Unit – IV

Alternating Current

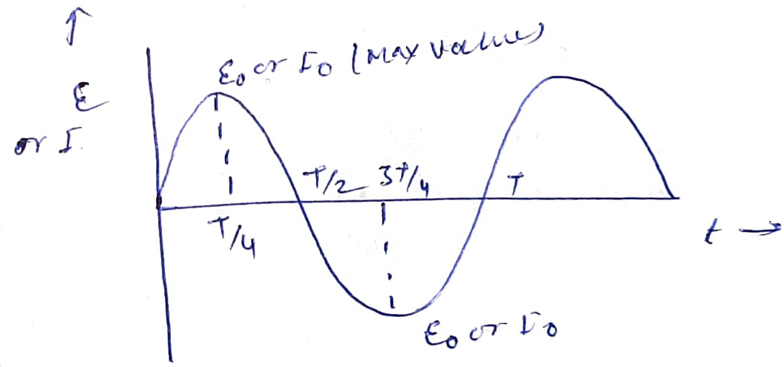
Alternating current

It is in general a current, that is a periodic function of time. [OR]

It is one which changes in magnitude and direction periodically. [OR]

It is one which passes through a cycle of changes at regular intervals. Each cycle consists of two half cycles during one of which the current is entirely positive ~~and~~ whereas during following half cycle the current is entirely negative.

In alternating current circuit, we are concerned with a steady state current and voltage, which are oscillating ~~simultaneously~~ sinusoidally without change in magnitude



The alternating e.m.f E at any ~~stat~~ instant t is

$$E = E_0 \sin \omega t = E_0 e^{j\omega t}$$

and current

$$I = I_0 \sin \omega t = I_0 e^{j\omega t}$$

where E_0 and I_0 are the peak values of e.m.f and current respectively.

j is complex no. $j = \sqrt{-1}$

ω is angular freq. of alternating voltage

$f = \frac{\omega}{2\pi}$ freq. of alternating voltage, ωt is phase angle

and time period of alternating voltage

$$T = \frac{1}{f} = \frac{2\pi}{\omega}$$

The alternating current in the circuit, applied by alternating e.m.f, may be controlled by inductance L, resistance R and capacitance C.

Due to the presence of L and C, the current is not necessarily in the phase with the applied e.m.f

therefore the alternating current is

$$I = I_0 \sin(\omega t + \phi)$$

where ϕ is the phase, which may be positive, negative or zero depending on the nature of circuit.

(A) ~~Average value of~~

Let value of alternating current at any instant is

$$I = I_0 \sin \omega t$$

(i) The average value of a sinusoidal wave over one complete cycle is given by

$$I_{av} = \frac{\int_0^T I\,dt}{\int_0^T dt} = \frac{\int_0^T I_0 \sin \omega t\,dt}{T}$$

$$I_{av} = \frac{I_0}{T}\left[\frac{\cos\omega t}{\omega}\right]_0^T$$

$$= \frac{I_0}{\omega T}\left[-\cos\frac{2\pi t}{T}\right]_0^T \quad \text{Since } \omega = \frac{2\pi}{T}$$

$$= -\frac{I_0}{\frac{2\pi}{T}\times T}\left[\cos 2\pi - \cos 0\right]$$

$$= -\frac{I_0}{2\pi}\left[1-1\right] = 0$$

Thus the average value of A.C. over one complete cycle is zero. The same is also true for alternating e.m.f.

(ii) Average value of A.C. or voltage during half cycle

The average over one half cycle is finite

$$(I_{av})_{\text{half cycle}} = \frac{\int_0^{T/2} I\,dt}{\int_0^{T/2} dt} = \frac{\int_0^{T/2} I_0 \sin\omega t\,dt}{T/2}$$

$$= \frac{2I_0}{T}\left[-\frac{\cos\omega t}{\omega}\right]_0^{T/2}$$

$$= -\frac{2I_0}{T\omega}\left[\cos\frac{2\pi t}{T}\right]_0^{T/2}$$

$$= -\frac{2I_0}{T\omega}\left[\cos\pi - \cos 0\right]$$

$$= -\frac{2I_0}{\frac{2\pi}{\omega}\omega}\left[-2\right]$$

$$= \frac{2I_0}{\pi}$$

$$(I_{av})_{\text{half cycle}} = \frac{2}{3.14} I_0 = 0.637\, I_0$$

Similarly for sinusoidal alternating voltage, average value over half cycle is

$$(E_{av})_{\text{I half cycle}} = \frac{2E_0}{\pi} = 0.637 E_0$$

The average value during the second half cycle is

$$(I_{av})_{\text{II half cycle}} = \frac{\int_{T/2}^{T} I\,dt}{\int_{T/2}^{T} dt} = \frac{I_0 \int_{T/2}^{T} \sin\omega t\,dt}{(T - T/2)}$$

$$= \frac{I_0}{T/2}\left[-\frac{\cos\omega t}{\omega}\right]_{T/2}^{T}$$

$$= -\frac{2I_0}{T\omega}\left[\cos \frac{2\pi}{T} t\right]_{T/2}^{T}$$

$$= -\frac{2I_0}{\frac{2\pi}{\omega}\cdot\omega}\left[\cos 2\pi - \cos\pi\right]$$

$$= -\frac{2I_0}{2\pi}\left[1+1\right]$$

$$(I_{av})_{\text{II half cycle}} = \frac{2I_0}{\pi} = 0.637 I_0$$

The average value of alternating current (or voltage) during the first and second half cycles are equal and opposite in sign. i.e. they are alternately positive and negative so that the average value over one complete cycle is zero.

(iii) Root mean square or virtual or effective value of alternating current.

The R.M.S value of alternating current is defined as the square root of mean square current during complete cycle.

$$I_{rms} = \sqrt{\overline{I^2}} = \sqrt{\langle I^2 \rangle}$$

Then
$$\overline{I^2} = \frac{\int_0^T I^2 dt}{\int_0^T dt} = \frac{\int_0^T I_0^2 \sin^2 \omega t \, dt}{T}$$

$$= \frac{I_0^2}{T} \int_0^T \sin^2 \omega t \, dt$$

$$= \frac{I_0^2}{T} \int_0^T \left(\frac{1 - \cos 2\omega t}{2} \right) dt$$

$$= \frac{I_0^2}{2T} \left[t - \frac{\sin 2\omega t}{2\omega} \right]_0^T$$

$$= \frac{I_0^2}{2T} \left[T - \frac{\sin 2\omega T}{2\omega} - 0 + \frac{\sin 0}{2\omega} \right]$$

$$= \frac{I_0^2}{2T} \left[T - \frac{\sin 4\pi}{2\omega} + \frac{0}{2\omega} \right]$$

$$= \frac{I_0^2}{2T} \left[T - 0 - 0 \right]$$

$$= \frac{I_0^2}{2T} \times T$$

$$\overline{I^2} = \frac{I_0^2}{2} \quad \text{or} \quad \langle \sin^2 \omega t \rangle = \frac{1}{2}$$

$$I_{rms} = \sqrt{\langle I^2 \rangle} = \sqrt{\frac{I_0^2}{2}} = \frac{I_0}{\sqrt{2}}$$

$$I_{rms} = \frac{I_0}{\sqrt{2}} = 0.707 I_0$$

The effective or RMS value of alternating current is $\frac{1}{\sqrt{2}}$ times its peak value.

