Unit - I

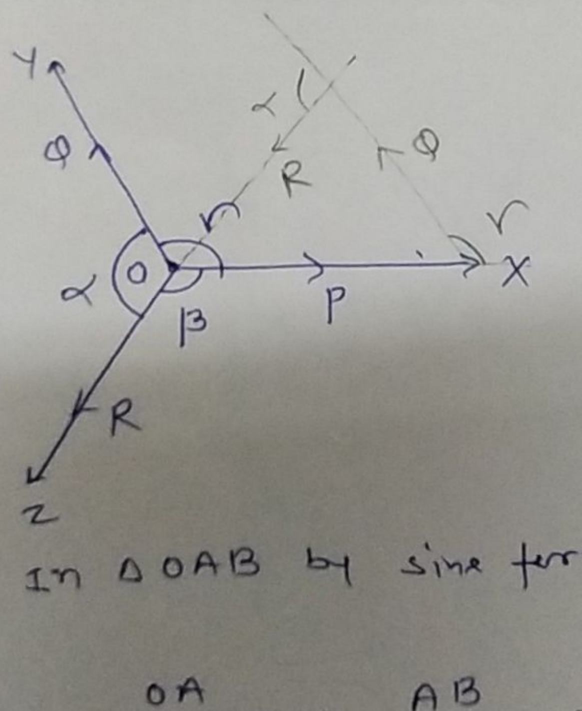
Equilibrium of a rigid body under Three forces:

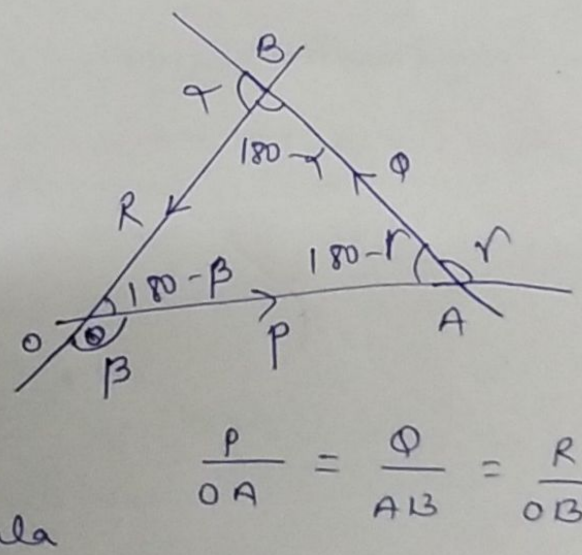
If three forces acting in one plane upon a rigid body, keep it in equilibrium, they must either meet in a point or be parallel.

Lami's theorem: If three forces acting at a point be in equilibrium then each force is proportional to the sine of the angle between the other two.

Proof: Let three forces P, Q, R acting at a point O in direction OX, OY & OZ respectively.

Let $\angle XOY = r$, $\angle YOZ = \alpha$, $\angle ZOX = \beta$





$$\frac{P}{OA} = \frac{Q}{AB} = \frac{R}{OB} \quad -(1)$$

In ΔOAB by sine formula

$$\frac{OA}{\sin B} = \frac{AB}{\sin O} = \frac{OB}{\sin A}$$

$$\Rightarrow \frac{BA}{\sin(180-\alpha)} = \frac{AB}{\sin(180-\beta)} = \frac{OB}{\sin(180-r)}$$

$$\Rightarrow \frac{OA}{\sin\alpha} = \frac{AB}{\sin\beta} = \frac{OB}{\sin r} \quad -(2)$$

by (1) & (2) $\boxed{\dfrac{P}{\sin\alpha} = \dfrac{Q}{\sin\beta} = \dfrac{R}{\sin\gamma}} = \lambda$ (say)

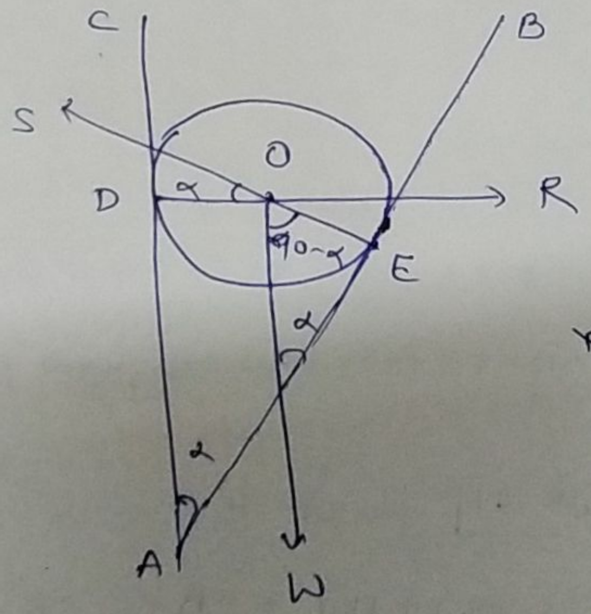
$\Rightarrow P = \lambda \sin\alpha \qquad\qquad P \propto \sin\alpha$

$Q = \lambda \sin\beta \quad \Rightarrow \quad Q \propto \sin\beta$

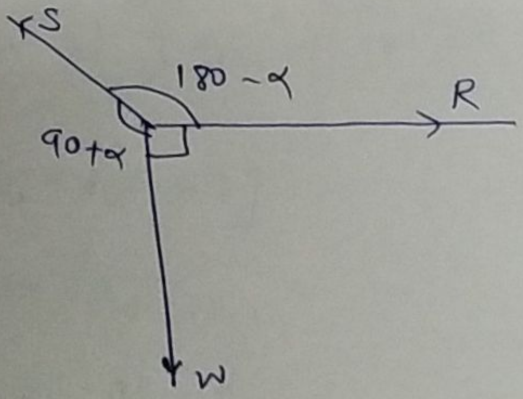
$R = \lambda \sin\gamma \qquad\qquad R \propto \sin\gamma$

Q.1 A sphere of given weight W rests between two smoth planes one vertical and the other inclined at an angle α to the vertical, find the reaction of the planes.

Sol. Let a smoth plane AB inclined at an angle α to the vertical smoth plane AC and a sphere of weight W rests between them i.e.



Let Reaction of planes AC & AB are R & S respect. then at O



by Lami's theorem

$$\frac{R}{\sin(90+\alpha)} = \frac{W}{\sin(180-\alpha)} = \frac{S}{\sin 90^\circ}$$

$\Rightarrow \quad R = W\cot\alpha, \quad S = W\operatorname{cosec}\alpha$

Q.2 A uniform rod can turn freely about one of its ends is pulled aside from the ~~veritical~~ vertical by a horizontal force acting at the other end of the rod and equal to half its weight, At what inclination to the veritical will the rod rest ?

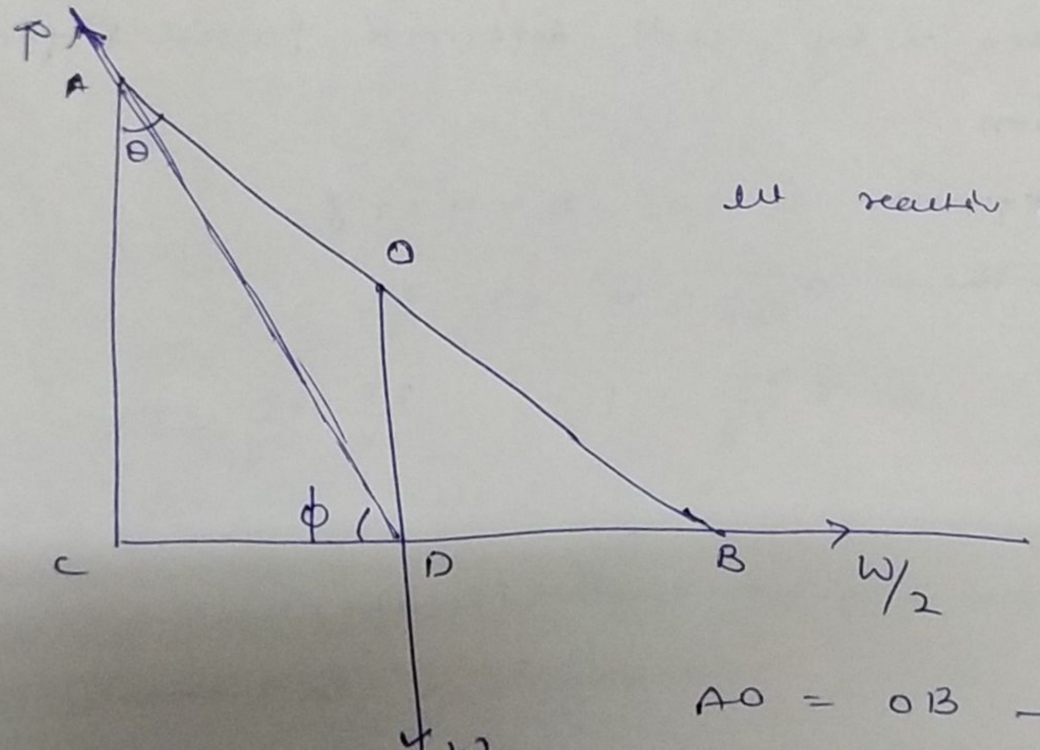
Sol^n: let ~~a~~ uniform rod AB of weight W turn freely about A and pulled aside from the vertical $\overset{AC}{~~by~~}$

~~Solution~~ ~~Question~~

AC by a horizontal force $W/2$ ~~at~~ acting at B.

