Unit - IV

Resisting medium

Q. Discuss the motion when particle ~~is~~ is moving vertically downwards from rest through a medium whose resistance varying as velocity.

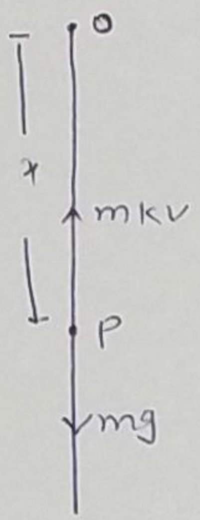
Sol: Let a particle is moving vertically downwards from rest at O under the medius whose resistance varying as velocity (i.e. mkv)

let after some time particle is at P, when OP = x and the velocity of particle is v, then two forces are acting on particle at P

~~(1) weigh of particle (Gravitational~~ Gravitational force) ↓

(1) weigh of particle (Gravitational force mg) ↓

(ii) Resistance of force (mkv) ↑ i.e.



~~ie~~ Now eqⁿ of motion at P

$$m\frac{d^2x}{dt^2} = mg - mkv$$

$$\Rightarrow \frac{d^2x}{dt^2} = g - kv \quad \text{——— (1)}$$

If the terminal velocity of particle is V

then $\frac{d^2x}{dt^2} = 0 = g - kV \Rightarrow \boxed{k = \frac{g}{V}}$ — (2)

$$\because \frac{d^2x}{dt^2} = \frac{dv}{dt}$$

$\therefore$ by (1) $\frac{dv}{dt} = g - kv \Rightarrow \frac{dv}{dt} + kv = g$ — (3)

$\Rightarrow \cancel{\frac{1}{g-kv}dv = dt}$ which is the LDE of first order

IF e^{kt}

Sol of (3) is

$$v \cdot e^{kt} = \int g \cdot e^{kt}\, dt$$

$$= \frac{g}{k} e^{kt} + C_1$$

at o, $t=0, v=0 \Rightarrow C_1 = -g/k$

then the solution (3) is

$$\boxed{V = \frac{g}{k}\left[1 - e^{-kt}\right]} \quad \text{——— (4)}$$

or

$$\boxed{v = V\left[1 - e^{-\frac{g}{V}t}\right]} \quad \text{by (2)}$$

$\because v = \frac{dx}{dt}$

$\therefore$ by (4) $\quad \frac{dx}{dt} = \frac{g}{k}\left[1 - e^{-kt}\right]$

on int

$$\Rightarrow x = \frac{g}{k}\left[t + \frac{1}{k}e^{-kt}\right] + C_2$$

at O, $t=0$, $x=0 \Rightarrow C_2 = -\frac{g}{k^2}$

then

$$\boxed{x = \frac{g}{k}t + \frac{g}{k^2}\left[e^{-kt} - 1\right]} \quad \text{——— (5)}$$

or

$$\boxed{x = Vt + \frac{V^2}{g}\left[e^{-\frac{g}{V}t} - 1\right]}$$

again $\because \frac{d^2x}{dt^2} = v\frac{dv}{dx}$

$\therefore$ by (1) $\quad v\frac{dv}{dx} = g - kv$

$$\Rightarrow \frac{v}{g - kv}\, dv = dx$$

$$\Rightarrow \left[\frac{g}{g-kv} - 1\right] dv = k\,dx$$

on int

$$-\frac{g}{k}\log(g-kv) - v = kx + c_3 \quad \text{--- (6)}$$

at 0, $x=0,\ v=0 \Rightarrow c_3 = -\frac{g}{k}\log(g)$

then (6) becomes

$$-\frac{g}{k}\log(g-kv) - v = kx - \frac{g}{k}\log(g)$$

$$\Rightarrow kx = \frac{g}{k}\left[\log g - \log(g-kv)\right] - v$$

$$\Rightarrow kx = \frac{g}{k}\log\left(\frac{g}{g-kv}\right) - v$$

$$\Rightarrow \boxed{x = \frac{g}{k^2}\log\left(\frac{g/k}{g/k - v}\right) - \frac{1}{k}v} \quad \text{--- (7)}$$

or

$$\boxed{x = \frac{V^2}{g}\log\left(\frac{V}{V-v}\right) - \frac{V}{g}v}$$

Hence eq of motion is $\frac{d^2x}{dt^2} = g - kv$

terminal velocity $V = g/k$

~~the~~ velocity in term of t

$$v = \frac{g}{k}(1-e^{-kt})$$

~~the~~ Displacement in term of t

$$x = \frac{g}{k}t + \frac{g}{k^2}\left[e^{-kt} - 1\right]$$

velocity in term of x or x in term of v

$$x = \frac{g}{k^2}\log\left(\frac{g}{g-kv}\right) - \frac{1}{k}v$$

§3.5. वेग के वर्ग-समानुपाती प्रतिरोध के अधीन एक कण की ऊर्ध्वाधर गति (Vertical Motion of a particle under resistance varies as the square of velocity)

एक कण विरामावस्था से गुरुत्वाकर्षण के अधीन एक ऐसे माध्यम में होकर गिरता है जिसका प्रतिरोध उसके वेग के वर्ग के समानुपाती है। इसकी गति की विवेचना करना :

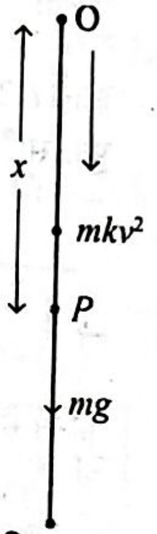
A particle is moving vertically downwards from rest through a medium whose resistance varying as the square of the velocity. To discuss its motion : **[Kota 08, 15; Ajmer 10; Udaipur 12, 14; Raj. 14]**

माना कि कण को विरामावस्था से O से उस माध्यम में गिराया जाता है जिसका प्रतिरोध वेग के वर्गानुपाती है।

यह भी माना कि t समय पर कण की स्थिति P है जहां, $OP = x$ तथा P पर वेग $v\left(=\dfrac{dx}{dt}\right)$ तथा त्वरण $\dfrac{dv}{dt}\left(=\dfrac{d^2x}{dt^2}\right)$ है। अब कण पर कार्यशील बल निम्न है :

(i) कण का भार mg ऊर्ध्वाधर नीचे की ओर।

(ii) प्रतिरोधी बल mkv^2 ऊर्ध्वाधर ऊपर की ओर।



चित्र 3.3

अतः t समय पर कण का गति समीकरण होगा

$$m\frac{d^2x}{dt^2} = mg - mkv^2$$

$$\Rightarrow \quad \frac{d^2x}{dt^2} = g - kv^2 \qquad \textbf{[Udaipur 13; Ajmer 14]} \quad ...(1)$$

यदि कण का अन्तिम वेग (Terminal Velocity) V है, तब $\ddot{x} = 0, v = V$

$$\Rightarrow \quad 0 = g - kV^2 \Rightarrow V^2 = (g/k)$$

$\therefore$ कण का गति समीकरण का नया रूप होगा

$$\frac{d^2x}{dt^2} = g - \frac{g}{V^2}v^2 = \frac{g}{V^2}(V^2 - v^2) \Rightarrow \frac{dv}{dt} = \frac{g}{V^2}(V^2 - v^2) \qquad ...(2)$$