Similarly R.M.S. value of alternating voltage ~~E = E0 sin~~ $E = E_0 \sin \omega t$ is

$$E_{rms} = \sqrt{\langle E^2 \rangle} = \frac{E_0}{\sqrt{2}}$$

$$E_{rms} = \frac{E_0}{\sqrt{2}} = 0.707 E_0$$

When an AC $I = I_0 \sin \omega t$ passes through the resistance R, the instantaneous rate of heating is $I^2 R$

Hence average rate of heating for a full cycle is

$$H = \frac{\int_0^T I^2 R\, dt}{\int_0^T dt} = \frac{R \int_0^T I_0^2 \sin^2 \omega t\, dt}{\int_0^T dt}$$

$$= R I_0^2 \overline{\sin^2 \omega t}$$

$$= R I_0^2 \cdot \frac{1}{2}$$

$$= \frac{I_0^2}{2} R$$

$$H = I_{rms}^2 R$$

Thus the average rate of heating would be produced by a direct current of value $I = I_{rms}$

form factor

It is defined as the ratio of r.m.s value of alternating current ~~or~~ or voltage to average value of alternating current or voltage

$$\text{form factor} = \frac{I_{rms}}{I_{av}} = \frac{E_{rms}}{E_{av}}$$

$$= \frac{I_0}{\sqrt{2}} \cdot \frac{\pi}{2I_0}$$

$$= \frac{\pi}{2\sqrt{2}} = 1.11$$

The form factor gives an indication of the wave ~~sp~~shape of an alternating voltage or current

$$E = E_0 \sin(\omega t + \phi)$$

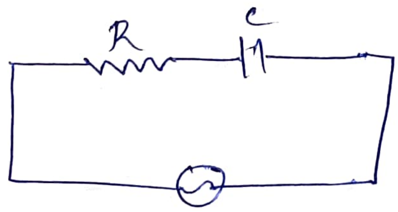
and

$$I = I_0 \sin(\omega t + \phi)$$

where ϕ is phase angle and added to the phase ωt.

AC Circuit

An interesting and perhaps more important problem is to determine what happens when an alternating current generator is used in the circuit as battery is used DC circuit.



The new problem in the AC case is to determine the steady state solution, which is, ~~one~~ of course, the particular solution of differential eqn. for the given circuit containing R and C connected in series with AC generator. Here the steady state current is not zero as in case of direct current. In one cycle, this capacitor is charged, discharged, recharged oppositely and the discharged once more. This happens again and again.

There are two method used to determine the current in such circuit.

(i) Vector method

(ii) Complex no. method.

<u>Complex number method for AC circuit analysis</u>

The A.C. network analysis becomes the simple and convenient by the complex no. representation.

A complex no. can be written as

$$\vec{Z} = x + jy$$

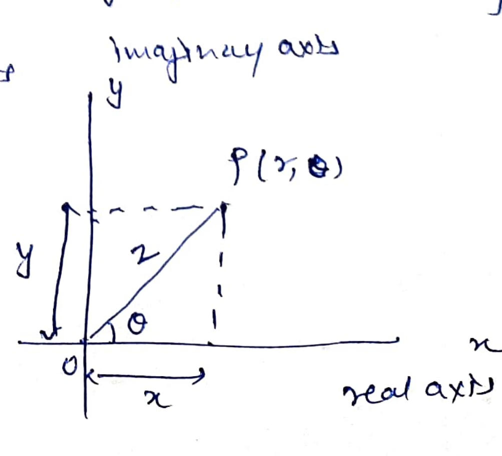
where x and y are real quantities

and $j = \sqrt{-1}$ is imaginary quantity

$\vec{Z}$ comprises real quantity and an imaginary quantity

Z is complex quantity

The complex no.'s are represented in cartesian coordinates by a vector in a complex plane by choosing real axis along x axis and imaginary axis along y-axis as shown in figure.



The complex no. is defined by point P since projection along real axis OP gives real part and its projection along y-axis gives Imaginary part of no.

The length of line ($\vec{OP} = \vec{z}$) represent the absolute value or modulus of complex no. and angle θ is the phase of this no.

Thus the vector properties of complex quantities suggest that it may be ~~convenient~~ convenient to express them in terms of amplitude and direction as well as in real and Imaginary part

If point p has polar coordinates (r, θ) then

$$x = r\cos\theta$$

$$y = r\sin\theta$$

Then $\vec{z} = r\cos\theta + jr\sin\theta$

$$= r(\cos\theta + j\sin\theta)$$

$$= re^{j\theta}$$

where $r = \sqrt{x^2 + y^2}$ is called modulus

and $\theta = \tan^{-1} y/x$ gives the phase of complex quantity

Important point about Complex no.

(i) Equality

Two complex no.'s are said to be equal if both their real parts and their Imaginary parts are equal