Let rod AB inclined at an ~~with~~ θ with vertical AC

ie

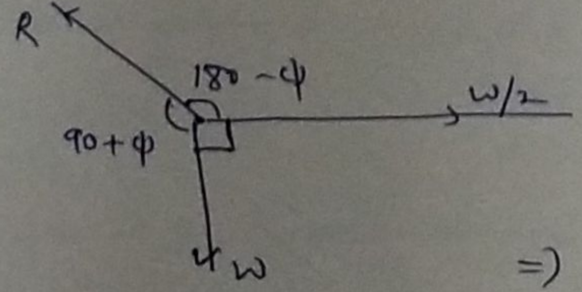


let reaction at A is R

$AO = OB$ —— (1)

$CD = DB$ —— (2)

then at D



by Lamis theorem

$$\frac{W}{\sin(180-\phi)} = \frac{W/2}{\sin(90+\phi)} = \frac{R}{\sin 90^\circ}$$

$$\Rightarrow \frac{1}{\sin\phi} = \frac{1/2}{\cos\phi} = \frac{R}{1}$$

$$\Rightarrow \boxed{\tan\phi = 2} \text{ —— (3)}$$

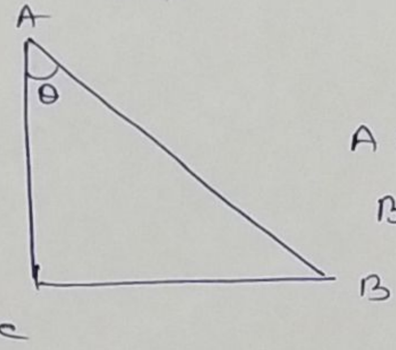
for ϕ & θ by Δ ACD $\tan\phi = \frac{AC}{CD}$

$$\tan\phi = \frac{AC}{\frac{1}{2}BC} \quad \text{by (2)}$$

$$\tan\phi = 2\cdot\frac{AC}{BC}$$

$$\Rightarrow \frac{AC}{BC} = 1 \quad -(4) \qquad \text{by (3) } \tan\phi = 2$$

in ΔACB



$AC = AB\cos\theta$

$BC = AB\sin\theta$

$$(4) \Rightarrow \frac{AB\cos\theta}{AB\sin\theta} = 1$$

$$\Rightarrow \cot\theta = 1$$

$$\Rightarrow \boxed{\theta = 45^\circ}$$

Hence rod AB incline at an angle 45° with verticle.

m-n theorem

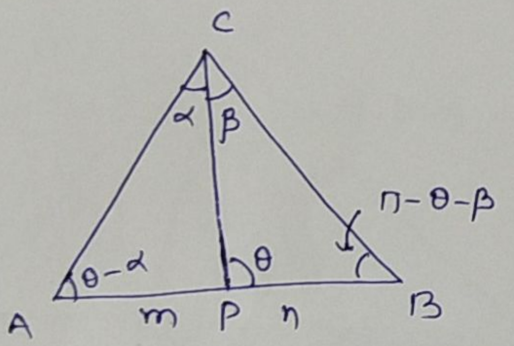
In a ΔABC, P be any point in the base AB, dividing it ~~two~~ into two segments m and n, if $\angle ACP = \alpha$, $\angle BCP = \beta$ and $\angle CPB = \theta$, then.

(i) $(m+n)\cot\theta = m\cot\alpha - n\cot\beta$

(ii) $(m+n)\cot\theta = n\cot A - m\cot B$

prof: ~~let P, d~~

Let point P divide base AB of ΔABC in to two segments m and n, $\angle ACP = \alpha$, $\angle BCP = \beta$ & $\angle CPB = \theta$ ~~then~~ ie



In ΔAPC

$$\cancel{\frac{AP}{AP}} = \frac{AP}{\sin\alpha} = \frac{CP}{\sin(\theta-\alpha)} \quad \text{—(1)}$$

In ΔBPC

$$\frac{CP}{\sin(\pi-\theta-\beta)} = \frac{BP}{\sin\beta}$$

$$\Rightarrow \frac{CP}{\sin(\theta+\beta)} = \frac{BP}{\sin\beta} \quad \text{—(2)}$$

also

$$\frac{AP}{m} = \frac{BP}{n}$$

$$\Rightarrow \frac{m}{n} = \frac{AP}{BP} \quad \text{—(3)}$$

$$= \frac{AP}{CP} \cdot \frac{CP}{BP}$$

$$= \frac{\sin\alpha}{\sin(\theta-\alpha)} \cdot \frac{\sin(\theta+\beta)}{\sin\beta} \quad [\text{by (1) \& (2)}]$$

$$= \frac{\sin\alpha[\sin\theta\cos\beta + \cos\theta\sin\beta]}{\sin\beta[\sin\theta\cos\alpha - \cos\theta\sin\alpha]}$$

$$\frac{m}{n} = \frac{\sin\alpha \sin\theta \cos\beta + \sin\alpha \cos\theta \sin\beta}{\sin\beta \sin\theta \cos\alpha - \sin\beta \cos\theta \sin\alpha}$$

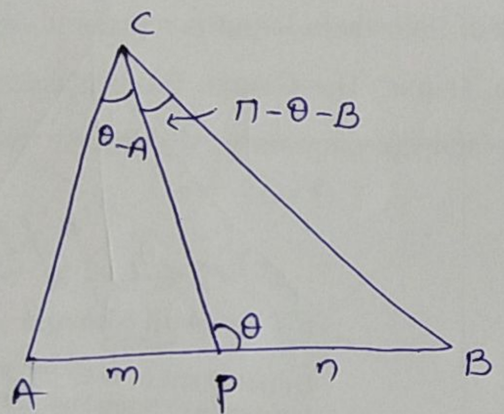
divide by $\sin\alpha \sin\beta \sin\theta$

$$= \frac{\cot\beta + \cot\theta}{\cot\alpha - \cot\theta}$$

$$\Rightarrow \quad m(\cot\alpha - \cot\theta) = n(\cot\beta + \cot\theta)$$

$$\Rightarrow \quad \boxed{(m+n)\cot\theta = m\cot\alpha - n\cot\beta}$$

for (ii)



In $\triangle ACP$

$$\frac{AP}{\sin(\theta-A)} = \frac{CP}{\sin A}$$

$$\frac{AP}{CP} = \frac{\sin(\theta-A)}{\sin A} \quad \text{—(4)}$$

In $\triangle BCP$

$$\frac{CP}{\sin B} = \frac{BP}{\sin(\pi-\theta-B)}$$

$$\Rightarrow \quad \frac{CP}{BP} = \frac{\sin B}{\sin(\theta+B)} \quad \text{—(5)}$$

by (3)

$$\frac{m}{n} = \frac{AP}{CP} \cdot \frac{CP}{BP}$$

$$= \frac{\sin(\theta-A)}{\sin A} \cdot \frac{\sin B}{\sin(\theta+B)}$$

$$= \frac{\sin B(\sin\theta \cos A - \cos\theta \sin A)}{\sin A(\sin\theta \cos B + \cos\theta \sin B)}$$

$$= \frac{\cot A - \cot\theta}{\cot B + \cot\theta} \quad (\text{divide by } \sin A \sin B \sin\theta)$$

$$\Rightarrow \quad \boxed{(m+n)\cot\theta = n\cot A - m\cot B}$$

[Udaipur B.Sc., 02; Raj. B.Sc., 05]

8. It is required to place a heavy uniform rod in equilibrium with one end against a smooth vertical wall and the other against a smooth inclined plane inclined to the wall at an angle θ. Prove that the inclination of the rod to the horizontal is $\tan^{-1}(\frac{1}{2}\tan\theta)$.

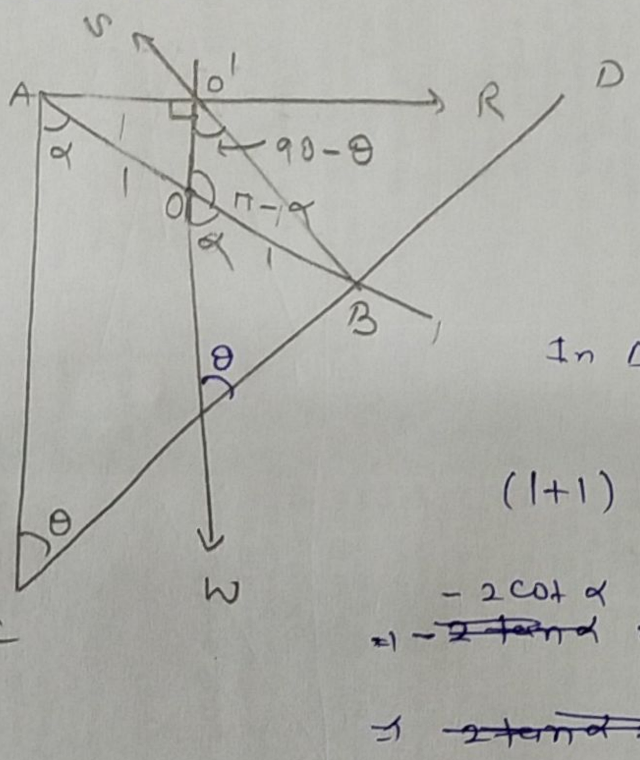
(एक एकसमान भारी छड़ का एक सिरा चिकनी ऊर्ध्वाधर दीवार के साथ तथा दूसरा सिरा एक चिकने झुके हुए समतल पर है जो दीवार से कोण θ बनाता है। सिद्ध कीजिए कि साम्यावस्था में छड़ का क्षैतिज से झुकाव $\tan^{-1}(\frac{1}{2}\tan\theta)$ होगा)

9. A beam whose $C.G.$ divides it into portions of length a and b rests in equilibrium with its ends resting on two smooth planes inclined at angles α and β to the horizontal respectively, the planes intersecting in a horizontal line. Find the inclination θ of the beam to the horizon and reactions of the planes.

