{Q-P} \right) AB = \left(\frac{2P}{2P-Q} \right) AB$$

$$\Rightarrow 2P-Q = 2(Q-P)$$

$$\Rightarrow 4P = 3Q$$

Resultant of many forces acting at a point :

Let many forces $F_1, F_2, F_3, \dots$ acting at a point O , which make an angle $\alpha_1, \alpha_2, \alpha_3, \dots$ respectively with OX .



their resolution parts along OX are $F_1 \cos \alpha_1, F_2 \cos \alpha_2, F_3 \cos \alpha_3, \dots$ and along OY are $F_1 \sin \alpha_1, F_2 \sin \alpha_2, F_3 \sin \alpha_3, \dots$

Let the resultant of forces is R at an angle θ with OX .

resolution of R in OX direction is $R \cos \theta$

and in OY direction is $R \sin \theta$

As per the theorem - The algebraic sum of the resolved parts of any two given forces acting at a point in any given direction in their plane is equal to the resolved part of their resultant in that direction,

$$\text{i.e. } R \cos \theta = F_1 \cos \alpha_1 + F_2 \cos \alpha_2 + F_3 \cos \alpha_3 + \dots = X \quad \text{--- (1)}$$

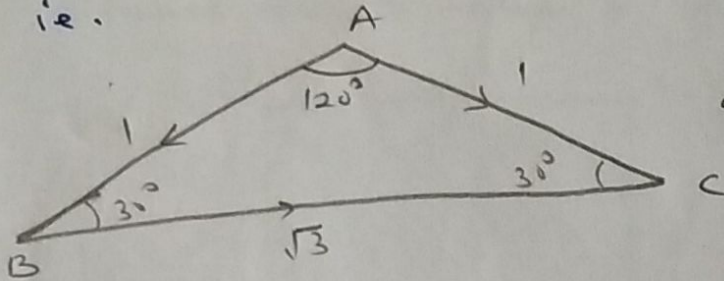
$$\& R \sin \theta = F_1 \sin \alpha_1 + F_2 \sin \alpha_2 + F_3 \sin \alpha_3 + \dots = Y \quad \text{--- (2)}$$

$$\text{by (1) \& (2) } R = \sqrt{X^2 + Y^2} \& \theta = \tan^{-1}(Y/X)$$

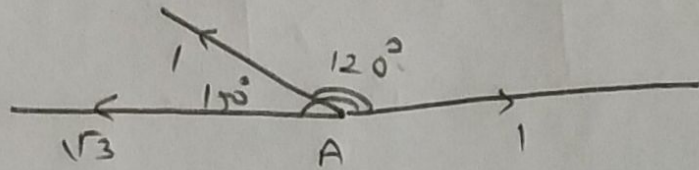
Q.1 ABC is an isosceles triangle whose angle A is 120° and forces of magnitude 1, 1 and $\sqrt{3}$ kg wt. act along AB, AC and BC respectively. Find the magnitude of the resultant.

Sol: Forces 1, 1 and $\sqrt{3}$ kg wt. act along AB, AC & BC resp. of isosceles triangle ABC when $\angle A = 120^\circ$

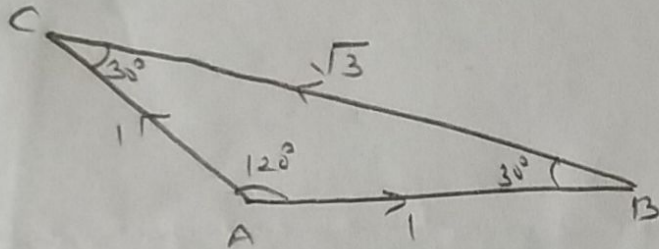
ie.



or



All forces shifted at A



$$\begin{aligned} F_1 &= 1, \alpha_1 = 0 \\ F_2 &= 1, \alpha_2 = 120^\circ \\ F_3 &= \sqrt{3}, \alpha_3 = 150^\circ \end{aligned}$$

$$\begin{aligned} X &= F_1 \cos \alpha_1 + F_2 \cos \alpha_2 + F_3 \cos \alpha_3 \\ &= 1 \cdot \cos 0^\circ + 1 \cdot \cos 120^\circ + \sqrt{3} \cdot \cos 150^\circ \\ &= 1 + 1 \cdot \cos (90^\circ + 30^\circ) + \sqrt{3} \cos (90^\circ + 60^\circ) \\ &= 1 - \sin 30^\circ - \sqrt{3} \sin 60^\circ \\ &= 1 - \frac{1}{2} - \sqrt{3} \cdot \frac{\sqrt{3}}{2} \\ &= \frac{1}{2} - \frac{3}{2} = -1 \end{aligned}$$