$$z_1 = x_1 + jy_1$$

$$z_2 = x_2 + jy_2$$

then equality $z_1 = z_2$ gives the two real relation

$$x_1 = x_2$$

and $y_1 = y_2$

(ii) Complex conjugate

the complex conjugate of $z = x+iy$ is

$$z^* = x - iy$$

then its product

$$zz^* = (x+iy)(x-iy)$$

$$zz^* = x^2+y^2 \text{ is it is real}$$

(iii) Addition and substraction

the complex no's are added and substracted like vector quantities

let

$$\vec{z}_1 = x_1 + iy_1$$

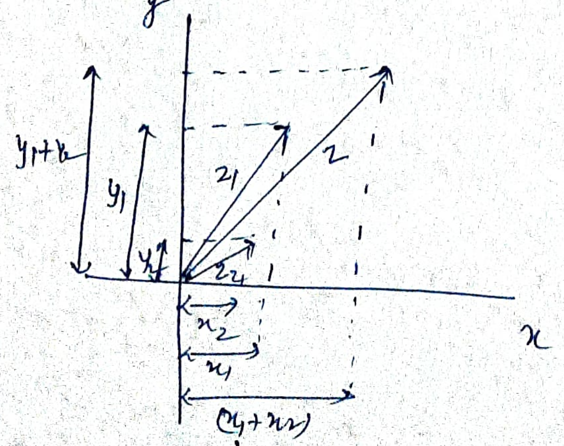
$$\vec{z}_2 = x_2 + jy_2$$

then

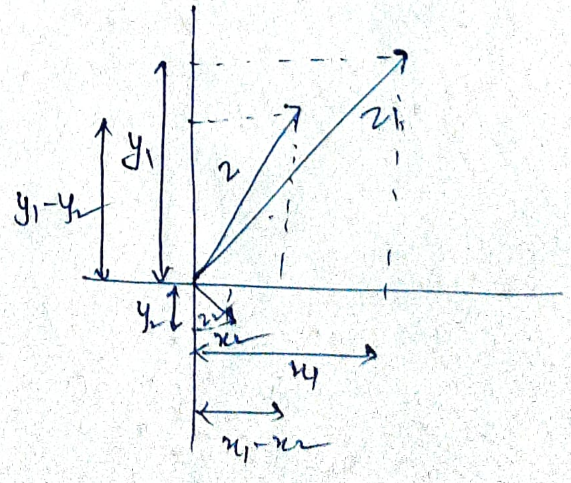
$$\vec{Z}_{add} = \vec{z}_1 + \vec{z}_2 = (x_1+x_2) + j(y_1+y_2)$$

$$\vec{Z}_{sub} = \vec{z}_1 - \vec{z}_2 = (x_1-x_2) + j(y_1-y_2)$$

graphical representation



addition



Substraction

The magnitude of z is

$$z_{add} = \sqrt{(x_1+x_2)^2 + (y_1+y_2)^2}$$

$$z_{sub} = \sqrt{(x_1-x_2)^2 + (y_1-y_2)^2}$$

and phase is

$$\alpha = \tan^{-1} \frac{y_1+y_2}{x_1+x_2} \quad \text{for addition}$$

$$\beta = \tan^{-1} \frac{y_1-y_2}{x_1-x_2} \quad \text{for subtraction}$$

The ~~sum~~ sum of complex no. is a complex no. whose real part is the algebraic sum of real part of added no's and its imaginary part is the algebraic sum of imaginary part of added no's. The same is also true for subtraction

Multiplication and Division

Multiplication

The products of complex no.'s are defined according to the usual algebraic rules.

The product of z_1 and z_2 is

$$\vec{z}_{multi} = \vec{z}_1 \vec{z}_2 = (x_1+iy_1)(x_2+iy_2)$$

$$= (x_1x_2 - y_1y_2) + j(x_1y_2 + y_1x_2)$$

The product of complex no's is also complex no.

Division

Division can be defined as inverse of multiplication.
When complex no's are divided, it is first necessary to eliminate the imaginary unit contained in the denominator and then to

... the complex no. termwise. Thus:

$$\vec{z}_{div} = \frac{\vec{z}_1}{\vec{z}_2} = \frac{x_1 + jy_1}{x_2 + jy_2}$$

then $$\vec{z}_d = \frac{\vec{z}_1}{\vec{z}_2} \cdot \frac{\vec{z}_2^*}{\vec{z}_2^*} = \frac{(x_1+jy_1)}{(x_2+jy_2)} \cdot \frac{(x_2-jy_2)}{(x_2-jy_2)}$$

$$= \frac{(x_1x_2 + y_1y_2) + j(x_2y_1 - y_2x_1)}{x_2^2 + y_2^2}$$

$$= \frac{x_1x_2 + y_1y_2}{x_2^2 + y_2^2} + j\,\frac{x_2y_1 - x_1y_2}{x_2^2 + y_2^2}$$

and Alternatively

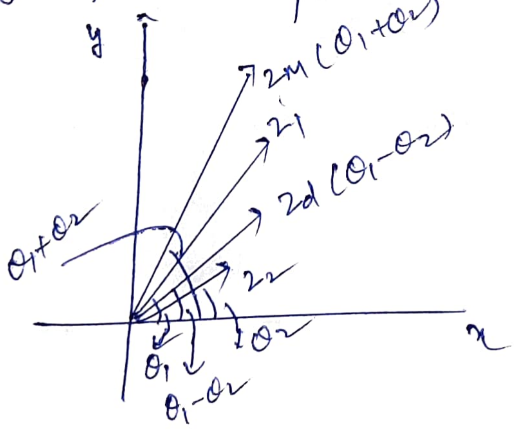
$$\vec{z}_1 = r_1 e^{j\theta_1} \text{ and } \vec{z}_2 = r_2 e^{j\theta_2}$$

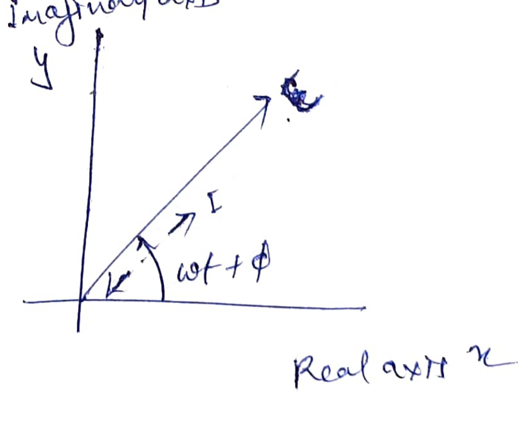
then $$\vec{z}_M = \vec{z}_1 \cdot \vec{z}_2 = r_1 r_2 e^{j(\theta_1 - \theta_2)}$$

and $$z_d = \frac{r_1}{r_2} e^{j(\theta_1 - \theta_2)}$$

Thus the product as well as division of complex no's are the complex no's

graphical representation





The magnitude of product of two complex no.s is product of magnitudes of given complex no's. and phase is algebraic sum of the cofactors

The magnitude of quotient (division) of two complex no's is the magnitudes of the dividend and divisor and phase is algebraic difference of phase of dividend and divisor

Complex representation of A.C. circuit quantity

AC network analysis can be simplified by expressing sinusoidal voltages and currents in complex no. representation rather than vector form.

Hence sinusoidal voltage ~~and~~ $E = E_0 \sin(\omega t + \phi)$ and current $I = I_0 \sin(\omega t + \phi)$ as expressed that is general forms are nothing but the imaginary parts of $E_0 e^{j(\omega t+\phi)}$ and $I_0 e^{j(\omega t+\phi)}$ respectively.