दोनों पक्षों के चर पृथक कर समाकलन करने पर

$$V^2\int\frac{dv}{V^2 - v^2} = g\int dt$$

या $\quad V^2\dfrac{1}{2V}\log\dfrac{V+v}{V-v} = gt + C_1$, जहां C_1 अचर है।

प्रारम्भ में $\quad v = 0, t = 0, \Rightarrow C_1 = 0$

फलतः $\quad \log\dfrac{V+v}{V-v} = \dfrac{2gt}{V}$

$$\Rightarrow \quad \frac{V+v}{V-v} = e^{2gt/V} = \frac{e^{gt/v}}{e^{-gt/V}} \Rightarrow \frac{(V+v)-(V-v)}{(V+v)+(V-v)} = \frac{e^{gt/V}-e^{-gt/V}}{e^{gt/V}+e^{-gt/V}}$$

(योगान्तर अनुपात लगाने पर)

$$\Rightarrow \quad v = V\tanh\left(\frac{gt}{V}\right) \Rightarrow \frac{dx}{dt} = V\tanh\left(\frac{gt}{V}\right) \quad \ldots(3)$$

दोनों पक्षों को t के सापेक्ष समाकलन करने पर

$$x = \frac{V^2}{g}\log\cosh\frac{g_t}{V} + C_2,$$ जहां C_2 अचर है।

प्रारम्भ में $t = 0, x = 0, \Rightarrow C_2 = 0$

फलतः $$x = \frac{V^2}{g}\log\cosh\left(\frac{g_t}{V}\right)$$

इससे t समय पर कण के द्वारा नीचे गिरी गई दूरी प्राप्त होती है।

पुनः समीकरण (2) से

$$v\frac{dv}{dx} = \frac{g}{V^2}(V^2 - v^2)$$

या $$\int\frac{v\,dv}{V^2 - v^2} = \int\frac{g}{V^2}\,dx + C_3,$$ जहां C_3 अचर है

या $$-\frac{1}{2}\log(V^2 - v^2) = \frac{g}{V^2}x + C_3,$$

प्रारम्भ में $v = 0, x = 0, \quad \Rightarrow C_3 = -\frac{1}{2}\log V^2$

फलतः $$x = \frac{V^2}{2g}\log\frac{V^2}{V^2 - v^2}$$

$$\Rightarrow \quad V^2 - v^2 = V^2 e^{(-2gx/V^2)} \Rightarrow v^2 = V^2\left(1 - e^{(-2gx/V^2)}\right) \quad \ldots(4)$$

जो कि v तथा x में एक सम्बन्ध है।

initially at O, $t=0$, $v=U \Rightarrow c_1 = \frac{1}{\sqrt{gk}} \tan^{-1}\left(U\sqrt{\frac{k}{g}}\right)$

then $\frac{1}{\sqrt{gk}} \tan^{-1}\left(v\sqrt{\frac{k}{g}}\right) = -t + \frac{1}{\sqrt{gk}} \tan^{-1}\left(U\sqrt{\frac{k}{g}}\right)$ — (3)

$$\Rightarrow t = \frac{1}{\sqrt{gk}}\left[\tan^{-1}\left(U\sqrt{\frac{k}{g}}\right) - \tan^{-1}\left(v\sqrt{\frac{k}{g}}\right)\right]$$

$$= \frac{1}{\sqrt{gk}} \tan^{-1}\left[\frac{(U-v)\sqrt{\frac{k}{g}}}{1+Uv\frac{k}{g}}\right]$$

$$\Rightarrow \boxed{t = \frac{1}{\sqrt{gk}} \tan^{-1}\left[\frac{(U-v)\sqrt{gk}}{g+Uvk}\right]} \quad \text{— (4)}$$

again by (3)

$$\frac{1}{\sqrt{gk}} \tan^{-1}\left(v\sqrt{\frac{k}{g}}\right) = \frac{1}{\sqrt{gk}} \tan^{-1}\left(U\sqrt{\frac{k}{g}}\right) - t$$

$$\Rightarrow v = \sqrt{\frac{g}{k}} \tan\left[\cancel{t}\ \tan^{-1}\left(U\sqrt{\frac{k}{g}}\right) - (\sqrt{gk})t\right] \quad \text{— (5)}$$

$$\Rightarrow v = \frac{dx}{dt} = \sqrt{\frac{g}{k}} \tan\left(\tan^{-1}\left(U\sqrt{\frac{k}{g}}\right) - (\sqrt{gk})t\right)$$

$$\Rightarrow \int dx = \sqrt{\frac{g}{k}} \int \left[\tan\left\{\tan^{-1}\left(U\sqrt{\frac{k}{g}}\right) - (\sqrt{gk})t\right\}\right] dt$$

$$\cancel{\Rightarrow x = \sqrt{\frac{g}{k}}\left[\left(U\sqrt{\frac{k}{g}}\right) t \ (\sqrt{gk})\right]}$$

$$\Rightarrow x = \sqrt{\frac{g}{k}}\ \frac{\log\sec\left[\tan\left(U\sqrt{\frac{k}{g}}\right) - (\sqrt{gk})t\right]}{-\sqrt{gk}} + c_2$$

initially at O, $t=0$, $x=0 \Rightarrow c_2 = \frac{1}{k}\log\sec\left(\tan^{-1}\left(U\sqrt{\frac{k}{g}}\right)\right)$

then $x = \frac{1}{k}\left[\log\sec\left\{\tan^{-1}\left(U\sqrt{\frac{k}{g}}\right)\right\} - \log\sec\left\{\tan^{-1}\left(U\sqrt{\frac{k}{g}}\right) - (\sqrt{gk})t\right\}\right]$ — (6)

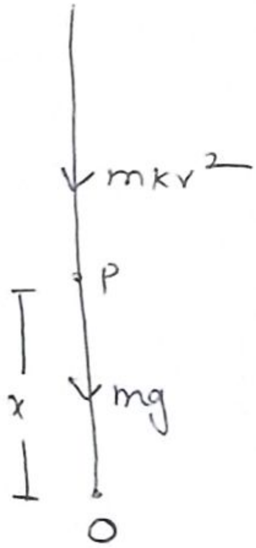
Q: Discuss the motion of particle is projected upwards with velocity U under gravity in a resisting medium due to which resistance varies as the square of velocity.

Sol: Let a particle is projected upwards with initial velocity U from O. ~~and after some time particle is at P when OP = x~~

under resisting medium which varies as the square of velocity and after some time particle is at P when OP = x

i.e.