$$\begin{aligned} R^2 &= X^2 + Y^2 \\ &= 1 + 3 \end{aligned}$$

$$\boxed{R = 2 \text{ kg}}$$

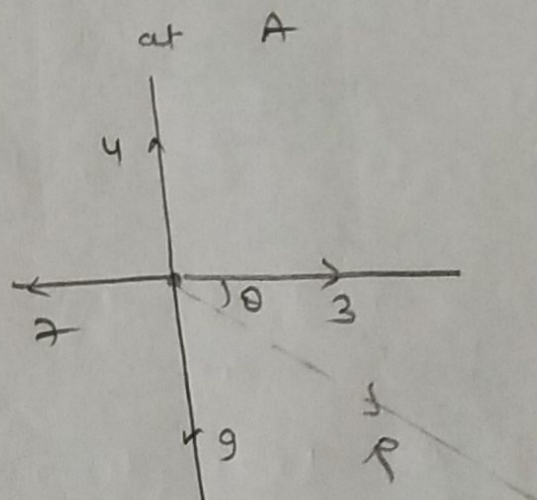
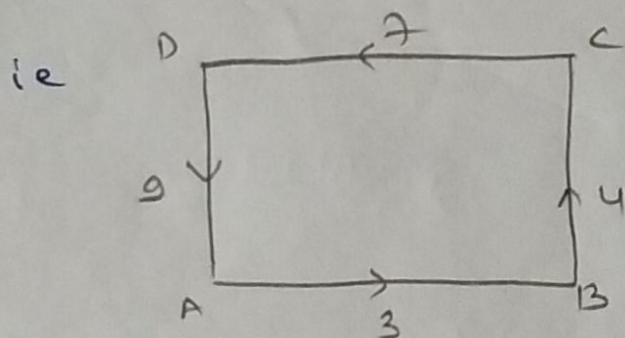
$$\begin{aligned} Y &= F_1 \sin \alpha_1 + F_2 \sin \alpha_2 + F_3 \sin \alpha_3 \\ &= 1 \cdot \sin 0^\circ + 1 \cdot \sin 120^\circ + \sqrt{3} \sin 150^\circ \\ &= 1 \cdot 0 + 1 \cdot \sin (90^\circ + 30^\circ) + \sqrt{3} \sin (90^\circ + 60^\circ) \\ &= 0 + \cos 30^\circ + \sqrt{3} \cos 60^\circ \\ &= \frac{\sqrt{3}}{2} + \sqrt{3} \cdot \frac{1}{2} \\ &= \sqrt{3} \end{aligned}$$

$$\begin{aligned} \theta &= \tan^{-1} \left(\frac{Y}{X} \right) \\ &= \tan^{-1} \left(\frac{\sqrt{3}}{-1} \right) \\ &= \tan^{-1} (-\sqrt{3}) \\ &= \pi - \tan^{-1} (\sqrt{3}) \\ &= 180^\circ - 60^\circ \end{aligned}$$

$$\boxed{\theta = 120^\circ}$$

2. ABCD is a square of having a side of length 3m. If forces 3, 4, 7 and 9 kg wt act along the sides of the square taken in order. Find the magnitude & direction of the resultant.

Sol Forces 3, 4, 7 & 9 kg wt act along the sides of the square ABCD taken in order.



$$F_1 = 3, \quad \alpha_1 = 0$$

$$F_2 = 4, \quad \alpha_2 = 90^\circ$$

$$F_3 = 7, \quad \alpha_3 = 180^\circ$$

$$F_4 = 9, \quad \alpha_4 = 270^\circ$$

$$X = 3 \cdot 1 + 4 \cdot 0 - 7 \cdot 1 + 9 \cdot 0$$

$$= 3 - 7 = -4$$

$$Y = 3 \cdot 0 + 4 \cdot 1 + 7 \cdot 0 - 9 \cdot 1$$

$$= 4 - 9 = -5$$

$$R^2 = X^2 + Y^2$$

$$= 16 + 25$$

$$R = \sqrt{41} \text{ kg.}$$

$$\theta = \tan^{-1}(Y/X)$$

$$= \tan^{-1}\left(\frac{5}{4}\right)$$

$\therefore$ both x & y are negative

$$\therefore \theta = 2\pi - \tan^{-1}\left(\frac{5}{4}\right)$$

$\therefore R$ in IV quadrant.