Impedance, reactance and admittance

AC in a circuit may be controlled by resistance, inductance and capacitance, while the DC current may be controlled only by resistance.

In AC circuit the ratio of potential difference across the circuit and the current flowing therein is termed as impedance and denoted as

$$Z = \frac{E}{I}$$

The impedance of ~~a~~ A.C. circuit is the hindrance offered by circuit to flow the A.C. through it.

Reactance (X)

The hindrance offered by inductance and capacitance to the flow of A.C. in the A.C. circuit is called the reactance and denoted by X

Thus when there is no ohmic resistance in the circuit, the reactance is equal to the impedance.

$$X = Z$$

The reactance due to inductance is called inductive reactance and denoted by X_L

The reactance due to capacitance is called capacitive reactance and denoted by X_C

The reactance due the capacitance and inductance is denoted by $X = X_L + X_C$

Admittance (Y)

The inverse of impedance is called the admittance and is denoted by Y such as

$$Y = \frac{1}{Z}$$

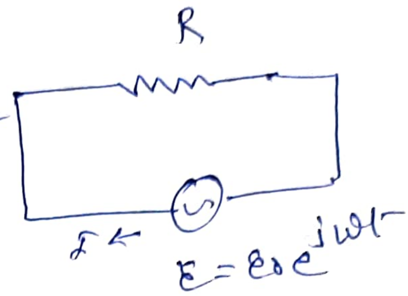
for any element in a circuit the product of admittance and voltage across it gives the current through that element

$$I = YE = YV$$

The real part of admittance is known as conductance and imaginary part is known as susceptance.

The circuit containing pure resistance only

Let a circuit containing a pure R with alternating voltage $E = E_0 \sin \omega t$ applied across it.



This voltage is the imaginary part of complex no. $E_0 e^{j\omega t}$

then $$E = E_0 e^{j\omega t}$$

If I is the current in the circuit then

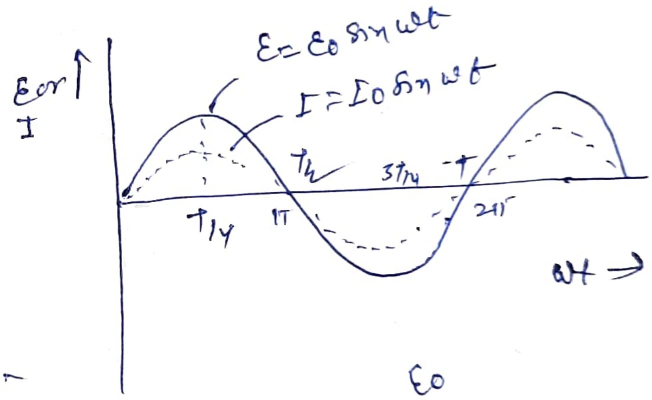
$$RI = E_0 e^{j\omega t}$$

$$I = \frac{E_0}{R} e^{j\omega t}$$

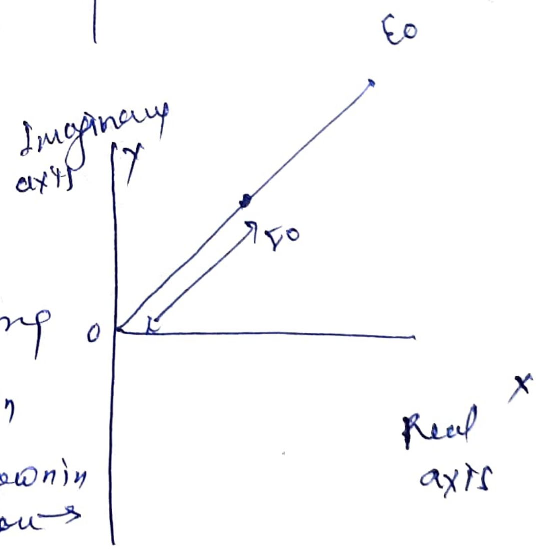
$$I = I_0 e^{j\omega t} \quad -(1)$$

where $I_0 = \frac{E_0}{R}$

max. value of current



Eqn (1) represents the variation of current with time t in circuit.



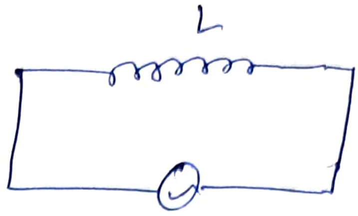
The current has the same form and freq as applied voltage. Thus current is in the phase with applied voltage as shown in figure →

Here R is impedance of resistance

$Z = R$ then

$$Y = 1/Z = 1/R$$

The circuit containing pure inductance only



Let circuit containing an inductor of L with an alternating voltage $\mathcal{E} = \mathcal{E}_0 \sin\omega t$ applied across it.

Then voltage in complex no. representation

$$\mathcal{E} = \mathcal{E}_0 e^{j\omega t}$$

If I is the instantaneous current in the circuit. Then e.m.f induced in L is

$$\mathcal{E}_{ind} = -L\frac{dI}{dt}$$

Where $\frac{dI}{dt} \rightarrow$ rate of change of current

To maintain the current flow through the inductor, the applied voltage must be equal to and opposite to the induced voltage. The emf eq^n of circuit

$$\mathcal{E} = \mathcal{E}_{ind} = L\frac{dI}{dt} \quad \text{or}$$

$$\mathcal{E} - L\frac{dI}{dt} = 0$$

$$\frac{dI}{dt} = \frac{\mathcal{E}_0 e^{j\omega t}}{L}$$

$$dI = \frac{\mathcal{E}_0 e^{j\omega t}}{L} dt$$

$$I = \frac{\mathcal{E}_0}{L}\left[\frac{1}{j\omega} e^{j\omega t}\right]$$

$$I = \frac{\mathcal{E}_0}{L\omega}(-j) e^{j\omega t}$$

$$I = \frac{\mathcal{E}_0}{\omega L} e^{-j\pi/2} e^{j\omega t}$$

$$I = \frac{\mathcal{E}_0}{\omega L} e^{j(\omega t - \pi/2)}$$

$$I = \frac{\mathcal{E}_0}{L}\frac{e^{j\omega t}}{j\omega} + A$$

at $t = 0$, $I = 0$

$$0 = \frac{\mathcal{E}_0}{L}\frac{1}{j\omega} + A$$

$$= -\frac{\mathcal{E}_0}{L}\frac{1}{j\omega}$$

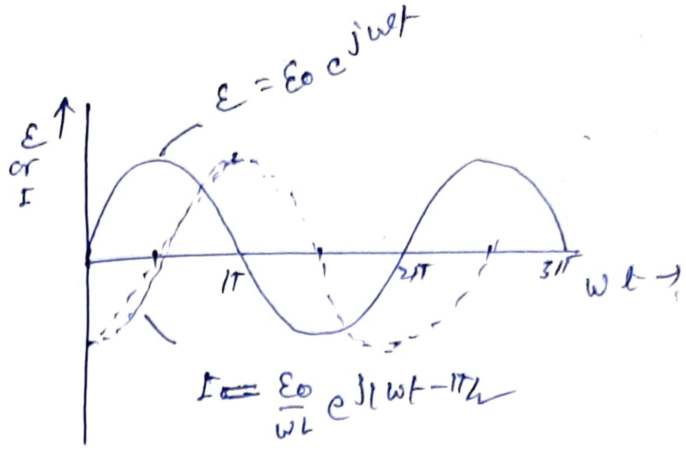
$$I = -\frac{\mathcal{E}_0}{L}\cdot\frac{1}{j\omega}\left(1 - e^{j\omega t}\right)$$