Now at this time two forces acting on particle at P



(i) Gravitational force (mg) ↓

(ii) Resistance (mkv^2) ↓

equation of motion

$$m\frac{d^2x}{dt^2} = -mg - mkv^2$$

$$\Rightarrow \frac{d^2x}{dt^2} = -(g + kv^2) \quad \text{——} \quad (1)$$

Let the terminal velocity of particle is V then

by (1) $\frac{d^2x}{dt^2} = 0 = g - kV^2$ ~~$-(g + kV^2)$~~ (when particle return)

$$\Rightarrow \boxed{V^2 = \frac{g}{k}} \quad \text{——} \quad (2)$$

$\because \frac{d^2x}{dt^2} = \frac{dv}{dt}$

$\therefore$ by (1) $\frac{dv}{dt} = -(g + kv^2)$

$$\Rightarrow \int \frac{1}{g + kv^2}\, dv = -\int dt$$

$$\Rightarrow \frac{1}{\sqrt{gk}} \tan^{-1}\left(v\sqrt{\frac{k}{g}}\right) = -t + c_1$$

$\because \frac{d^2x}{dt^2} = v\frac{dv}{dx}$

$\therefore$ again by (1) $\quad v\frac{dv}{dx} = -(g+kv^2)$

$$\Rightarrow \int \frac{v}{g+kv^2}\,dv = -\int dx$$

~~on inte~~

$$\Rightarrow \frac{1}{2k}\log(g+kv^2) = -x + c_3$$

initi at 0, $x=0, v=U \Rightarrow c_3 = \frac{1}{2k}\log(g+kU^2)$

then $\frac{1}{2k}\log(g+kv^2) = -x + \frac{1}{2k}\log(g+kU^2)$ — (7)

$$\Rightarrow x = \frac{1}{2k}\left[\log(g+kU^2) - \log(g+kv^2)\right]$$

$$\Rightarrow \boxed{x = \frac{1}{2k}\log\left[\frac{g+kU^2}{g+kv^2}\right]} \quad \text{— (8)}$$

Greatest Hight : when $v=0$, $x=H$ by (8)

$$H = \frac{1}{2k}\log\left(1+\frac{k}{g}U^2\right)$$

$$\boxed{H = \frac{V^2}{2g}\log\left(1+\frac{U^2}{V^2}\right)} \quad \text{by (2)} \quad \text{— (9)}$$

Time to flight for Greatest Hight: when $v=0$, $t=T$ by (4)

$$T = \frac{1}{\sqrt{gk}}\tan^{-1}\left[\frac{U(\sqrt{gk})}{g}\right]$$

$$\boxed{T = \frac{V}{g}\tan^{-1}\left(\frac{U}{V}\right)} \quad \text{by (2)} \quad \text{— (10)}$$

by (9) & (10) $\quad H = \frac{V^2}{2g}\log\left[1+\tan^2\left(\frac{Tg}{V}\right)\right]$

$$\boxed{H = \frac{V^2}{g}\log\sec\left(\frac{Tg}{V}\right)}$$

§3.4. m द्रव्यमान के एक कण को वेग U से ऊर्ध्वाधर ऊपर की ओर ऐसे माध्यम में फेंका जाता है, जिसका प्रतिरोध वेग के समानुपाती है। इसकी गुरुत्वाकर्षण अचर (माना) के अधीन गति की विवेचना करना :

A particle of mass m is projected upwards with velocity U under *gravity in a resisting medium due to which resistance varies* as the *velocity. To discuss the motion, supposing gravitional force to be constant :*

[Ajmer 15; Udaipur 06, 13, Raj. 04, 05; Alwar 11]

माना कि एक कण को बिन्दु O से ऊर्ध्वाधर ऊपर की ओर U वेग से फैंका जाता है। माना कि t समय पर कण की स्थिति P है, जहां $OP = x$. इस अवस्था में कण पर निम्न बल कार्यरत हैं :

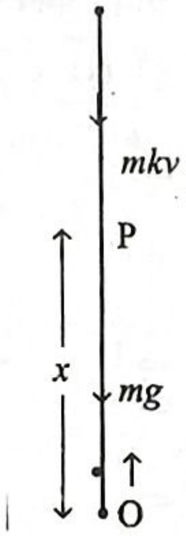
(i) बल mg ऊर्ध्वाधर नीचे की ओर

(ii) प्रतिरोध बल mkv ऊर्ध्वाधर नीचे की ओर

$\therefore$ समय t पर कण की गति समीकरण होगी

$$m\frac{d^2x}{dt^2} = -mg - mkv$$

$$\Rightarrow \quad \frac{dv}{dt} = \frac{d^2x}{dt^2} = -g - kv \qquad ....(1)$$



चित्र 3.2

यदि कण का अन्तिम वेग V , जो कि कण जब नीचे की ओर आएगा तब ही प्राप्त होगा तो समीकरण $\frac{d^2x}{dt^2} = -g + kv$ से

$k = g/V$

पुनः (1) से $-\int\frac{dv}{g+kv} = \int dt$ [चर पृथक विधि से]

$\Rightarrow -\frac{1}{k}\log(g+kv) = t + C_1$, जहां C_1 समाकलन अचर है।

गति के प्रारम्भ में $v = U, t = 0, \Rightarrow C_1 = -\frac{1}{k}\log(g+kU)$

$$\therefore \quad t = \frac{1}{k}\log\frac{g+kU}{g+kv}$$

$$\Rightarrow \quad \frac{g+kU}{g+kv} = e^{kt}$$

$$\Rightarrow \quad g + kv = (g+kU)e^{-kt}$$

$$\Rightarrow \quad v = -\frac{g}{k} + \frac{1}{k}(g+kU)e^{-kt} \qquad ....(2)$$

$$\Rightarrow \quad v = (U+V)e^{-gt/V} - V, \quad \text{जहां } k = g/V$$

इस समीकरण से t समय पर कण का वेग v प्राप्त होता है।

पुनः $\quad v = \frac{dx}{dt} = -\frac{g}{k} + \frac{1}{k}(g+kU)e^{-kt}$

इसका t के सापेक्ष समाकलन करने पर

$$x = -\frac{g}{k}t - \frac{1}{k^2}(g+kU)e^{-kt} + C_2, \quad \text{जहां } C_2 \text{ अचर है।}$$

प्रारम्भ में $t=0, x=0, \Rightarrow C_2 = \frac{1}{k^2}(g+kU)$

$$\therefore \quad x = -\frac{g}{k}t + \frac{1}{k^2}(g+kU)\left[1-e^{-kt}\right] \quad ...(3)$$

$$\Rightarrow \quad x = -Vt + \frac{V}{g}(U+V)\left(1-e^{-\frac{gt}{V}}\right) \quad ...(4)$$

इससे t समय पर कण की ऊंचाई x प्राप्त होती है

पुनः (1) से $\frac{d^2x}{dt^2} = -g - kv \Rightarrow v\frac{dv}{dx} = -g - kv,$

$$\Rightarrow \quad \int \frac{-v\,dv}{g+kv} = \int dx$$

[चर पृथक् करके दोनों पक्षों का समाकलन करने पर]

$$\Rightarrow \quad -\frac{1}{k}\int\left(1-\frac{g}{g+kv}\right)dv = \int dx$$

$$\Rightarrow \quad -\frac{v}{k} + \frac{g}{k^2}\log(g+kv) = x + C_3, \text{ जहां } C_3 \text{ अचर है।}$$

प्रारम्भ में $v=U, x=0, \quad \Rightarrow C_3 = \frac{g}{k^2}\log(g+kU) - \frac{U}{k}$

$$\therefore x = \frac{g}{k^2}\log\frac{g+kv}{g+kU} + \frac{1}{k}(U-v) \text{ या } \frac{V^2}{g}\log\frac{V+v}{V+U} + \frac{V}{g}(U-v) \quad ...(5)$$

यह समीकरण x तथा v का सम्बन्ध बताता है।

यदि कण अधिकतम ऊंचाई H तक जाता है तो अधिकतम ऊंचाई पर $v=0$, तो

(5) से,

$$H = \frac{g}{k^2}\log\frac{g}{g+kU} + \frac{U}{k}$$

अब $k = g/V$ रखने पर, $H = \frac{UV}{g} + \frac{V^2}{g}\log\frac{V}{V+U}$

अधिकतम ऊंचाई H तक का उड्डयन काल T (Time of flight T for the greatest height H)