(एक दण्ड का गुरुत्व केन्द्र उसे a और b भागों में विभाजित करता है। दण्ड विरामावस्था में है, जिसके सिरे क्षैतिज के साथ कोण α और β पर झुके हुए दो समतलों पर स्थित है, जो एक क्षैतिज रेखा में प्रतिच्छेदित है। दण्ड का क्षैतिज से झुकाव θ तथा समतलों की प्रतिक्रियाएँ ज्ञात कीजिए)

Q.8 It is required to place a heavy uniform rod in equilibrium with one end against a smoth vertical wall and the other against a smoth inclined plane inclined to the wall at an angle θ. Prove that the inclination of the rod to the horizontal is $\tan^{-1}(\frac{1}{2}\tan\theta)$.

Sol: Let CD be a smoth inclined plane inclined to the smothe vertical plane AC at an angle θ.
a rod AB is placed in equilibrium with one end A against AC & other B against CD at an angle α with AC i.e.



Let R & S are reaction of planes AC & CD resp. and W weigh of rod meet at O'

In $\Delta AO'B$ by m.n theorem

$$(1+1)\cot(\pi-\alpha) = 1\cdot\cot 90^\circ - 1\cdot\cot(90-\theta)$$

$$\Rightarrow -2\cot\alpha = 0-\tan\theta$$

$$\Rightarrow \cancel{2\tan\alpha = \tan\theta}$$

$$\Rightarrow 2\cot\alpha = \tan\theta$$

$$\Rightarrow \cot\alpha = \frac{1}{2}\tan\theta$$

$$\Rightarrow \tan(90-\alpha) = \frac{1}{2}\tan\theta$$

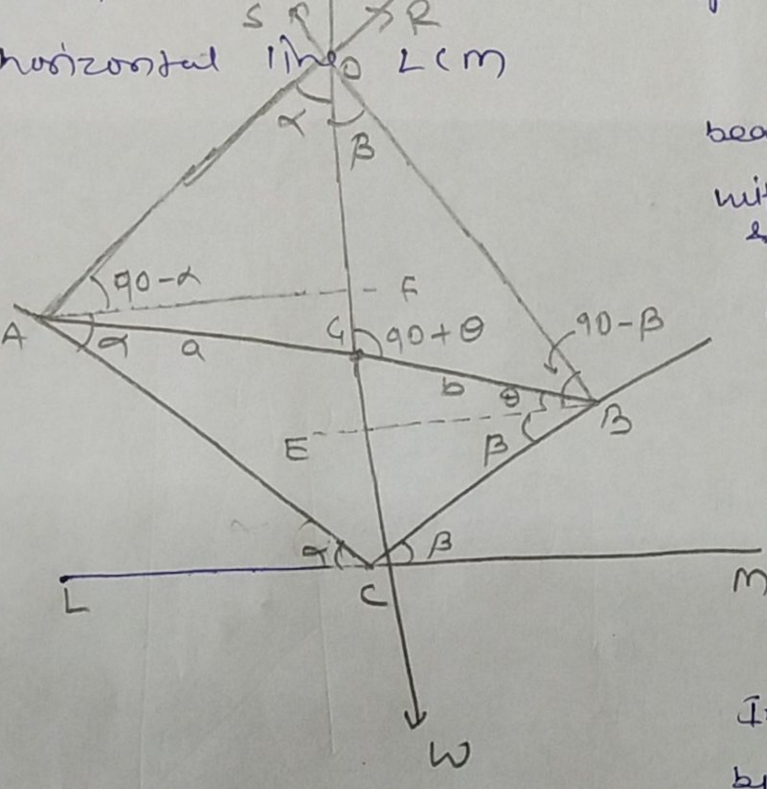
$$\Rightarrow \boxed{90-\alpha = \tan^{-1}\left(\frac{1}{2}\tan\theta\right)}$$

$\because$ α is angle of inclination of rod AB with vertical AC

$\therefore$ $90-\alpha$ is an angle of inclination of rod AB with horizontal.

Q.9 A beam whose C.G. divides in into portions of length a & b, rests in equilibrium with its ends resting on two smoth planes inclined at angles α & β to the horizontal respectively, the planes intersecting in a horizontal line. Find the inclination θ of the beam to the horizon and reactions of the planes.

Sol. Let two smoth planes AC & BC inclined at angles α & β to the horizontal resp. and intersecting at C in horizontal line LCM



beam AB rest in equilibrium with its ends resting on AC & BC, whose C.G divides into parts a & b.

Let R & S are reactions of plane AC & BC respect and W is the weight of beam meet at O

In ΔAOB

by m-n theorem

$(a+b)\cot(90+\theta) = a\cot\alpha - b\cot\beta$

$\Rightarrow -(a+b)\tan\theta = a\cot\alpha - b\cot\beta$

$\Rightarrow \boxed{(a+b)\tan\theta = b\cot\beta - a\cot\alpha}$

$\because \angle ACL = \angle CAF = \alpha$

$\angle BCM = \angle CBE = \beta$

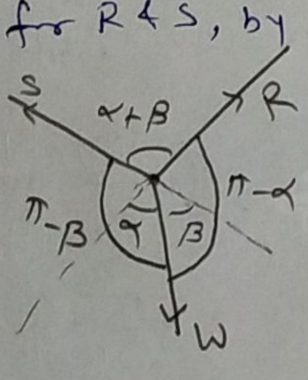
$\angle EBG = \theta$

in ΔBEG

$\because \angle BGE = 90-\theta$

$\therefore \angle BGO = 90+\theta$

for R & S, by Lami's theorem at O



$$\frac{R}{\sin(\pi-\beta)} = \frac{W}{\sin(\alpha+\beta)} = \frac{S}{\sin(\pi-\alpha)}$$

$$\Rightarrow R = W.\frac{\sin\beta}{\sin(\alpha+\beta)}$$

$$\& \ S = W.\frac{\sin\alpha}{\sin(\alpha+\beta)}$$

भार का आधार है। ज्ञात कीजिए कि संतुलन की अवस्था में छड़ ऊर्ध्वाधर से कितना कोण बनाती है?)

3. A uniform bar AB of weight W and length l is hinged at its upper end A and horizontal force is applied to the end B so that the bar is in equilibrium with B at a distance 'a' from the vertical through A, show that the reaction at the hinge is:

(एक एकसमान दण्ड AB का भार W तथा लम्बाई l है। इसके ऊपरी सिरे A पर एक कब्जा है तथा दूसरे सिरे B पर एक क्षैतिज बल लगाया जाता है। दण्ड की साम्यावस्था में B सिरे की A से गुजरने वाली ऊर्ध्व रेखा से दूरी a रहती हो तो सिद्ध कीजिए कि कब्जे की प्रतिक्रिया है) :

$$\frac{1}{2}W\sqrt{\left(\frac{4l^2-3a^2}{l^2-a^2}\right)}$$

[Jodhpur B.Sc., 01]

4. A uniform rod AB of weight W is movable in a vertical plane about a hinge A and is sustained in equilibrium by a weight P attached to the string BCP passing over a smooth peg C, AC being vertical; if $AC=AB$, prove that $P=W\cos ACB$.

(एक W भार की एकसमान दण्ड AB ऊर्ध्वाधर तल में A पर स्थित एक कब्जे पर घूम सकती है। उसको संतुलित अवस्था में रखने वाला भार P एक चिकनी खूँटी C के ऊपर से जाती हुई डोरी BCP से लटका हुआ है, AC ऊर्ध्वाधर है। यदि $AC = AB$, तो सिद्ध कीजिए : $P = W\cos ACB$.)

[Jodhpur B.Sc., 04; Udaipur, B.Sc., 02]

5. ABC is a uniform rod of weight W; it is supported (B being uppermost) with its end A against a smooth vertical wall AD by means of a string CD. DB being horizontal and CD inclined to the wall at an angle of $30°$. Find the tension of the string and the reaction of the wall and prove that $AC = (1/3)AB$.

(W भार का ABC एकसमान दण्ड है। इसे एक रस्सी CD के द्वारा एक चिकनी ऊर्ध्वाधर दीवार AD के सहारे रखा गया है। B ऊपर की तरफ है, DB क्षैतिज है तथा CD दीवार के साथ $30°$ पर झुकी है। रस्सी में तनाव तथा दीवार की प्रतिक्रिया ज्ञात कीजिए और यह भी सिद्ध कीजिए कि $AC = (1/3)AB$).

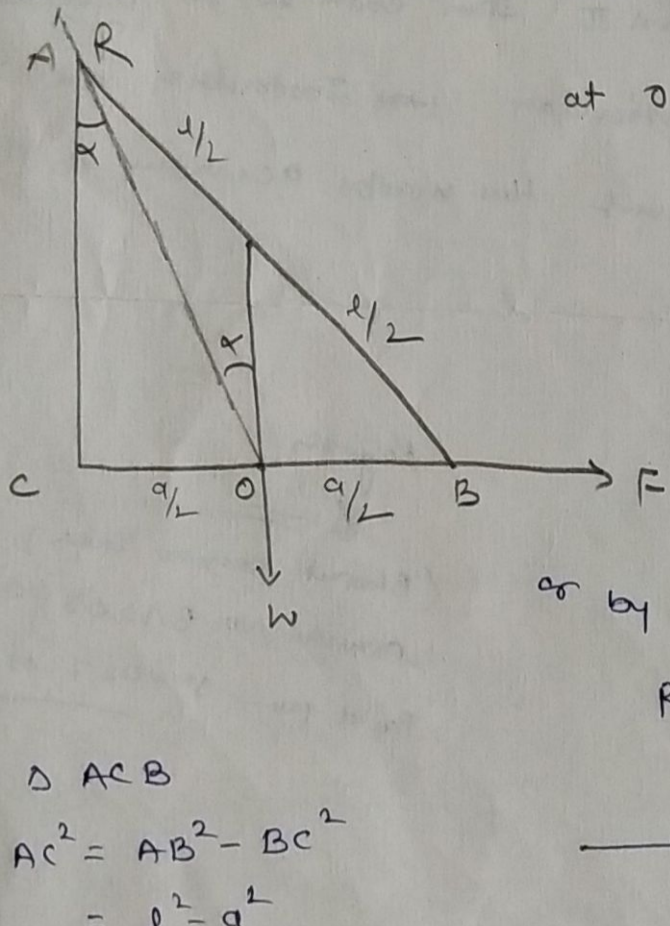
[Raj. B.Sc., 99; Jodhpur B.Sc., 02]