$$\frac{1}{j} = -j = e^{-j\pi/2}$$

$$= \cos\pi/2 - j\sin\pi/2 = -j$$

The quantity ωL is called inductive reactance of inductance and denoted by X_L. Thus

$$X_L = \omega L$$



Then

$$I = \frac{\mathcal{E}_0}{X_L} e^{j(\omega t - \pi/2)}$$

$$= I_0 e^{j(\omega t - \pi/2)}$$

where $I_0 = \frac{\mathcal{E}_0}{X_L} = \frac{\mathcal{E}_0}{\omega L}$

is the peak value of current

$$X_L = \omega L = \frac{\mathcal{E}_0}{I_0}$$

Thus inductive reactance is obtained by dividing the peak value of voltage by peak value of current.

Again $X_L = \omega L = 2\pi f L$ shows that X_L is directly proportional to freq of A.C. circuit

$X_L \propto f$

Its value for DC supply is zero ~~since~~

$$X_L = \frac{\mathcal{E}_0}{I_0} = \frac{\mathcal{E}_0/\sqrt{2}}{I_0/\sqrt{2}} = \frac{\mathcal{E}_{eff}}{I_{eff}}$$

since ~~or~~ L freq for D.C. source is zero.

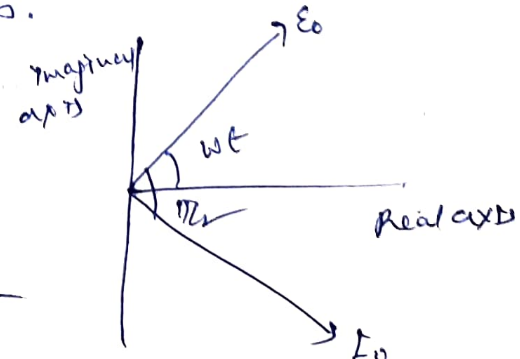
Thus X_L may be defined as ratio of r.m.s of e.m.f across the inductance to r.m.s of current flowing through it.

The effect of inductance reactance $X_L = \omega L$ in AC circuit is same as the resistance R in DC circuit to reduce the current in it, but without power loss.

The current I in the L is obtained by

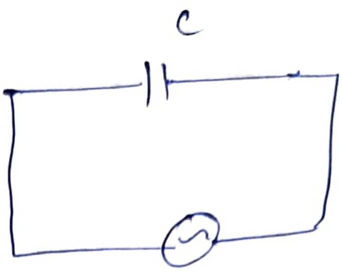
$$I = \frac{\text{voltage}}{j\omega L}$$

The current lags behinds the voltage by $\pi/2$ as shown in figure



the circuit containing only capacitance

Let the circuit containing a capacitor of capacitance with alternating voltage $\mathcal{E} = \mathcal{E}_0 \sin \omega t$ i.e. in complex no. representation



$$\mathcal{E} = \mathcal{E}_0 e^{j\omega t} \quad \text{(Imaginary part)}$$

the instantaneous charge on C at time t is

$$q = C\mathcal{E} = C\mathcal{E}_0 e^{j\omega t}$$

instantaneous current in the circuit is

$$I = \frac{dq}{dt} = C\mathcal{E}_0 \cdot j\omega e^{j\omega t}$$

$$= \frac{\mathcal{E}_0}{1/\omega C} j e^{j\omega t}$$

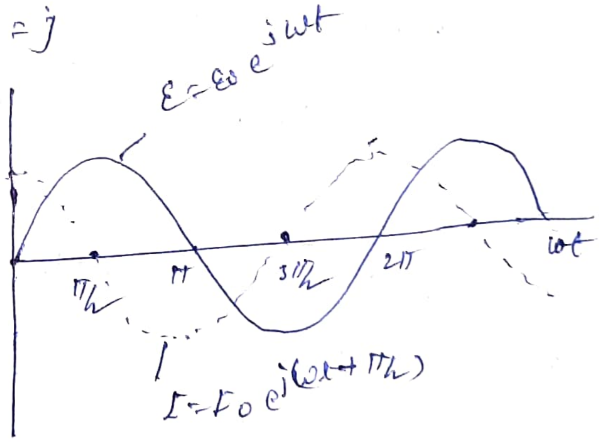
$$I = \frac{\mathcal{E}_0}{1/j\omega C} e^{j\omega t}$$

But $e^{j\pi/2} = \cos \pi/2 + j \sin \pi/2 = j$

Hence $I = \dfrac{\mathcal{E}_0}{1/\omega C} e^{j(\omega t + \pi/2)}$

$$I = \frac{\mathcal{E}_0}{X_C} e^{j(\omega t + \pi/2)}$$

$$I = I_0 e^{j(\omega t + \pi/2)}$$



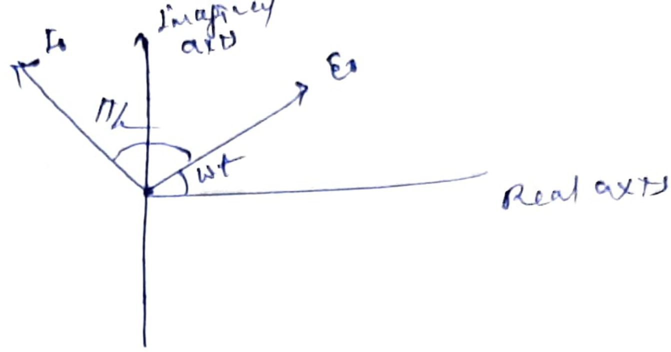
where $X_C = \dfrac{1}{\omega C}$ is known as capacitive reactance and

$I_0 = \dfrac{\mathcal{E}_0}{X_C}$ is peak value of current

the current leads the voltage by $\pi/2$ as shown in fig.