(2) में $v=0$ तथा $t=T$ रखने पर (जो H ऊंचाई तक का उड्डयन काल है)

$$0 = -\frac{g}{k} + \frac{1}{k}(g+kU)\,e^{-kT} \Rightarrow e^{kT} = \frac{g+kU}{g}$$

$$\Rightarrow \quad T = \frac{1}{k}\log\left(1+\frac{kU}{g}\right)$$

अब $k = g/V$ रखने पर, $T = \frac{V}{g}\log\left(1+\frac{U}{V}\right)$ ....(6) **[Udaipur 13]**

प्रश्नावली III

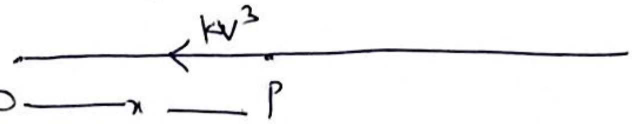
1. kv^3 प्रति इकाई संहति वाले प्रतिरोधी माध्यम में एक कण एक सरल रेखा में चलता है। प्रदर्शित कीजिए कि यदि t समय पर इसका वेग v व दूरी s है, तो $v = \frac{u}{1+2kus}$ तथा $t = \frac{s}{u} + \frac{1}{2}ks^2$, जहाँ u कण का प्रारम्भिक वेग है।

   A particle is moving along a straight line in a resisting medium whose resistance is kv^3 per unit mass. Show that if v is the velocity at time t and distance is s in that time, then $v = \frac{u}{1+2kus}$ and $t = \frac{s}{u} + \frac{1}{2}ks^2$, where u is the initial velocity of the particle.

   [Raj. 03; Jodhpur 03; Udaipur 04; Kota 05]

2. m द्रव्यमान का एक कण स्थिर अवस्था से क्षैतिज सरल रेखा में एक अचर बल के प्रभाव में ऐसे माध्यम में गतिमान होता है जिसकी गति का प्रतिरोध $m(a + bv)$ है, जबकि a तथा b अचर राशियाँ हैं एवं v किसी समय t पर कण का वेग है। यदि कण का अन्तिम वेग V हो, तो प्रदर्शित कीजिए कि कण द्वारा t समय में तय की गई दूरी x, $bx = V(bt - 1 + e^{-bt})$ के द्वारा दी जाएगी।

Q.1 (sol): Let a particle of mass m moving from O with initial velocity u along a straight line in a medium where resistance is kv^3 per unit mass, after some time particle is at P, where OP = x and velocity of particle is v ie



then eq of motion at P is

$$m\frac{d^2x}{dt^2} = -mkv^3 \Rightarrow \frac{d^2x}{dt^2} = -kv^3 \quad \text{—— (1)}$$

$$\because \frac{d^2x}{dt^2} = \frac{dv}{dt}$$

$$\therefore \frac{dv}{dt} = -kv^3 \Rightarrow -\frac{1}{v^3}dv = k\,dt$$

on inte.

$$\frac{1}{2v^2} = kt + c_1$$

initial at O, $t=0$, $v=u \Rightarrow c_1 = \frac{1}{2u^2}$

then $\frac{1}{2v^2} = kt + \frac{1}{2u^2}$

$$\Rightarrow v^2 = \frac{u^2}{2ku^2t+1}$$

$$\Rightarrow v = \frac{u}{\sqrt{2ku^2t+1}} \quad \text{—— (2)}$$

$$\Rightarrow dx = \frac{u}{\sqrt{2ku^2t+1}}\,dt \qquad \because v = \frac{dx}{dt}$$

on integ

$$x = \frac{\sqrt{2ku^2t+1}}{ku} + c_2$$

initi at O, $x=0, t=0 \Rightarrow c_2 = -\frac{1}{ku}$

then $x = \frac{\sqrt{2ku^2t+1}}{ku} - \frac{1}{ku}$

$$\Rightarrow \quad xku+1 = \sqrt{2ku^2t+1}$$

$$\Rightarrow \quad x^2k^2u^2+1+2xku = 2ku^2t+1$$

$$\Rightarrow \quad x^2ku+2x = 2ut$$

$$\Rightarrow \quad t = \frac{kx^2}{2} + \frac{x}{u}$$

$\because$ at P, $x = s$

$$\therefore \quad \boxed{t = \frac{ks^2}{2} + \frac{s}{u}}$$

and $$V = \frac{u}{\sqrt{\left[2ku^2\left(\frac{ks^2}{2}+\frac{s}{u}\right)+1\right]}} = \frac{u}{\sqrt{\left[k^2u^2s^2+2kus+1\right]}}$$

$$= \cancel{\frac{u}{\sqrt{\left[ku(ks^2u+2s)+2u\right]}}} \qquad \boxed{V = \frac{u}{1+kus}}$$

$$\cancel{= \frac{u}{\sqrt{\left[k^2s^2u^2-2sku+2u\right.}}}$$

Q-2 (Sol)

$$\frac{d^2x}{dt^2} = -(a+bv) \quad \text{—(1)}$$

$$\frac{dv}{dt} = -(a+bv)$$

$$\frac{1}{b}\ln(a+bv) = -t+C_1$$

at O, $t=0$, $v=0$, $C_1 = \frac{1}{b}\ln(a+bv)$

$$\frac{1}{b}\ln\left(1+\frac{b}{a}v\right) = -t$$

$$v = \frac{a}{b}\left(e^{-bt}-1\right)$$

$$x = \frac{a}{b}\left(\frac{e^{-bt}}{-b}-t\right)+C_2$$

$t=0$, $x=0$, $\Rightarrow C_2 = \frac{a}{b^2}$

$$x = -\frac{a}{b^2}\left(bt+e^{-bt}\right)+\frac{a}{b^2}$$

$$bx = -\frac{a}{b}\left(bt+e^{-bt}-1\right) \quad \text{—— (2)}$$

$\because$ terminal vel is V

from (1) $\quad -(a+bV) = 0$

$$V = -\frac{a}{b}$$

$$bx = V\left(bt-1+e^{-bt}\right)$$

A particle of mass m moves from rest in a horizontal straight line under the action of a contant force in a medium whose resistance to the motion is $m\ (a + bv)$, where a and b are contants and v is the velocity at time t. If V is the terminal velocity, show that distance x moved by the particle in time t is given by $bx = V\,(bt - 1 + e^{-bt})$.