6. A rigid wire, without weight, in the form of the arc of a circle subtending an angle α at its centre and having two weights P and Q at its extremities, rests with its concavity downwards upon a horizontal plane; show that θ be the inclination to the vertical of the radius to the end at which P is suspended, then:

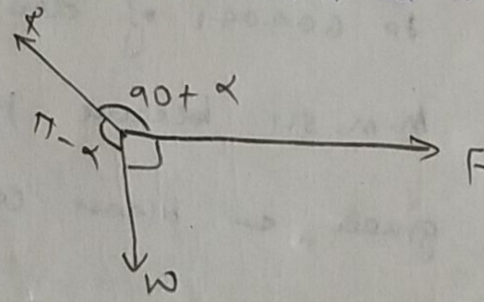
(एक भारहीन वृत्त की चाप के आकार का तार जो केन्द्र पर α कोण बनाता है, एक क्षैतिज धरातल को अपने किसी बिन्दु पर स्पर्श करता है। तार के सिरों से P और Q वजन के दो भार लटकाये जाते हैं। जिस सिरे से भार P लटका हुआ है, उससे होकर जाने वाली त्रिज्या यदि ऊर्ध्वाधर से कोण θ बनाती है, तो सिद्ध कीजिए कि) :

Q3 A uniform bar AB of weight W and length l is hinged at its upper end A and horizontal force is applied to the end B so that the bar is in equilibrium with B at a distance 'a' from vertical through A, show that the reaction at the hinge is: $\frac{W}{2}\sqrt{\left(\frac{4l^2-3a^2}{l^2-a^2}\right)}$

Sol. Let A uniform bar AB of weight W and length l is hinged at upper end A and horizontal force F is applied to the end B so that bar is in equilibrium with B at a distance 'a' from vertical AC. Let the reaction at A is R



at O



$$\frac{R}{\sin 90^\circ} = \frac{W}{\sin(90+\alpha)}$$

$$R = \frac{W}{\cos\alpha} \quad \text{---} (1)$$

or by horizontal component of R

$$R\cos\alpha = W$$

$$\boxed{R = \frac{W}{\cos\alpha}}$$

in Δ ACB

$$AC^2 = AB^2 - BC^2$$
$$= l^2 - a^2$$

$$\cos\alpha = \frac{AC}{AL} = \sqrt{\frac{4(l^2-a^2)}{4l^2-3a^2}} \quad \text{---} (2)$$

in Δ ACO

$$AO^2 = AC^2 + OC^2$$
$$= l^2 - a^2 + \frac{a^2}{4}$$
$$= l^2 - \frac{3a^2}{4}$$

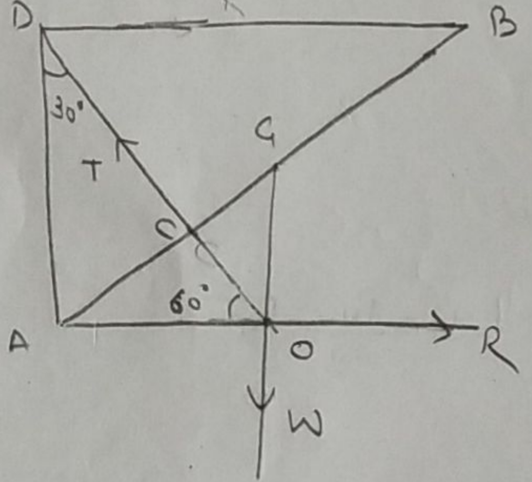
by (1) & (2)

$$R = \frac{W}{2\sqrt{\frac{l^2-a^2}{4l^2-3a^2}}} = \frac{W}{2}\sqrt{\left(\frac{4l^2-3a^2}{l^2-a^2}\right)}$$

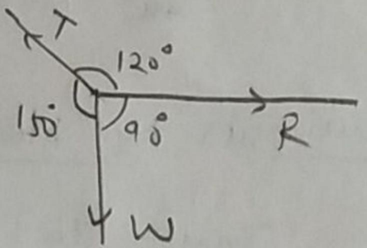
Q5/ ABC is a uniform rod of weight W, it is supported (B being uppermost) with its end A against a smooth vertical wall AD by means of a string ~~CD~~ CD, DB being horizontal and CD inclined to the wall at an angle of 30°. Find the tension of the string and the reaction of the wall and prove that $AC = \frac{1}{3} AB$.

Sol: Let AD is a smooth vertical wall and ACB is a uniform rod of weight W, it is supported by a string CD, which inclines at an 30° with wall AD.

Let tension in string CD is T & Reaction of wall is R.



at O



by Lami's theorem

$$\frac{T}{\sin 90°} = \frac{W}{\sin 120°} = \frac{R}{\sin 150°}$$

$$T = \frac{2W}{\sqrt{3}}, \quad R = \frac{W}{\sqrt{3}}$$

[or by horizontal & vertical component of T

$T\cos 60° = R \Rightarrow \frac{T}{2} = R$ —(1)

$T\sin 60° = W \Rightarrow T.\frac{\sqrt{3}}{2} = W \xrightarrow{(2)} T = \frac{2W}{\sqrt{3}}$

$\& \ R = \frac{W}{\sqrt{3}}$]

In ΔADB, $\angle D = 90°$

$\therefore \angle ADC = 30°$

$\angle CBB = 60°$

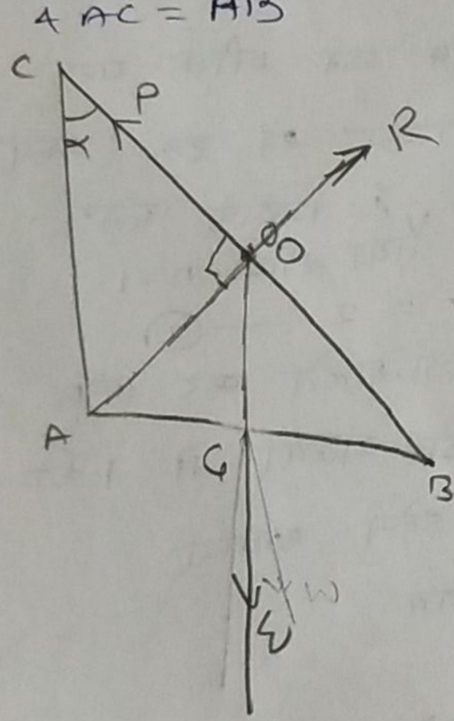
$\therefore \frac{AC}{CB} = \frac{1}{2}$

$\Rightarrow \frac{AC}{AB} = \frac{1}{2} \Rightarrow AC = \frac{1}{3} AB$

Q4 A uniform rod AB of weight W is movable in a vertical plane about a hinge A and is sustained in equilibrium by a weight P attached to the string BCP passing over a smooth pege C, AC being vertical: if AC = AB, prove that $P = W\cos(ACB)$.

Sol. let AC is vertical & rod AB of weight W is movable in a vertical plane about a hinge A for equilibrium weight P attache to the strine BC passing over a smooth pege.

& AC = AB



∵ G is mid point of AB

& AC || OG

∴ AO ⊥ BC

let $\angle ACO = \alpha$

at

$$\frac{P}{\sin(90+\alpha)} = \frac{W}{\sin 90^\circ}$$

$$P = W\cos\alpha$$

Hence $\boxed{P = W\cos(ACB)}$

$$\tan\theta = \frac{Q\sin\alpha}{P+Q\cos\alpha}$$

7. A rod whose centre of gravity divides it into two portions, whose lengths are a and, b has a string of length l, tied to its two ends and the string is slung over a small smooth peg. Find the position of equilibrium of the rod in which it is not vertical.

(एक छड़ का गुरुत्व केन्द्र उसे a और b भागों में विभाजित करता है। छड़ के दोनों सिरों को l लम्बाई की रस्सी से बाँधकर उसे एक चिकनी खूंटी के ऊपर लटका दिया जाता है। छड़ की साम्यावस्था की स्थिति ज्ञात कीजिए जबकि वह ऊर्ध्वाधर नहीं हो)

[Udaipur B.Sc., 02; Raj. B.Sc., 05]

8. It is required to place a heavy uniform rod in equilibrium with one end against a smooth vertical wall and the other against a smooth inclined plane inclined to the wall at an angle θ. Prove that the inclination of the rod to the horizontal is $\tan^{-1}(\frac{1}{2}\tan\theta)$.

(एक एकसमान भारी छड़ का एक सिरा चिकनी ऊर्ध्वाधर दीवार के साथ तथा दूसरा सिरा एक चिकने झुके हुए समतल पर है जो दीवार से कोण θ बनाता है। सिद्ध कीजिए कि साम्यावस्था में छड़ का क्षैतिज से झुकाव $\tan^{-1}(\frac{1}{2}\tan\theta)$ होगा)

9. A beam whose $C.G.$ divides it into portions of length a and b rests in equilibrium with its ends resting on two smooth planes inclined at angles α and β to the horizontal respectively, the planes intersecting in a horizontal line. Find the inclination θ of the beam to the horizon and reactions of the planes.

(एक दण्ड का गुरुत्व केन्द्र उसे a और b भागों में विभाजित करता है। दण्ड विरामावस्था में है, जिसके सिरे क्षैतिज के साथ कोण α और β पर झुके हुए दो समतलों पर स्थित है, जो एक क्षैतिज रेखा में प्रतिच्छेदित है। दण्ड का क्षैतिज से झुकाव θ तथा समतलों की प्रतिक्रियाएँ ज्ञात कीजिए)

10. A bowl is formed from a hollow sphere of radius a and is so placed that the radius of the sphere drawn to any point in the rim makes an angle α with the vertical. P is a point of the bowl such that the radius drawn to it makes an angle β with the vertical; if a smooth uniform rod remains at rest with one extremity at P and with a point of its length in contact with the rim, show that the length of the rod is $4a\sin\beta\sec\frac{1}{2}(\alpha-\beta)$.