the complex no. representation is shown in figure. the current I in circuit is obtained

$$I = \frac{\mathcal{E}}{1/j\omega C}$$



Since $X_C \propto \frac{1}{\omega C} \propto \frac{1}{2\pi f C}$

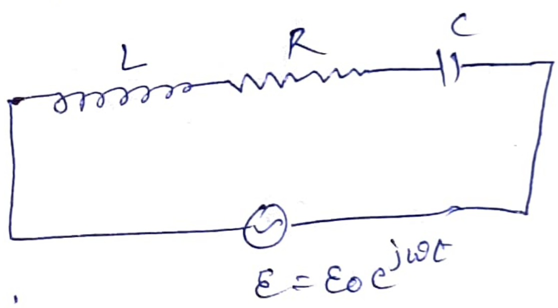
then $X_C \propto \frac{1}{f}$

Hence capacitor offers negligible reactance at high freq. and work as bypass capacitor and very high reactance at low freq. and work as blocking capacitor.

reactance for DC is infinity

A.C. Circuit with resistance, capacitance and inductance

let alternating e.m.f. $\mathcal{E} = \mathcal{E}_0 e^{j\omega t}$ is applied to the series combination of L and R.



The impedance of combination is sum of three parts

(i) R due to resistance R

(ii) $j\omega L$ due to inductance L

(iii) $\frac{1}{j\omega C} = -j/\omega C$ due to capacitance C

Then resultant e.m.f eqn of circuit is

$$\left[R + j\left(\omega L - \frac{1}{\omega C}\right)\right] I = \mathcal{E}_0 e^{j\omega t}$$

where I is instantaneous current then

$$I = \frac{\mathcal{E}_0 e^{j\omega t}}{R + j\left(\omega L - \frac{1}{\omega C}\right)}$$

Then .

$$\frac{1}{R+j(\omega L-1/\omega C)} = \frac{R-j(\omega L-1/\omega C)}{[R+j(\omega L-1/\omega C)][R-j(\omega L-1/\omega C)]}$$

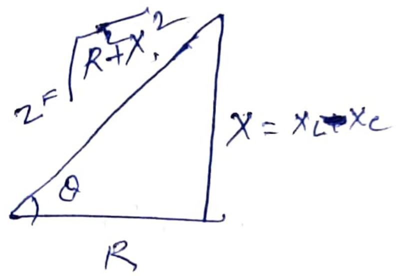
$$= \frac{R-j(\omega L-1/\omega C)}{R^2+(\omega L-1/\omega C)^2} = \alpha - jB \quad (\text{say})$$

where

$$\alpha = \frac{R}{R^2+(\omega L-1/\omega C)^2}$$

and

$$B = \frac{(\omega L-1/\omega C)}{R^2+(\omega L-1/\omega C)^2}$$



If $\tan\theta = \frac{\omega L - 1/\omega C}{R}$

Then $\cos\theta = \frac{R}{\sqrt{R^2+(\omega L-1/\omega C)^2}}$

and $\sin\theta = \frac{\cancel{R}(\omega L-1/\omega C)}{\sqrt{R^2+(\omega L-1/\omega C)^2}}$

Hence

$$\frac{1}{R+j(\omega L-1/\omega C)} = \frac{1}{\sqrt{R^2+(\omega L-1/\omega C)^2}}(\cos\theta - j\sin\theta)$$

$$= \frac{1}{\sqrt{R^2+(\omega L-1/\omega C)^2}}\, e^{-j\theta}$$

Then

$$I = \frac{E_0}{\sqrt{R^2+(\omega L-1/\omega C)^2}}\, e^{j(\omega t-\theta)}$$

$$I = I_0\, e^{j(\omega t-\theta)}$$

where $I_0 = \frac{E_0}{\sqrt{R^2+(\omega L-1/\omega C)^2}}$ peak value of current and $\sqrt{R^2+(\omega L-1/\omega C)^2} = Z$ (impedance)

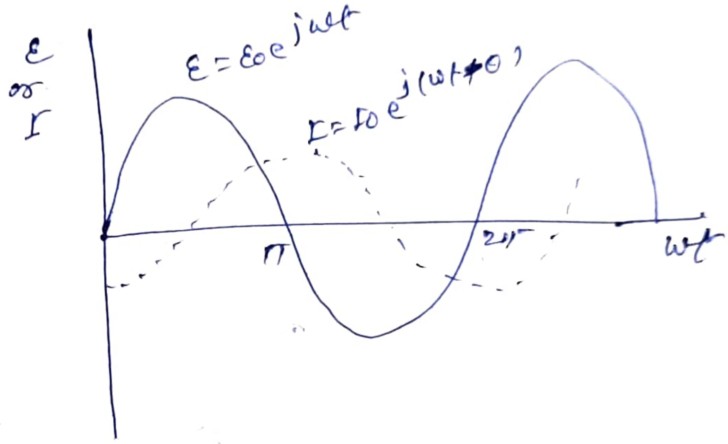
Z is ratio of peak ~~emf~~ rms of voltage to peak ~~emf~~ rms of current

thus

$$Z = \frac{E_0}{I_0} = \frac{E_0/\sqrt{2}}{I_0/\sqrt{2}} = \frac{E_{rms}}{I_{rms}}$$

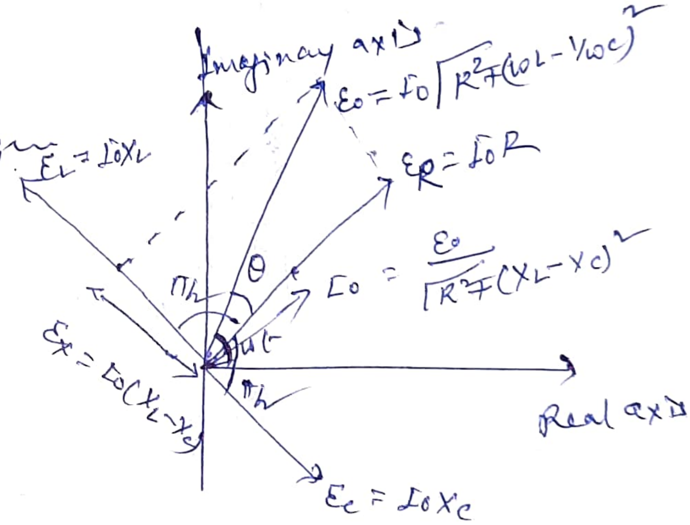
the current lags behind the applied voltage in phase by angle θ and given by

$$\theta = \tan^{-1}\left(\frac{\omega L - 1/\omega C}{R}\right)$$



the e.m.f. and current have the phase difference of θ.