3. इकाई संहति का एक कण V वेग से ऊर्ध्वाधर दिशा में ऊपर की ओर एक ऐसे माध्यम में फेंका जाता है जिसका प्रतिरोध kv है। सिद्ध करों कि कण प्रक्षेप बिन्दु पर U वेग से लौट कर आयेगा, जहां

A particle of unit mass is projected vertically upwards with velocity V in a medium whose resistance is kv. Prove that the particle will return to the point of projection with velocity U, where

$$U + V = \frac{g}{k}\log\frac{g + kV}{g - kU}$$ **[Raj. 2000, 07; Kota 10]**

4. एक भारी कण ऊर्ध्वाधर दिशा में ऊपर की ओर ऐसे माध्यम में फेंका जाता है कि जिसका प्रतिरोध वेग के समानुपाती है। यदि किसी बिन्दु पर कण का वेग क्रमशः v_1 तथा v_2 ऊपर जाते तथा नीचे आते समय हो तथा बिन्दु से गुजरने में समय का अन्तराल t हो, यदि V कण का अन्तिम वेग है, तो सिद्ध कीजिए

$$v_1 + v_2 = gt \text{ तथा } V - v_2 = (V + v_1)\, e^{-gt/V}$$

A heavy particle is projected vertically in a resisting medium, the resistance varying as the velocity. If v_1 and v_2 are its velocities at any point in its upward and downward paths and t the interval between its passage through this point, if V be the terminal velocity. Prove that $v_1 + v_2 = gt$ and $V - v_2 = (V + v_1)\, e^{-gt/V}$

[Ajmer 03; Bikaner 13; Raj. 02, 04, 07; Hons. 07; Kota 10]

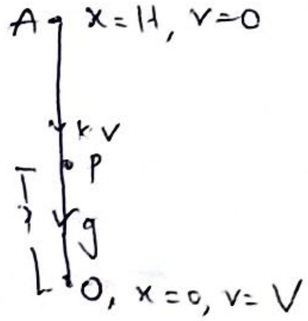
5. एक कण U वेग से ऊर्ध्वाधर दिशा में ऊपर की ओर एक ऐसे माध्यम में फेंका जाता है जिसका प्रतिरोध, वेग के वर्ग के समानुपाती है तो कण प्रक्षेप बिन्दु पर वेग $v = \dfrac{UV}{\sqrt{(U^2 + V^2)}}$ से लौट आयेगा तथा इसको लौटने में $\dfrac{V}{g}\left[\tan^{-1}\left(\dfrac{U}{V}\right) + \tanh^{-1}\left(\dfrac{v}{V}\right)\right]$ समय लगेगा, जहां V अन्तिम वेग है।

A particle, projected upwards with a velocity U in a medium whose resistance varies as the square of the velocity. Prove that the particle will return to the point of projection with velocity $v = \dfrac{UV}{\sqrt{(U^2 + V^2)}}$ after a time $\dfrac{V}{g}\left[\tan^{-1}\left(\dfrac{U}{V}\right) + \tanh^{-1}\left(\dfrac{v}{V}\right)\right]$

Where V is the terminal velocity.

[Kota 06, 08, 17; Udaipur 06; Jodhpur 03; Raj. 03, 05, 08, 15]

Q.3(iii) A particle of unit mass projected vertically upwards from O with velocity V in a medium of resistance kV.
let after some time particle is at P (OP = x) and velocity of particle is v. at P two forces acting



(i) gravitational force g (↓)

(ii) resistance kv (↓)

then eq of motion

$$\frac{d^2x}{dt^2} = -g - kv \quad -(1)$$

particle will return at the point of projection if he covers the maximum height therefore

$$v\frac{dv}{dx} = -(g+kv)$$

$$\Rightarrow \frac{v}{g+kv}\,dv = -\,dx$$

$$\Rightarrow \left(1 - \frac{g}{g+kv}\right)dv = -k\,dx$$

on int

$$\Rightarrow v - \frac{g}{k}\ln(g+kv) = -kx + c_1$$

initially at O, $x = 0$, $v = V$, $c_1 = V - \frac{g}{k}\ln(g-kV)$

then $v - \frac{g}{k}\ln(g+kv) = -kx + V - \frac{g}{k}\log(g-kV)$

for maximum height, $x = H$, $v = 0$

$$-\frac{g}{k}\ln(g) = -kH + V - \frac{g}{k}\ln(g-kV)$$

$$\Rightarrow kH = V - \frac{g}{k}\log(g-kV) + \frac{g}{k}\log(g) \quad -(2)$$

Now eq^n of motion when particle return

$$\frac{d^2x}{dt^2} = g - kv^2$$

$$\Rightarrow \quad v\frac{dv}{dx} = g - kv^2$$

$$\Rightarrow \quad \frac{v}{g-kv^2}\,dv = dx$$

$$\Rightarrow \left(1 - \frac{g}{g-kv^2}\right)dv = -k\,dx$$

on int

$$v + \frac{g}{k}\log(g-kv^2) = -kx + c_2$$

at maxim heig $x=0,\ v=0,\ c_2 = \frac{g}{k}\ln(g)$

then $\quad v + \frac{g}{k}\log(g-kv^2) = -kx + \frac{g}{k}\log(g)$

when particle return to point of project velo is U

ie $x = H,\ v = U$ then

$$U + \frac{g}{k}\log(g-kU^2) = -kH + \frac{g}{k}\log(g)$$

$$\Rightarrow \quad kH = \frac{g}{k}\log(g) - U - \frac{g}{k}\log(g-kU^2) \quad —(3)$$

$\because$ particle travel same dist

$\therefore$ by (2) & (3)

$$V - \frac{g}{k}\log(g+kV^2) + \frac{g}{k}\log(g) = \frac{g}{k}\log(g) - U - \frac{g}{k}\log(g-kU^2)$$

$$\Rightarrow \quad V + U = \frac{g}{k}\log(g+kV^2) - \frac{g}{k}\log(g-kU^2)$$

$$\Rightarrow \quad V + U = \frac{g}{k}\log\left(\frac{g+kV^2}{g-kU^2}\right)$$

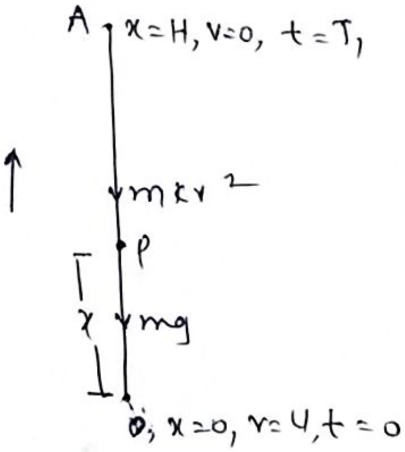
Q.5 (Soi)

Let A particle is projected upwards from O with a velocity U in a resisting medium varies as the square of the velocity, after some time t particle is at P (where OP = x) and velocity of particle is v i.e.

then at P acting two forces

(i) gravitational force (mg) ↓

(ii) Resistance (mkv^2) ↓



then eq^n of motion is

$$m\frac{d^2x}{dt^2} = -mg - mkv^2$$

$$\Rightarrow \frac{d^2x}{dt^2} = -(g + kv^2) \quad \text{——— (1)}$$

$$\Rightarrow v\frac{dv}{dx} = -(g + kv^2)$$

$$\Rightarrow \frac{v}{g + kv^2}dv = -dx$$

$$\Rightarrow \frac{1}{2k}\log(g + kv^2) = -x + c_1$$

at O, $x = 0, v = U \Rightarrow c_1 = \frac{1}{2k}\log(g + kU^2)$

then $\frac{1}{2k}\log(g + kv^2) = -x + \frac{1}{2k}\log(g + kU^2)$

particle will return if $v = 0$ then $x = H$ at A

$$\frac{1}{2k}\log(g) = -H + \frac{1}{2k}\log(g + kU^2)$$

$$\Rightarrow H = \frac{1}{2k}\log(g + kU^2) - \frac{1}{2k}\log(g)$$

$$\Rightarrow H = \frac{1}{2k}\log\left(1 + \frac{k}{g}U^2\right) \quad \text{——— (2)}$$