(एक प्याला जो त्रिज्या a के एक खोखले गोले का भाग है, इस प्रकार रखा गया है कि उसकी कोर (rim) के प्रत्येक बिन्दु से खींची गई त्रिज्या ऊर्ध्वाधर से कोण α बनाती है। प्याले में बिन्दु A से खींची गई त्रिज्या ऊर्ध्वाधर से कोण β बनाती है। इस प्याले में एक चिकनी एकसमान छड़ इस प्रकार टिकी है कि इसका निचला सिरा बिन्दु A पर है और इसका कुछ भाग प्याले के बाहर है। सिद्ध कीजिए कि छड़ की लम्बाई $4a\sin\beta\sec\frac{1}{2}(\alpha-\beta)$ है)

11. A smooth hemi spherical bowl of diameter a is placed so that its edges touches a smooth vertical wall, a heavy uniform rod is in equilibrium, inclined at 60° to the horizon with one end resting on the inner surface of the bowl and the other end resting against the wall, show that the length of the rod must be $a+(a/\sqrt{13})$.
(एक चिकने अर्धगोलीय प्याले का एक सिरा एक चिकनी ऊर्ध्वाधर दीवार को स्पर्श करता हुआ है। प्याले का व्यास a है। एक भारी एकसमान दण्ड क्षैतिज से 60° का कोण बनाती हुई संतुलित अवस्था में रहती है। यदि उसका एक सिरा प्याले के भीतरी पृष्ठ पर और दूसरा दीवार पर हो तो सिद्ध कीजिए कि दण्ड की लम्बाई $a+(a/\sqrt{13})$ है।)

12. A heavy wheel of diameter 1m and weight 100 kg is to be dragged over a stone of height 20 cm. Find the least horizontal force which should be applied at the centre to do so.
(एक गाड़ी का पहिया जिसका भार 100 किग्रा और व्यास 1 मी है, एक 20 सेमी ऊँचे पत्थर के ऊपर से खींचा जाता है। इसे खींचने के लिए केन्द्र पर क्षैतिज दिशा में लगने वाले कम से कम बल को ज्ञात कीजिए) **[Raj. B.Sc., 2000]**

13. A uniform beam of weight W, can move freely in a vertical plane about a hinge at one end A. To the other end B, a string is fastened which passes over a small fixed smooth pulley vertically above A, and supports a weight w at the other end. Prove that the beam can rest inclined to the horizontal at an angle θ where:
(एक W भार की एकसमान दण्ड AB एक सिरे A पर कब्जे के परित ऊर्ध्वाधर तल में स्वतंत्रतापूर्वक घूम सकती है। दूसरे सिरे B पर एक डोरी बँधी है, जो A के ऊर्ध्वाधर ऊपर एक चिकनी स्थिर घिरनी के ऊपर से जाती है, जिसके दूसरे छोर पर एक भार w लगा है। सिद्ध कीजिए कि दण्ड क्षैतिज से कोण θ पर विरामावस्था में होती है, जहाँ) :

$$\sin\theta=\frac{l}{2a}\left(1-\frac{4w^2}{W^2}\right)+\frac{a}{2l}$$

where a is the length of the beam, l the height of the pulley above A.
(जहाँ a दण्ड की लम्बाई तथा घिरनी की A से ऊँचाई l है)

14. A solid cone of height h and semivertical angle α, is placed with its base against a smooth vertical wall and is supported by a string attached to its vertex and to a point in the wall. Show that the greatest possible length of the string is:
(h ऊँचाई तथा अर्ध शीर्ष कोण α वाला ठोस शंकु का आधार एक चिकनी ऊर्ध्वाधर दीवार पर रखा है तथा शीर्ष को एक डोरी द्वारा दीवार के एक बिन्दु से बाँधा हुआ है। प्रदर्शित कीजिए कि डोरी की अधिकतम लम्बाई है) :

$$h\sqrt{(1+\frac{16}{9}\tan^2\alpha)}$$

Q.6 A rigid wire, without weight, in the form of the arc of a circle subtending an angle α at its centre and having two weights P & Q at its extremities, that θ be the inclination to the vertical of the radius to the rend at which P is suspend then prove that

$$\tan\theta = \frac{Q\sin\alpha}{P + Q\cos\alpha}$$

Sol: let ~~AB~~ ACB a rigid wire in the form of the arc of a circle subtending an angle α at centre O of circle, have two weights P & Q at A & B resp.

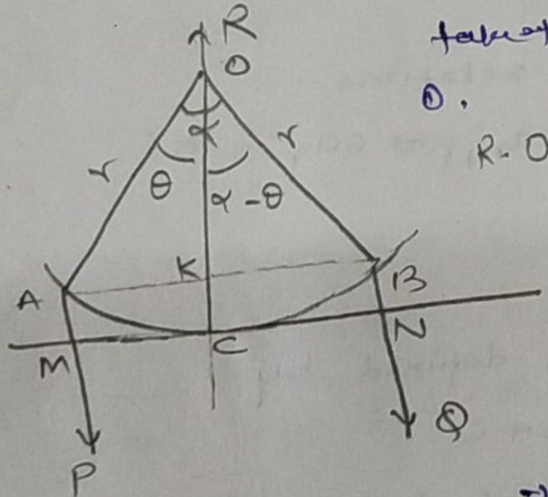
radius OA incline at an angle θ with vertical

~~also~~ :- P & Q are parallel forces

∴ R is the Resultant of forces

for θ

take the moment of forces P, Q & R about O.



$$R \cdot 0 + P \cdot CM - Q \cdot CN = 0$$

$$P \cdot AK - Q \cdot KB = 0$$

$$P \cdot r\sin\theta - Q \cdot r\sin(\alpha - \theta) = 0$$

$$P\sin\theta - Q[\sin\alpha\cos\theta - \cos\alpha\sin\theta] = 0$$

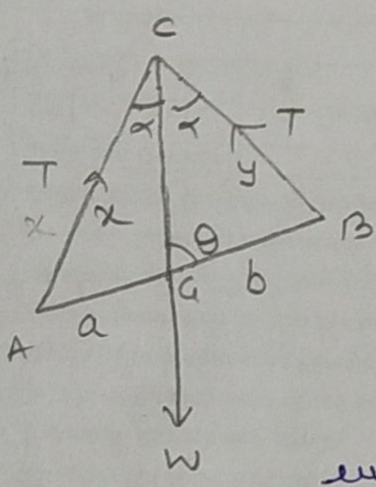
$$\Rightarrow (P + Q\cos\alpha)\sin\theta - Q\sin\alpha\cos\theta = 0$$

$$\Rightarrow \tan\theta = \frac{Q\sin\alpha}{P + Q\cos\alpha}$$

Q.7 A rod whose centre of gravity divides it into two portions, whose length are a & b has a string of length l, tied to its ends and the strings is stung over a small smoth peg. Find the position of equilibrium of the rod in which it is not vertical.

sol. let ~~A~~ a rod AB of weight w divide into two portions a & b by its centre of gravity. A string ACB of length l tied to its ends and stung at C.

let rod inclined at an angle θ with vertical



let $AC = x$ & $BC = y$

$\Rightarrow x + y = l$ —— (1)

$\because$ W is the resultant of two equal forces T & T for equilibrium.

~~$\therefore$ CG divides $\angle$ACB in two~~.

$\therefore$ CG bisect $\angle ACB$

let $\angle ACG = \alpha = \angle BCG$

by m-n theorem.

$$(a+b)\cot\theta = a\cot\alpha - b\cot\alpha$$

$$\Rightarrow \cot\theta = \frac{(a-b)\cot\alpha}{(a+b)} \quad \text{—— (2)}$$

for α by cosine formula in $\triangle ACB$

$$AB^2 = AC^2 + BC^2 - 2AC\cdot BC\cdot\cos(2\alpha)$$

$$\Rightarrow (a+b)^2 = x^2 + y^2 - 2x\cdot y\cdot\cos(2\alpha)$$

$$= (x+y)^2 - 2xy(1+\cos 2\alpha)$$

$$= l^2 - 2xy\cdot(2\cos^2\alpha) \quad \text{by (1)}$$

$$\Rightarrow (a+b)^2 = l^2 - 4xy\cos^2\alpha \quad \text{—— (3)}$$

for x & y by ΔACG & ΔBCG

$$\frac{x}{y} = \frac{AC}{BC} = \frac{AG}{BG} = \frac{a}{b}$$

$$\Rightarrow \frac{x}{a} = \frac{y}{b} = \frac{x+y}{a+b} = \frac{l}{a+b}$$

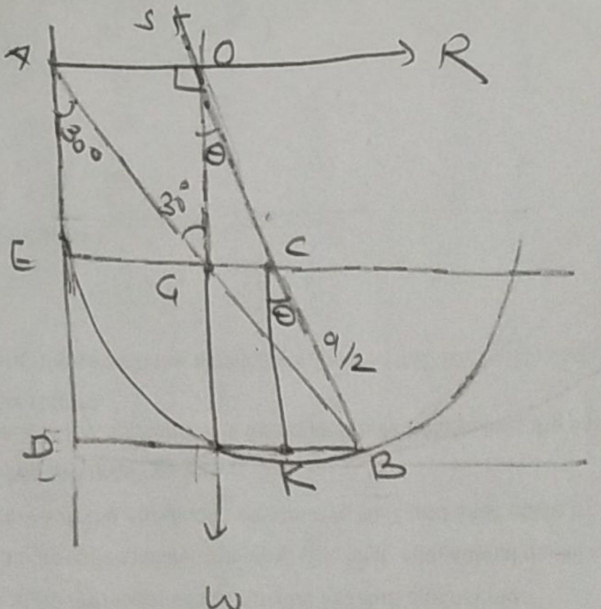
$$\Rightarrow x = \frac{al}{a+b}, \quad y = \frac{bl}{a+b}$$

$$(3) \Rightarrow (a+b)^2 = l^2 - \frac{4 \cdot abl^2}{(a+b)^2} \cos^2\alpha \quad \text{--- (4)}$$

by (4) we get α and by (2) we get angle θ.