the phase difference is shown in figure in complex representation



the voltage across the R (E_R) in phase with current

$$E_X = E_L - E_C$$

which is net voltage across the combination of L and C

and ~~lags behind~~ ~~leads~~ Leads the current by angle $\pi/2$.

the total voltage ~~volt~~ E_0 is shown in figure leading the current by angle

$$\theta = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$$

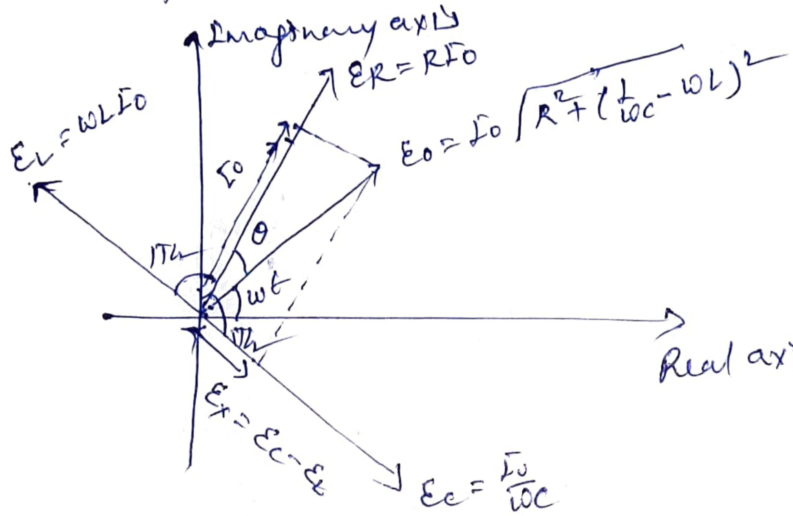
<u>Special ~~gas~~ case</u>

the phase difference depends on the value of θ, which depends on values of X_L and X_C

Case I

If $\omega L > 1/\omega C$ when impressed freq ($f = \frac{\omega}{2\pi}$) is very large then phase angle θ will be positive and current will lag behind the applied e.m.f.

The potential difference $I_0 X_L = I_0 \omega L$ across the inductor is greater than potential difference $I_0/\omega C$ across the capacitor and circuit behaves as inductive circuit. The vector voltage in complex plane is shown above



Case II If $\omega L < 1/\omega C$ then phase angle θ will be negative and current will lead the applied voltage. The circuit behaves as capacitive circuit. The vector voltage in complex plane is shown above

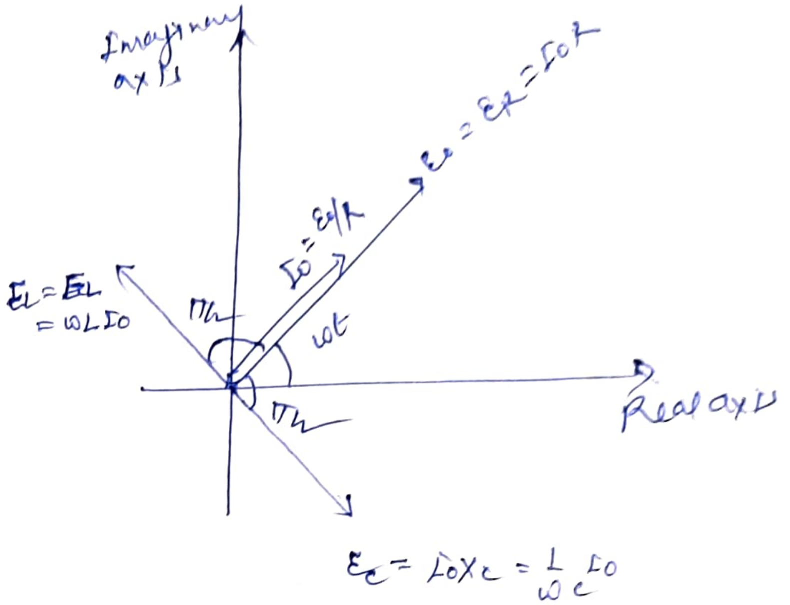
Case III

If $\omega L = \frac{1}{\omega C}$ (Important and interesting case)

then $\theta = \tan^{-1}\left(\frac{\omega L - 1/\omega C}{R}\right) = 0$

$\theta = 0$

Therefore current and e.m.f will be in phase. The potential differences across the L and C are equal in magnitude but opposite in phase and therefore cancel out and whole voltage is dropped across the resistance. The vector diagram in complex plane is shown



using this condition the peak value will be max.

$$I_0 = \frac{E_0}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}$$

$$I_0 = \frac{E_0}{R}$$

when $\omega L = 1/\omega C$ the e.m.f and current will be in phase and current is max. This circuit is said to be a series resonant circuit and phenomenon of maximum current is called resonance

At the resonance

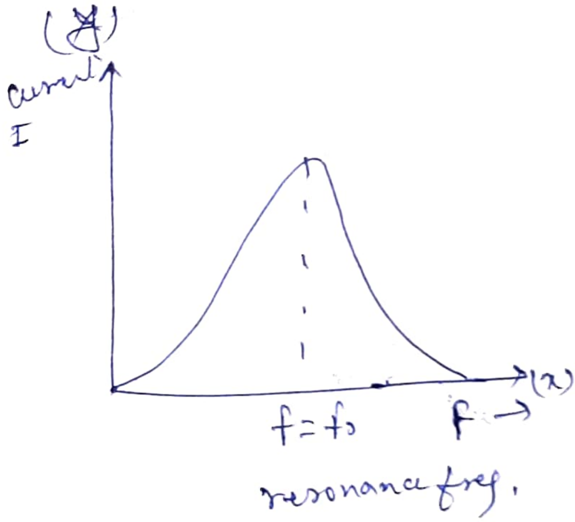
$$\omega_r L = \frac{1}{\omega_r C}$$

$$\omega_r^2 = \frac{1}{LC}$$

$$\omega_r = \sqrt{\frac{1}{LC}} = \frac{1}{\sqrt{LC}}$$

$$2\pi f_r = \frac{1}{\sqrt{LC}}$$

$$f_r = \frac{1}{2\pi\sqrt{LC}}$$



~~for~~ fr is freq. is called resonance freq. The resonant freq. is the same as freq. of oscillatory discharge of a series LCR circuit when the resistance is low. Thus series LCR circuit is in resonance with applied voltage. The freq. of applied voltage ~~Conci~~ coincides with natural freq. of circuit. as shown in figure