Particle take time T_1 to reach A ~~at time~~ T_1 then

therefore for T_1 by (1)

$$\frac{dv}{dt} = -(g + kv^2)$$

$$\Rightarrow \frac{1}{g+kv^2}\,dv = -dt$$

$$\Rightarrow \frac{1}{\sqrt{gk}}\tan^{-1}\left(v\sqrt{\frac{k}{g}}\right) = -t + c_2$$

at O, $t=0$, $v=u \Rightarrow c_2 = \frac{1}{\sqrt{gk}}\tan^{-1}\left(u\sqrt{\frac{k}{g}}\right)$

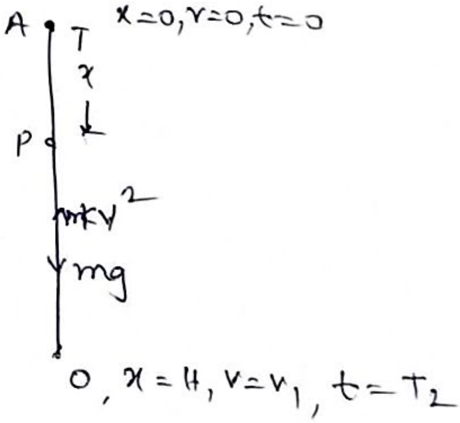
then
$$\frac{1}{\sqrt{gk}}\tan^{-1}\left(v\sqrt{\frac{k}{g}}\right) = -t + \frac{1}{\sqrt{gk}}\tan^{-1}\left(u\sqrt{\frac{k}{g}}\right)$$

at A, $t=T_1$, $v=0$

$$\frac{1}{\sqrt{gk}}\tan^{-1}(0) = -T_1 + \frac{1}{\sqrt{gk}}\tan^{-1}\left(u\sqrt{\frac{k}{g}}\right)$$

$$\Rightarrow T_1 = \frac{1}{\sqrt{gk}}\tan^{-1}\left(u\sqrt{\frac{k}{g}}\right) \quad \text{--- (3)}$$

when particle returns



eq^n of motion

$$m\frac{d^2x}{dt^2} = mg - mkv^2$$

$$\Rightarrow \frac{d^2x}{dt^2} = g - kv^2 \quad \text{--- (4)}$$

$$\Rightarrow v\frac{dv}{dx} = g - kv^2$$

$$\Rightarrow \frac{v}{g-kv^2}\,dv = dx$$

$$\Rightarrow -\frac{1}{2k}\log(g-kv^2) = x + c_3$$

at A, $x=0$, $v=0 \Rightarrow c_3 = -\frac{1}{2k}\log(g)$

then $-\frac{1}{2k}\log(g-kv^2) = x - \frac{1}{2k}\log(g)$

at O, $x=H$ then

$$H = \frac{1}{2k}\log(g) - \frac{1}{2k}\log(g-kv^2)$$

$$H = \frac{1}{2k}\log\left(\frac{1}{1 - \frac{v_1^2}{g/k}}\right) \quad \text{--- (5)}$$

let particle take time T_2 to return at O then by (4)

$$\frac{dv}{dt} = g - kv^2$$

$$\Rightarrow \frac{1}{g-kv^2} dv = dt$$

$$\Rightarrow \frac{1}{2\sqrt{gk}} \log\left(\frac{\sqrt{\frac{g}{k}} + v}{\sqrt{\frac{g}{k}} - v}\right) = t + c_4$$

at A; $t=0$, $v=0$ $\Rightarrow c_4 = 0$

~~at O, $t = T_2$~~

~~then $T_2 = \frac{1}{\sqrt{gk}} \log($~~

$$t = \frac{1}{2\sqrt{gk}} \log\left(\frac{\sqrt{\frac{g}{k}} + v}{\sqrt{\frac{g}{k}} - v}\right) \quad \text{--- (6)}$$

for terminal velocity $\frac{d^2x}{dt^2} = 0$, $v = V$ then by (4)

$$0 = g - kV^2 \Rightarrow V^2 = \frac{g}{k} \quad \text{--- (7)}$$

then (6) become

$$t = \frac{V}{2g} \log\left(\frac{V+v}{V-v}\right)$$

$$\Rightarrow \log\left(\frac{V+v}{V-v}\right) = \frac{2gt}{V}$$

$$\Rightarrow \frac{V+v}{V-v} = e^{\frac{2gt}{V}}$$

$$\Rightarrow v = V \tanh\left(\frac{gt}{V}\right)$$

$$\Rightarrow t = \frac{V}{g} \tanh^{-1}\left(\frac{v}{V}\right)$$

at O, $t = T_2$

$$T_2 = \frac{V}{g} \tanh^{-1}\left(\frac{v}{V}\right) \quad \text{--- (8)}$$

& (3) become $T_1 = \frac{V}{g} \tan^{-1}\left(\frac{u}{V}\right)$ --- (9)

Hence Particle take time to return at O is

$$T_1 + T_2 = \frac{V}{g}\left[\tan^{-1}\left(\frac{U}{V}\right) + \tanh^{-1}\left(\frac{v}{V}\right)\right)$$

and particle when return at O then distance travel by particle in upwards & downwards are same

ie by (2) & (5)

$$\frac{1}{2k}\log\left(1+\frac{k}{g}U^2\right) = \frac{1}{2k}\log\left(\frac{1}{1-\frac{v^2}{g/k}}\right)$$

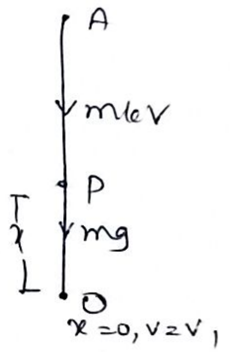
$$\frac{V^2+U^2}{V^2} = \frac{V^2}{V^2-v^2}$$

$$\Rightarrow \quad v = \frac{UV}{\sqrt{U^2+V^2}}$$

Hence particle return to point of projection with velocity

$$v = \frac{UV}{\sqrt{U^2+V^2}}$$

Q. 4 (Sol)^n: Let a particle of mass m projected vertically from O with velocity V_1 in medium of resistance varies as velocity, after some time particle is at P (where OP = x) i.e. at P, velocity of particle is v and acting two force (i) mg ↓ (ii) mkv ↓ then



eq^n of motion

$$m\frac{d^2x}{dt^2} = -mg - mkv$$

$$\Rightarrow \frac{d^2x}{dt^2} = -(g+kv) \quad \text{—(1)}$$

$$\Rightarrow v\frac{dv}{dx} = -(g+kv)$$

$\Rightarrow \quad \frac{v}{g+kv}\,dv = -dx$

$\Rightarrow \quad v - \frac{g}{k}\log(g+kv) = -kx + c_1$

at O, $x=0$, $v=v_1 \Rightarrow c_1 = v_1 - \frac{g}{k}\log(g+kv_1)$

then

$$v - \frac{g}{k}\log(g+kv) = -kx + v_1 - \frac{g}{k}\log(g+kv_1)$$

particle move up to A, when $v=0$ and let $x=H$

then

$$kH = v_1 - \frac{g}{k}\log(g+kv_1) + \frac{g}{k}\log(g)$$

$$kH = v_1 + \frac{g}{k}\log\left(\frac{g}{g+kv_1}\right) \quad \text{— (2)}$$

let particle take time t_1 to reach A then by (1)