Q.1~~φ~~. A smoth hemi spherical bowl of diameter a is placed so that its edges toches a smothe vertical wall, a heavy uniform rod is in equillibrium, inclined at 60° to the horizon with one end resting on the inner surface of the bowl and the other end resting against the wall, show that the length of the rod must be $a\left(1 + \frac{1}{\sqrt{13}}\right)$.

Sol: Let AB is a uniform rod of weight W is in equillibrium, incline at 60° ~~[illegible]~~ to the horizon with end B resting on the inner surface of the ~~[illegible]~~ smoth hemi sperical bowl of diameter a, so that its edge toche a smoth vertical wall AD. Other end A or rod resting against the wall ~~AB~~ AD.



let R & S are the reaction of ~~ro~~ wall & hemi sphere.

~~force~~ for equillibrium forces

W, R & S meet at O.

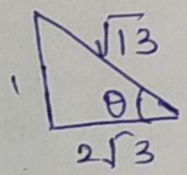
by m-n theorem,

$(AG + GB)\cot 30^\circ = BG \cdot \cot\theta - AG \cot 90^\circ$

$2 \cdot AG \cdot \sqrt{3} = AG \cot\theta$

$\boxed{\cot\theta = 2\sqrt{3}}$ — (1)

$\therefore$



CB is the radious of sphere ie $CB = a/2$

by Δ BKC

$BK = \frac{a}{2}\sin\theta$

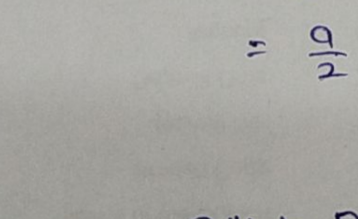
$= \frac{a}{2} \cdot \frac{1}{\sqrt{13}}$ by (1)

$BD = BK + DK (= CE)$

$= \frac{a}{2} \cdot \frac{1}{\sqrt{13}} + \frac{a}{2}$

$= \frac{a}{2}\left(1 + \frac{1}{\sqrt{13}}\right)$

by Δ ADB



$\frac{DB}{AB} = \sin 30^\circ$

$AB = \frac{\frac{a}{2}\left(1 + \frac{1}{\sqrt{13}}\right)}{1/2}$

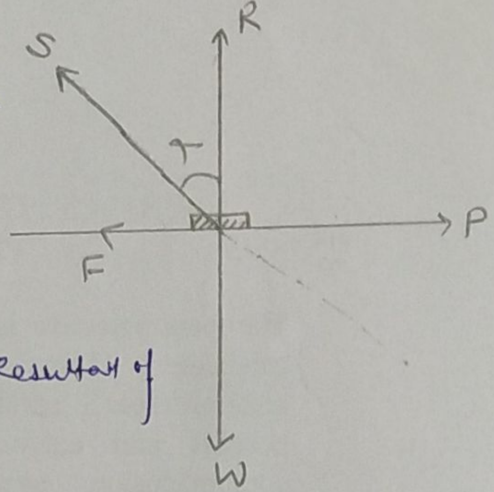
$= a\left(1 + \frac{1}{\sqrt{13}}\right)$

Hence length of rod is $a\left(1 + \frac{1}{\sqrt{13}}\right)$

Friction

Let a body of weight W is in ~~equilibrium~~ rest on rough horizontal table then in that case there are two forces
(i) W (↓) & (ii) Reaction of table R (↑)

Let a force P (very less) applied on body such as body will in rest, in that case three forces are acting
(i) W (↓), (ii) P (→) & (iii) Reaction of table S (↖) (in opposite of Resultant of P & W)



Let F is opposite force of body against P then by resolution of forces

$$F = S\sin\alpha = P$$
$$R = S\cos\alpha = W \quad \Rightarrow \quad \boxed{\tan\alpha = \frac{F}{R} = \mu}$$

$\boxed{F \text{ is called friction force}}$

μ : Coefficient of friction, where $0 < \mu < 1$, for smooth $\mu = 0$

Friction: If two bodies are in contact with each other, the property of two bodies, by virtue of which a force is exerted between them at their point of ~~contact~~ contact to prevent one body sliding on the other, is called friction, also the force exerted is called the force of friction.

Kinds of friction:

(i) Statical friction: when two bodies are in equilibrium with contact.

(ii) Limiting friction: when two bodies are in position of sliding with contact.

(iii) Dynamic friction: when one body slide on other body then friction at point of contact is called Dynamic frict.

Angle of friction: At the point of contact in limiting equilibrium angle between resultant reaction (S) & normal reaction (R) is called angle of friction, general denoted by λ

ie $\tan\lambda = \frac{F}{R}$

$\Rightarrow \boxed{\mu = \tan\lambda}$ $\quad \because \mu = \frac{F}{R}$

§ 2.6. Laws of Friction (घर्षण के नियम): [Raj. B.Sc., 01]

घर्षण के निम्न नियम, जो केवल अनुभव पर आधारित हैं, घर्षण बल का परिमाण, दिशा और प्रकृति तय करतें हैं।

Law I. *When two bodies are in contact, the direction of the friction on one of them at its point of contact is opposite to the direction in which this point of contact would commence to move.*

(जब दो पिण्ड परस्पर स्पर्श करते हैं तो स्पर्श बिन्दु पर घर्षण-बल की दिशा स्पर्श बिन्दु की गति (संभावित गति) की दिशा के विपरीत होती है)

Law II. *The magnitude of the friction is, when there is equilibrium, just sufficient to prevent the body from moving.*

(संतुलन की अवस्था में उसका परिमाण केवल उतना होता है जितना कि पिण्ड को गतिमान होने से रोकने को पर्याप्त हो)

यह घर्षण-बल **स्वसमायोजक (self adjusting)** बल कहलाता है।

Law III. *The magnitude of the limiting friction always bears a constant ratio to the normal reaction and this ratio depends only on the substance of which the bodies are composed.*

(चरम घर्षण बल और पिण्ड की अभिलम्ब प्रतिक्रिया का अनुपात अर्थात् घर्षण-गुणांक μ निश्चित होता है, जो पिण्डों के पदार्थों की प्रकृति पर निर्भर करता है)

Law IV. *The limiting friction is independent of the extent and shape of the surfaces in contact, so long as the normal reaction is unaltered.*

(चरम घर्षण स्पर्श करने वाले पृष्ठों के आधार पर निर्भर नहीं करता जब तक कि अभिलम्व प्रतिक्रिया अपरिवर्तित रहती हो)

Law V. *When motion ensures, by one body sliding over the other, the direction of friction is opposite to the direction of motion; the magnitude of the friction is independent of the velocity but the ratio of the friction to the normal reaction is slightly less than when the body is at rest and just on the point of motion.*

(जब पिण्ड गतिमान अवस्था में हो तब घर्षण गुणांक μ का परिमाण पिण्ड की स्थैतिक अवस्था की अपेक्षा कुछ कम होता है। पुनः μ पिण्डों के वेग पर निर्भर नहीं करता)

Illustrative Examples

Ex.1. *A ladder whose C.G. divides it into two positions of lengths a and b, rest with one end on a rough horizontal floor and the other end against a rough vertical wall. If the coefficient of friction at the floor and the wall be respectively μ and μ', show that the inclination of the ladder to the floor, when equilibrium is limiting, is :*

(एक सीढ़ी का गुरुत्व केन्द्र इसे दो भाग a और b में बाँटता है। सीढ़ी का एक सिरा रुक्ष क्षैतिज फर्श पर तथा दूसरा रुक्ष ऊर्ध्वाधर दीवार पर टिका है। यदि फर्श तथा दीवार का घर्षण गुणांक क्रमशः μ तथा μ' हो तो प्रदर्शित कीजिए कि सीमान्त संतुलन में सीढ़ी

Exercise II (*a*)

1. A uniform ladder rests in limiting equilibrium with one end against a rough vertical wall and the other on a rough horizontal plane, the angles of friction being λ and λ' respectively. Show that the inclination θ of the ladder to the horizon is given by:

(एक सर्वत्रसम सीढ़ी सीमांत संतुलन में है, जिसका एक सिरा रुक्ष ऊर्ध्वाधर दीवार पर आश्रित है और दूसरा रुक्ष क्षैतिज धरातल पर है। दीवार और धरातल पर घर्षण कोण क्रमशः λ और λ' हैं, तो सिद्ध कीजिए कि सीढ़ी का क्षैतिज से झुकाव कोण θ है, जहाँ):

$$\tan\theta = \frac{\cos(\lambda+\lambda')}{2\sin\lambda' \cdot \cos\lambda}$$

[Udaipur B.Sc. 99, Raj. B.Sc. Hons. 02]

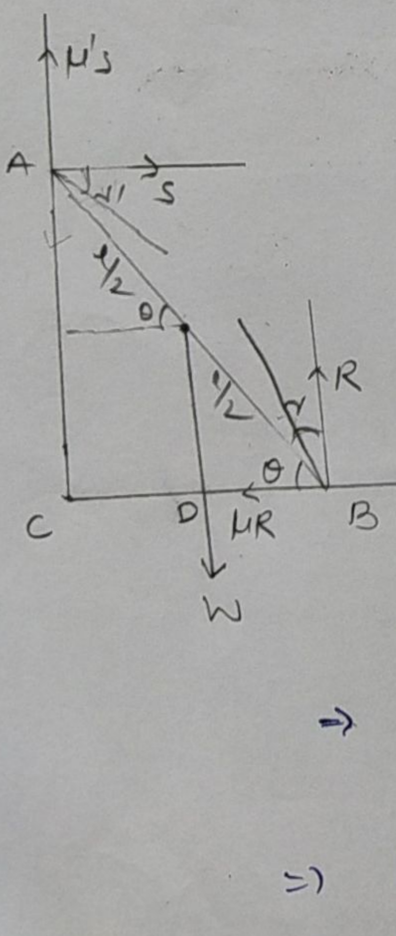
Q.1 A uniform ladder rests in limiting equilibrium with one end against a rough vertical wall and the other on a rough horizontal plane, the angles of friction being λ and λ' resp. Show that the inclination θ of the ladder to the horizon is given by $\tan\theta = \dfrac{\cos(\lambda+\lambda')}{2\sin\lambda'\cos\lambda'}$

Sol. Let AB is a uniform ladder of length l and weight W, rest in equilibrium with one end against a vertical wall AC & other on a rough horizontal plane BC.

normal reaction of horizontal plane and vertical wall are R & S and friction force are μR & $\mu' S$ resp.

where $\mu = \tan\lambda$, $\mu' = \tan\lambda'$

$\because$ λ & λ' are angles of friction, θ is the angle of inclination of ladder to the horizon



Now horizontal & vertical resolution of forces

$S = \mu R$ —(1)

& $R + \mu' S = W$ — (2)

by (1) & (2)

$W = R(1+\mu\mu')$ — (3)

taking sum of moments of all forces about A.

$R.BC - \mu R.AC - W.DC = 0$

$\Rightarrow R.l\cos\theta - \mu R.l\sin\theta - W.\frac{l}{2}\cos\theta = 0$

$\Rightarrow R\cos\theta - \mu.R\sin\theta - \frac{R(1+\mu\mu')}{2}\cos\theta = 0$

$\Rightarrow \left(\frac{1-\mu\mu'}{2}\right)\cos\theta = \mu\sin\theta$

$\Rightarrow \tan\theta = \dfrac{1-\mu\mu'}{2\mu} = \dfrac{1-\tan\lambda.\tan\lambda'}{2\tan\lambda}$

$= \dfrac{1-\frac{\sin\lambda}{\cos\lambda}.\frac{\sin\lambda'}{\cos\lambda'}}{2\sin\lambda/\cos\lambda} = \dfrac{\frac{\cos\lambda\cos\lambda'-\sin\lambda\sin\lambda'}{\cos\lambda\cos\lambda'}}{2\sin\lambda/\cos\lambda}$

$\tan\theta = \dfrac{\cos\lambda\cos\lambda'-\sin\lambda\sin\lambda'}{2\sin\lambda\cos\lambda'}$

$= \dfrac{\cos(\lambda+\lambda')}{2\sin\lambda\cos\lambda'}$,

2. A uniform ladder rests in limiting equilibrium with its lower end on a rough horizontal plane whose coeff. of friction is μ and its upper end against a smooth wall. Prove that the inclination of the ladder to the vertical is $\tan^{-1}(2\mu)$.

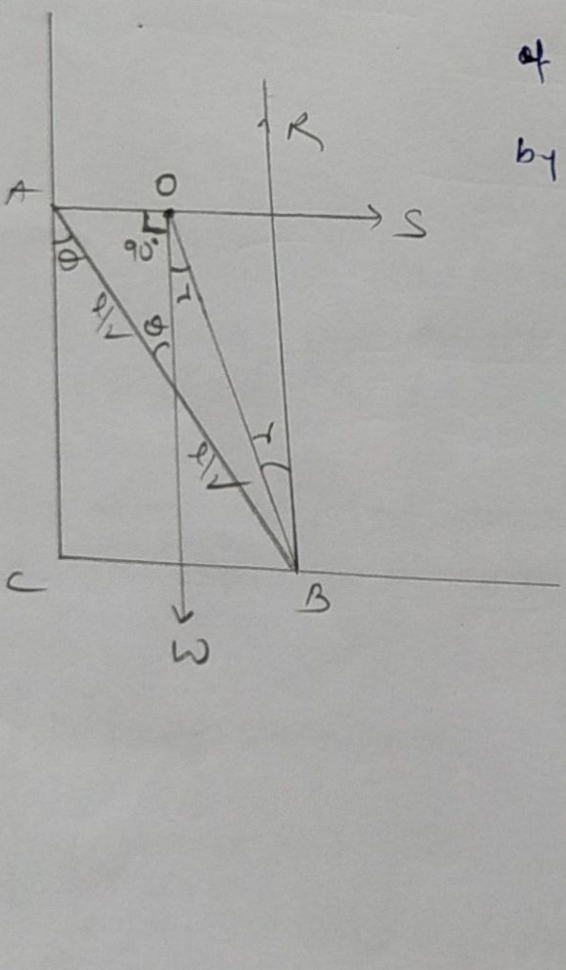
(एक एकसमान सीढ़ी सीमान्त संतुलन में स्थित है, जिसका एक सिरा μ घर्षण-गुणांक वाली रुक्ष भूमि पर है और दूसरा सिरा किसी चिकनी दीवार के सहारे हैं। सिद्ध कीजिए कि सीढ़ी का ऊर्ध्वाधर से झुकाव $\tan^{-1}(2\mu)$ है) **[Ajmer B.Sc.,01, Hons.03]**

Q.2 A uniform ladder rests in limiting equilibrium with its ~~to~~ lower end on a rough horizontal plane whose coefficient of friction is μ and its upper end against a smooth wall. Prove that the inclination of the ladder to the vertical wall is $\tan^{-1}(2\mu)$.

Sol. Let $AB = l$ be a uniform ladder ~~~~ of weight W rests in limiting equilibrium with its lower end on a rough horizontal plane BC whose coefficient of friction is μ and upper end against a smooth wall AC. Let the normal reaction are R & S and ~~coefficient~~ friction force in horizontal is μR

where ~~~~, $\mu = \tan\lambda$ —— (1)

Let θ is the angle of inclination of ladder to the vertical wall



by m-n theorem

$$\left(\frac{l}{2}+\frac{l}{2}\right)\cot\theta = \frac{l}{2}\cot\lambda - \frac{l}{2}\cot 90^\circ$$

$$l\cot\theta = \frac{l}{2}\cot\lambda$$

$$\cot\theta = \frac{\cot\lambda}{2}$$

$$\frac{1}{\tan\theta} = \frac{1}{2\tan\lambda}$$

$$\tan\theta = 2\tan\lambda$$

$$\tan\theta = 2\mu$$

$$\theta = \tan^{-1}(2\mu)$$

7. Two equal uniform rods AC, CB are freely jointed at C and rest in a vertical plane with the ends A and B in contact with a rough horizontal plane. If the equilibrium is limiting and the coefficient of friction is μ, show that :

(दो बराबर एकसमान दण्ड AC, CB बिन्दु C पर जुड़ी है तथा ऊर्ध्वाधर समतल में खड़ी है, जिसके सिरे A तथा B रुक्ष क्षैतिज समतल पर टिके हैं। यदि सीमान्त संतुलन में घर्षण गुणांक μ है तो सिद्ध कीजिए) :

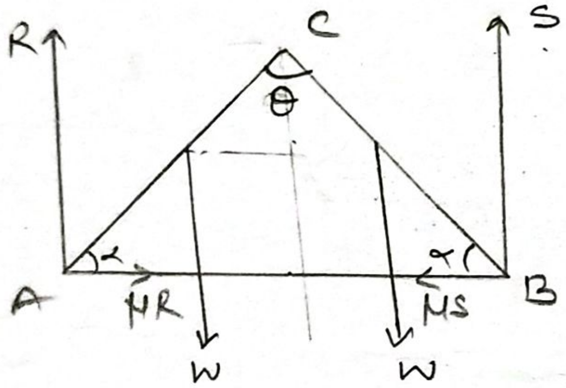
$$\sin \angle ACB = \frac{4\mu}{1 + 4\mu^2}$$

Q.7 Two equal uniform rods AC, CB are freely jointed at C rest in a vertical plane with ends A & B in contact with a rough horizontal plane. If the equilibrium is limiting and the coefficient of friction is μ, show that

$$\sin \angle ACB = \frac{4\mu}{1+4\mu^2}$$

Sol: let two equal rods AC = CB = 2a are joined at C with ends A & B in contact with a horizontal plane.

i.e.



let $\angle ACB = \theta$ &

$\angle CAB = \angle CBA = \alpha$

R & S are normal reactions at A & B resp.

∴ μR & μS are friction forces

by horizontal and vertical resolution of forces

$\mu R = \mu S \Rightarrow R = S$

$R + S = 2W \Rightarrow R = S = W$ —— (1)

taking sum of moment of forces about C

$$W \cdot a\cos\alpha + \mu R \cdot 2a\sin\alpha = R \cdot 2a\cos\alpha$$

$$\Rightarrow \cos\alpha + 2\mu\sin\alpha = 2\cos\alpha \qquad \because R = W$$

$$\Rightarrow \tan\alpha = \frac{1}{2\mu} \quad \text{—— (2)}$$

∵ In ΔACB

$\angle ACB = \theta = 180 - 2\alpha$

$$\therefore \sin\angle ACB = \sin\theta = \sin(180-2\alpha)$$

$$= \sin 2\alpha = \frac{2\tan\alpha}{1+\tan^2\alpha}$$

$$= \frac{2\cdot\left(\frac{1}{2\mu}\right)}{1+\left(\frac{1}{2\mu}\right)^2} = \frac{4\mu}{1+4\mu^2}$$

$= \frac{4\mu}{\cancel{1+4\mu}}$

2. A uniform ladder rests in limiting equilibrium with its lower end on a rough horizontal plane whose coeff. of friction is μ and its upper end against a smooth wall. Prove that the inclination of the ladder to the vertical is $\tan^{-1}(2\mu)$.