$$\frac{dv}{dt} = -(g+kv)$$

$$\Rightarrow \quad \frac{1}{g+kv}\,dv = -dt$$

$$\Rightarrow \quad \frac{1}{k}\log(g+kv) = -t + c_2$$

at O, $t=0$, $v=v_1 \Rightarrow c_2 = \frac{1}{k}\log(g+kv_1)$

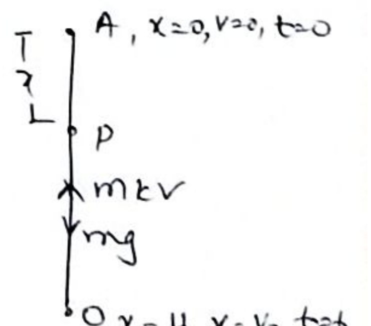
then $\frac{1}{k}\log(g+kv) = -t + \frac{1}{k}\log(g+kv_1)$

at A, $v=0$, $t=t_1$ then

$$t_1 = \frac{1}{k}\log(g+kv_1) - \frac{1}{k}\log(g)$$

$$t_1 = \frac{1}{k}\log\left(1+\frac{kv_1}{g}\right) \quad \text{— (3)}$$

when particle return, eq of motion



$$\frac{d^2x}{dt^2} = g - kv \quad \text{—(4)}$$

$$\Rightarrow \quad v\frac{dv}{dx} = g - kv$$

$$\Rightarrow \frac{v}{g-kv}\,dv = dx$$

$$\Rightarrow v + \frac{g}{k}\log(g-kv) = -kx + C_3$$

at A, $x=0, v=0 \Rightarrow C_3 = \frac{g}{k}\log(g)$

then $v + \frac{g}{k}\log(g-kv) = -kx + \frac{g}{k}\log(g)$

when particle return to O, $x=H, v=v_2$ then

$$KH = \frac{g}{k}\log(g) - v_2 - \frac{g}{k}\log(g-kv_2)$$

$$KH = \frac{g}{k}\log\left(\frac{g}{g-kv_2}\right) - v_2 \quad \text{———} (5)$$

let particle take time t_2 to return at O then by (4)

$$\frac{dv}{dt} = g-kv$$

$$\Rightarrow \frac{1}{g-kv}\,dv = dt$$

$$\Rightarrow -\frac{1}{k}\log(g-kv) = t + C_4$$

at A, $t=0, v=0 \Rightarrow C_4 = -\frac{1}{k}\log(g)$

then $-\frac{1}{k}\log(g-kv) = t - \frac{1}{k}\log(g)$

$\Rightarrow$ at O, $t=t_2$, $v=v_2$ then

$$t_2 = -\frac{1}{k}\log(g-kv_2) + \frac{1}{k}\log g$$

$$t_2 = \frac{1}{k}\log\left(\frac{g}{g-kv_2}\right)$$

$$\cancel{t_2 = -\frac{1}{k}\log\left(\frac{g-kv_2}{g}\right)} \quad \text{———} (6)$$

when particle return at O then distance travel by particle in upward & downward same therefore by (2) & (5)

$$v_1 + \frac{g}{k}\log\left(\frac{g}{g+kv_1}\right) = \frac{g}{k}\log\left(\frac{g}{g-kv_2}\right) - v_2$$

$$\Rightarrow v_1 + v_2 = \frac{g}{k}\log\left(\frac{g+kv_1}{g-kv_2}\right) \quad - (7)$$

and particle take time to return O is $t_1 + t_2 = t$

therefor by (3) & (6)

$$t_1 + t_2 = t = \frac{1}{k}\log\left(\frac{g + kv_1}{g}\right) + \frac{1}{k}\log\left(\frac{g}{g - kv_2}\right)$$

$$t = \frac{1}{k}\log\left(\frac{g + kv_1}{g - kv_2}\right) \quad \text{——} \; (8)$$

∵ Terminal velocity of particle is V then $\frac{d^2x}{dt^2} = 0$

by (4)

$$0 = g - kV \Rightarrow V = \frac{g}{k} \quad \text{——} \; (9)$$

then eq (7)

$$v_1 + v_2 = V\log\left(\frac{V + v_1}{V - v_2}\right) \quad \text{——} \; (10)$$

& (8)

$$t = \frac{V}{g}\log\left(\frac{V + v_1}{V - v_2}\right) \quad \text{——} \; (11)$$

$$\cancel{\Rightarrow \frac{V + v_1}{V - v_2} = e^{\frac{gt}{V}}}$$

$$\Rightarrow \cancel{V - v_2} =$$

by (10) & (11) $\boxed{v_1 + v_2 = gt}$ —(12)

& ~~by (11)~~ particle return with velocity V is given by (11)

$$\frac{V + v_1}{V - v_2} = e^{\frac{gt}{V}}$$

$$\Rightarrow \boxed{V - v_2 = (V + v_1)\, e^{-\frac{gt}{V}}}$$

6. कोई कण V वेग से एक चिकने क्षैतिज तल पर ऐसे माध्यम में फेंका जाता है जिसका प्रतिरोध प्रति इकाई संहति के लिए वेग के घन का μ गुणा है। सिद्ध करो कि t समय पश्चात् इसके द्वारा तय की गई दूरी $\frac{1}{\mu V}[\sqrt{(1+2\mu V^2 t)}-1]$ होगी और उस समय उसका वेग $\frac{V}{\sqrt{(1+2\mu V^2 t)}}$ होगा ।

A particle is projected with velocity V along a smooth horizontal plane in a medium whose resistance per unit mass is μ times the cube of the velocity. Show that the distance it has described in time t is $\frac{1}{\mu V}[\sqrt{(1+2\mu V^2 t)}-1]$ and its velocity at the time is $\frac{V}{\sqrt{(1+2\mu V^2 t)}}$.

[Ajmer 01, 09; Raj. 03; Jodhpur 01]

7. एक भारी कण वेग के वर्ग के समानुपाती प्रतिरोध उत्पन्न करने वाले माध्यम में ऊर्ध्वाधर दिशा में ऊपर की ओर फेंका जाता है। यदि अपने पथ में किसी बिन्दु पर इसकी गतिज ऊर्जा k हो तो सिद्ध करो कि जब वह अपने पथ पर नीचे गिरता हुआ उस बिन्दु से गुजरता है तो उसमें ऊर्जा का ह्रास $\frac{k^2}{k+k'}$ होगा, जहां k' उसकी नीचे की ओर गिरते समय अधिकतम (अन्तिम) ऊर्जा है।

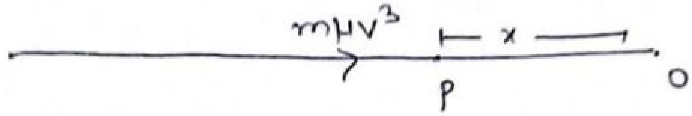
A heavy particle is projected vertically upwards in a medium the resistance of which varies as the square of the velocity. It has a kinetic energy k in its upward path at a given position. When it passes the same point on the way down, show that the loss of K.E. is $\frac{k^2}{k+k'}$; where k' is the limit to which the energy approaches in the downwards course.

[Raj. 02, 04, 06; Udaipur 05; Ajmer 13]

8. एक भारी कण ऊर्ध्वाधर ऊपर की ओर u वेग से ऐसे माध्यम में फेंका जाता है

Q 5 (sol). Let a particle of mass m with velocity V along a smooth horizontal plane, v in a medium whose resistance per unit mass is μ times the cube of the velocity, after some time from O particle is at P (where $OP = x$) and velocity at P is v,

i.e.



then equation of motion is

$$m\frac{d^2x}{dt^2} = -m\mu v^3$$

$$\Rightarrow \frac{d^2x}{dt^2} = -\mu v^3 \quad \text{--- (1)}$$

$$\because \frac{d^2x}{dt^2} = \frac{dv}{dt}$$

$$\therefore \frac{dv}{dt} = -\mu v^3$$

$$\Rightarrow -\frac{1}{v^3}\,dv = \mu\,dt$$

on integrating

$$\frac{1}{2v^2} = \mu t + c_1$$

initially at O, $t=0$, $v = V \Rightarrow c_1 = \frac{1}{2V^2}$

then $\frac{1}{2v^2} = \mu t + \frac{1}{2V^2}$

$$\Rightarrow 2v^2 = \frac{2V^2}{1+2\mu V^2 t}$$

$$\Rightarrow v = \frac{V}{\sqrt{(1+2\mu V^2 t)}} \quad \text{--- (2)}$$

velocity at the time t is $V/\sqrt{(1+2\mu V^2 t)}$

$\because \; v = \dfrac{dx}{dt}$

$\therefore$ by (2)

$$\frac{dx}{dt} = \frac{V}{\sqrt{(1+2\mu V^2 t)}}$$

$$\Rightarrow dx = \frac{V}{\sqrt{(1+2\mu V^2 t)}}\, dt$$

on int

$$x = \frac{1}{\mu V}\sqrt{(1+2\mu V^2 t)} + c_2$$

initi at O, $t=0,\ x=0 \Rightarrow c_2 = -\dfrac{1}{\mu V}$

then

$$x = \frac{1}{\mu V}\left[\sqrt{(1+2\mu V^2 t)} - 1\right]$$

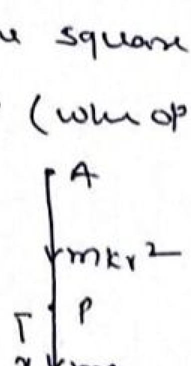
Hence particle described the distance

$$\frac{1}{\mu V}\left[\sqrt{(1+2\mu V^2 t)} - 1\right] \text{ is time } t.$$

Q 7 (sol): A particle of mass m is projected from O vertically upwards in a medium the resistance varies as the square of the velocity from O with velocity U

let after some time particle is at P (when OP = x) velocity at P is v

and the force acting at P i.e.



(i) Gravitational (mg) ↓

(ii) Resistance (mkv^2) ↓

then equation of motion at P is

$$m\frac{d^2x}{dt^2} = -mg - mkv^2$$

$$\Rightarrow \frac{d^2x}{dt^2} = -(g+kv^2) \quad \text{---} (1)$$

for kinetic energy we have to find velocity of particle.

$$\because \frac{d^2x}{dt^2} = v\frac{dv}{dx}$$

$$\therefore v\frac{dv}{dx} = -(g+kv^2)$$

$$\Rightarrow \frac{v}{g+kv^2}dv = -dx$$

$$\Rightarrow \frac{1}{2k}\log(g+kv^2) = -x + c_1$$

initially at O, $x=0$, $v=U \Rightarrow c_1 = \frac{1}{2k}\log(g+kU^2)$

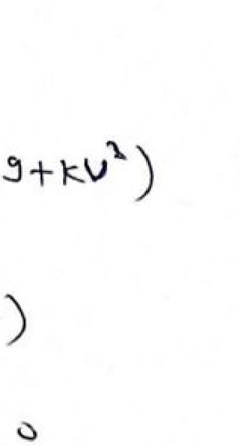
then

$$x = \frac{1}{2k}\log(g+kU^2) - \frac{1}{2k}\log(g+kv^2)$$

for maximum height at A, $x=H$, $v=0$

$$H = \frac{1}{2k}\log(g+kU^2) \cancel{-\frac{1}{2k}\log(g)}$$

when particle return eq^n of motion



$$m\frac{d^2x}{dt^2} = mg - mkv^2$$

$$\frac{d^2x}{dt^2} = g - kv^2 \quad \text{---} (3)$$

$$\Rightarrow v\frac{dv}{dx} = g - kv^2$$

$$\Rightarrow \frac{v}{g-kv^2}dv = dx$$

$$\Rightarrow -\frac{1}{2k}\log(g-kv^2) = x + c_2$$

at A, $x=0$, $v=0 \Rightarrow c_2 = -\frac{1}{2k}\log(g)$

then $x = \frac{1}{2k}\log(g) - \frac{1}{2k}\log(g-kv^2) \quad \text{---} (4)$

particle will return at the point of projection with velocity v_1 then $x = H$, $v = v_1$ by. (4)

$$H = \frac{1}{2k}\log(g) - \frac{1}{2k}\log(g - kv_1^2) \quad \text{---(5)}$$

by (2) & (5)

$$\frac{1}{2k}\log(g) - \frac{1}{2k}\log(g-kv_1^2) = \frac{1}{2k}\log(g+kU^2) - \frac{1}{2k}\log(g)$$

$$\Rightarrow \cancel{\log} \frac{g}{g-kv_1^2} = \frac{g+kU^2}{g}$$

$$\Rightarrow \frac{g}{g+kU^2} = \frac{g-kv_1^2}{g} = 1 - \frac{kv_1^2}{g}$$

$$\Rightarrow \frac{kv_1^2}{g} = 1 - \frac{g}{g+kU^2} = \frac{kU^2}{g+kU^2}$$

$$\Rightarrow \cancel{v_1^2 = \frac{g}{k}\left[1 - \frac{1}{\frac{g}{k}+U^2}\right]} \quad \text{---(6)}$$

$$v_1^2 = \frac{gU^2}{g+kU^2}$$

~~let~~ the terminal velocity of particle is V

then by (3) $\quad 0 = g - kV^2$

$$\Rightarrow V^2 = \frac{g}{k}$$

then $\cancel{v_1^2 = V^2\left[1 - \frac{1}{V^2+U^2}\right]}$

$$v_1^2 = \frac{g\,V^2U^2}{V^2+U^2}$$

Now K.E of particle when particle project $K = \frac{1}{2}mU^2$

K.E of particle when particle return to O $K_1 = \frac{1}{2}mv_1^2$

K.E of particle ~~when at~~ terminal vel $K' = \frac{1}{2}mV^2$

Loss of K.E is $k - k_1 = \frac{1}{2}mu^2 - \frac{1}{2}mv_1^2$

$$= \frac{1}{2}m\left[u^2 - v_1^2\right]$$

$$= \frac{1}{2}m\left[u^2 - \frac{v^2u^2}{u^2+v^2}\right]$$

$$= \frac{1}{2}m\left[\frac{u^4}{u^2+v^2}\right]$$

$$= \frac{\left(\frac{1}{2}mu^2\right)\left(\frac{1}{2}mu^2\right)}{\frac{1}{2}mu^2 + \frac{1}{2}mv^2}$$

$$= \frac{k^2}{k+k^1}$$

Hence the loss of, kello is $\frac{k^2}{k+k^1}$