(एक एकसमान सीढ़ी सीमान्त संतुलन में स्थित है, जिसका एक सिरा μ घर्षण-गुणांक वाली रुक्ष भूमि पर है और दूसरा सिरा किसी चिकनी दीवार के सहारे हैं। सिद्ध कीजिए कि सीढ़ी का ऊर्ध्वाधर से झुकाव $\tan^{-1}(2\mu)$ है) **[Ajmer B.Sc.,01, Hons.03]**

3. A uniform ladder rests in limiting equilibrium with one end on a rough horizontal floor and the other against a smooth vertical wall. A man then ascends the ladder. Show that, whatever his weight, he can not go more than half way up. What happens if the horizontal plane be also smooth?

(एक एकसमान सीढ़ी सीमांत संतुलन में स्थित है, जिसका एक सिरा रुक्ष क्षैतिज फर्श पर है और दूसरा सिरा एक चिकनी ऊर्ध्वाधर दीवार के सहारे है; तब एक मनुष्य सीढ़ी पर चढ़ता है। सिद्ध कीजिए कि वह आधे से अधिक दूरी तक नहीं पहुँच सकता। यदि क्षैतिज फर्श भी चिकना हो, तो क्या होगा?)

4. A uniform ladder of weight W, inclined to the horizon at $45°$, rest with its upper extremity against a rough vertical wall and its lower extremity on the ground, prove that the least horizontal force which will move the lower end towards the wall is just greater than:

(W भार की एकसमान सीढ़ी क्षैतिज से $45°$ पर स्थित है, जिसका ऊपरी सिरा एक रुक्ष ऊर्ध्वाधर दीवार के सहारे है और जिसका नीचे वाला सिरा रुक्ष भूमि पर है। यदि सीमान्त संतुलन में सीढ़ी और भूमि तथा सीढ़ी और दीवार के मध्यस्थ घर्षण-गुणांक क्रमशः μ, μ' हों, तो सिद्ध कीजिए कि वह न्यूनतम क्षैतिज बल जो नीचे वाले सिरे को दीवार की ओर ले जायेगा; निम्न से जरा सा अधिक है):

$$\frac{W}{2}\left(\frac{1+2\mu-\mu'}{1-\mu'}\right)$$ **[Jodhpur B.Sc., 01]**

5. A uniform rod rests inside a fixed vertical circle which subtends an angle 2α at the centre. The upper end of the rod is smooth and lower end is rough. The coeff. of friction is $\tan\lambda$. Prove that the angle which the rod makes with the horizon can not exceed the angle θ where:

(एक एकसमान छड़ किसी स्थित ऊर्ध्वाधर वृत्त के भीतर स्थित है और केन्द्र पर कोण 2α अन्तरित करती है। छड़ का ऊपरी सिरा चिकना और नीचे वाला रुक्ष है घर्षण-गुणांक, $\tan\lambda$ है। सिद्ध कीजिए कि वह कोण जो छड़ क्षैतिज से बनाती है θ से अधिक नहीं हो सकता, जहाँ):

$$\tan\theta = \frac{\sin\lambda}{\cos\lambda + \cos(\lambda + 2\alpha)}$$ **[Ajmer B.Sc. Hons., 04]**

6. A straight uniform beam of length $2h$ rests in limiting equilibrium, in contact with a rough vertical wall of height h, with one end on a rough horizontal plane and with the other end projecting beyond the wall. If both the wall and the plane be equally rough, prove that λ, the angle of friction is given by $\sin 2\lambda = \sin\alpha \sin 2\alpha$, where α is the inclination of the beam to the horizon.

(एक सीधी एकसमान $2h$ लम्बाई की दण्ड सीमान्त साम्यावस्था में है, जिसका एक सिरा h ऊँचाई की ऊर्ध्वाधर दीवार से बाहर निकला हुआ है तथा दूसरा सिरा क्षैतिज फर्श पर टिका है। यदि फर्श और दीवार समान रुक्ष के हों तो सिद्ध कीजिए कि घर्षण कोण λ, $\sin 2\lambda = \sin\alpha \sin 2\alpha$ से प्राप्त होता है, जहाँ दण्ड का क्षैतिज से झुकाव α है)

[Bikaner B.Sc., 06]

7. Two equal uniform rods AC, CB are freely jointed at C and rest in a vertical plane with the ends A and B in contact with a rough horizontal plane. If the equilibrium is limiting and the coefficient of friction is μ, show that :

(दो बराबर एकसमान दण्ड AC, CB बिन्दु C पर जुड़ी है तथा ऊर्ध्वाधर समतल में खड़ी है, जिसके सिरे A तथा B रुक्ष क्षैतिज समतल पर टिके हैं। यदि सीमान्त संतुलन में घर्षण गुणांक μ है तो सिद्ध कीजिए) :

$$\sin\angle ACB = \frac{4\mu}{1+4\mu^2}$$

[Raj. B.Sc. (Hons.) 02; Kota B.Sc., 05; Ajmer B.Sc., 04]

8. A ladder of length $2l$ is in contact with a vertical wall and horizontal floor, the angle of friction being λ at each contact. If the weight of the ladder acts at a point distant kl below the mid point, prove that its limiting inclination θ to the vertical is given by:

(2l लम्बाई की एक सीढ़ी एक ऊर्ध्वाधर दीवार तथा क्षैतिज फर्श के सम्पर्क में है। प्रत्येक सम्पर्क में घर्षण कोण λ है। यदि सीढ़ी का भार इसके मध्य बिन्दु से kl दूरी नीचे क्रियाशील हो तो सिद्ध कीजिए कि ऊर्ध्वाधर से सीमांत झुकाव θ निम्न से प्राप्त होता है) :

$$\cot\theta = \cot 2\lambda - k\,\mathrm{cosec}\,2\lambda.$$

9. A uniform ladder rests with its lower end on a rough horizontal ground and its upper end against a smooth vertical wall. Prove that a horizontal force applied at the foot of the ladder to make it move towards the wall must be atleast $W(\mu + ½\tan\theta)$, where W is the weight of the ladder, θ its inclination to the vertical and μ is the coefficient of friction at the foot of the ladder.

(एकसमान सीढ़ी का नीचे का सिरा एक रुक्ष क्षैतिज धरातल पर तथा ऊपर का सिरा एक चिकनी दीवार पर टिका हुआ है। सिद्ध कीजिए कि सीढ़ी को दीवार की ओर गतिमान करने के लिए इसके पाद पर एक क्षैतिज बल कम से कम $W(\mu + ½\tan\theta)$ होना चाहिए जहाँ, W सीढ़ी का भार, θ सीढ़ी का ऊर्ध्वाधर से झुकाव तथा μ सीढ़ी के पाद पर घर्षण गुणांक है)

10. A uniform rod of weight W is placed with its lower end on a rough horizontal floor and its upper end against an equally rough vertical wall. The rod makes an angle α with the wall and is just prevented from slipping down by a horizontal force P applied at its middle point. Prove that $P = W\tan(\alpha - 2\lambda)$, where λ is the angle of friction and $\lambda \le ½\,\alpha$.

(W भार की एकसमान छड़ का नीचे का सिरा एक रुक्ष क्षैतिज फर्श पर तथा ऊपरी सिरा समान रुक्ष वाली ऊर्ध्वाधर दीवार पर है। दण्ड दीवार से कोण α बनाती है तथा इसके मध्य बिन्दु पर एक क्षैतिज बल P लगाकर फिसलने से रोका हुआ है। सिद्ध कीजिए $P = W\tan(\alpha - 2\lambda)$, जहाँ λ घर्षण कोण तथा $\lambda \le ½\,\alpha$ है)

11. If a uniform beam of length $2h$ can rest with one end on a rough horizontal plane and against the top of a wall of height h, in a vertical plane perpendicular to the wall and at any inclination to the wall which is geometrically possible. Show that the angle of friction between the beam and both wall and ground supposed to be equally rough, must be not less than $½\sin^{-1}(4/3\sqrt{3})$.

($2h$ लम्बाई की एकसमान दण्ड का एक सिरा रुक्ष क्षैतिज समतल तथा h ऊँचाई की दीवार के ऊपरी सिरे पर टिकी है। इसका दीवार से झुकाव ऐसा है जो कि ज्यामितीय सम्भव है। सिद्ध कीजिए कि दण्ड तथा दोनों समान रुक्ष दीवार एवं धरातल का घर्षण कोण $½\sin^{-1}(4/3\sqrt{3})$ से कम नहीं होना चाहिए)

12. A uniform rod rests with one extremity against a rough vertical wall, the other being supported by a string of equal length fastened to a point in the wall. Prove that the least angle which the string can make with the wall is $\tan^{-1}(3/\mu)$.

(एकसमान छड़ का एक सिरा रुक्ष ऊर्ध्वाधर दीवार पर स्थित है तथा दूसरे सिरे को समान लम्बाई की डोरी से बाँध कर दीवार के एक बिन्दु पर बाँधा हुआ है। सिद्ध कीजिए कि डोरी दीवार से कम से कम $\tan^{-1}(3/\mu)$ कोण बना सकती है)

[Udaipur B.Sc. 99]

13. A uniform rod AB is supported at an inclination α to the horizontal, with its lower end B on a rough horizontal plane by a light string attached to A. Prove that the greatest inclination θ of the string to the vertical is given by $\cot\theta = (1/\mu) \pm 2\tan\alpha$, according to the direction in which the end B is about to move.

(एकसमान छड़ AB क्षैतिज से झुकाव α पर है जिसका निचला सिरा B एक रुक्ष क्षैतिज समतल पर तथा A पर एक हल्की डोरी बंधी है, सिद्ध कीजिए कि ऊर्ध्वाधर से डोरी का अधिकतम झुकाव θ, $\cot\theta = (1/\mu) \pm 2\tan\alpha$ से प्राप्त होगा, जो सिरे B के गतिमान होने की दिशा के अनुरूप है)

[Ajmer B.Sc. Hons., 03]