Sharpness of resonance of a series LCR circuit

The current amplitude is given by

$$I_0 = \frac{E_0}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}$$

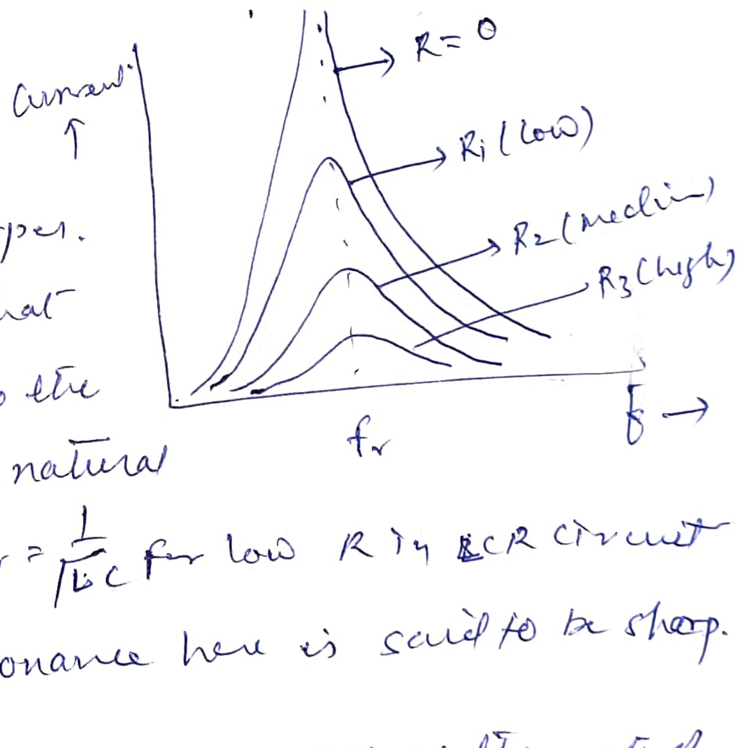
At resonance

$$\omega L = \frac{1}{\omega C}$$

then

$$I_{0\,max} = \frac{E_0}{R}$$

The variation of resistance on the current with varying freq. is shown in figure. The curve obtained by plotting amplitude of current against ω or f are known as resonance curve



As the R is reduced the resonance curve becomes sharper. The peak value of I_0 shows that the circuit responds only to the freq. exactly equal to its natural freq. of the circuit ω_r, $\omega_r = \frac{1}{\sqrt{LC}}$ for low R in LCR circuit and to none other. The resonance here is said to be sharp.

Thus the sharpness of resonance is a measure of the rate of fall of amplitude of current from its max. value ~~to~~ at resonant freq. on either side of it.

The more quickly the current amplitude falls for changes of freq. on both side of the resonance freq. f_r, the sharper is said to be resonance.

If the amplitude remains more or less at it peak value over an ~~apprec~~ appreciable range of freq. on either side of resonance freq. Thus the circuit responds to a no. of freq's near about f_r on either side of it. The resonance in this case is said to be flat.

Quality factor of circuit (Q)

Qualitatively the sharpness of resonance curve is determin by a quality factor 'Q' of the circuit.

It is defined as the ratio of reactance of either the inductance or capacitance at resonance freq. to the total resistance of circuit. Thus

$$Q = \frac{X_L}{R} = \frac{\omega_r L}{R}$$

$$Q = \frac{X_C}{R} = \frac{1}{\omega_r C R}$$

Since at resonance

$$X_L = X_C \Rightarrow \omega_r L = \frac{1}{\omega_r C}$$

How the sharpness depends on Q.

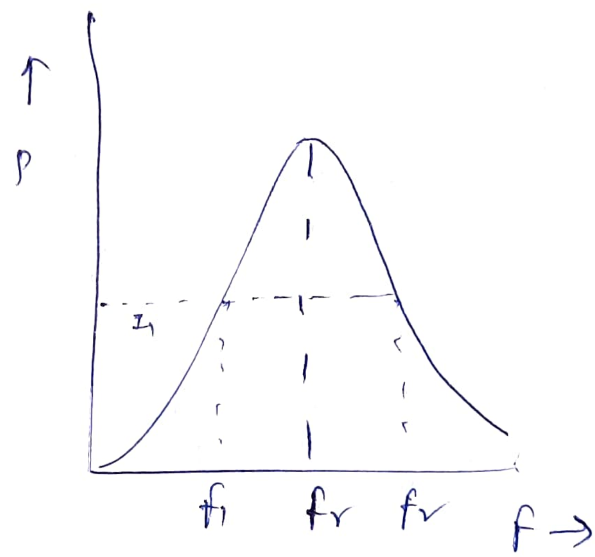
At the resonance current is inversely proportional to the resistance $(I_0 = \frac{E}{R})$. so directly proportional to Q. The value of current for freq. appreciably different from resonance freq. is mainly determined by reactance and not by the resistance. Thus the ~~circuit~~ current is nearly independ of resistance of circuit thus results of increasing the resistance of circuit (lowering Q) reduces the ~~respond~~ response at the resonant freq. but leaves the other freq nearly unaffected. Therefore if Q is large the resonance curve is sharp.

Q is also defined in terms of lower and upper half power freq. The half power freq. are the freq. at which the power dissipation in the circuit drops to one half its resonance value.

If f_1 and f_2 are lower and upper half power freq and f_r is resonant freq

so ~~$Q = f_2 - f_1$~~

$$Q = \frac{\omega_r}{\omega_2 - \omega_1} = \frac{f_r}{f_2 - f_1}$$



The power at resonant freq

$$= I_{0max}^2 R$$

$$= \left(\frac{E_0}{R}\right)^2 R$$

The power dissipation at freq. f_1 is

$$I_1^2 R = \frac{1}{2}\left(\frac{E_0}{R}\right)^2 R$$

where I_1 is current at lower half power freq.

$$I_1 = \frac{1}{\sqrt{2}} \frac{E_0}{R} = \frac{1}{\sqrt{2}} \text{ current at resonance}$$

Similarly for the current at upper half power freq. is

$$I_2 = \frac{1}{\sqrt{2}} \frac{E_0}{R} = \frac{1}{\sqrt{2}} \text{ current at resonane}$$

Thus at lower or upper half power freq. the current amplitude becomes $\frac{1}{\sqrt{2}}$ times the current amplitude at resonance.

the current at lower ~~and~~ or upper half power freq.

$$\frac{E_0}{\sqrt{R^2+(\omega L-1/\omega C)^2}} = \frac{1}{\sqrt{2}}\frac{E_0}{R}$$

$$R^2+(\omega L-1/\omega C)^2 = 2R^2$$

$$(\omega L-1/\omega C)^2 = R^2$$

This eqn has two solution one for lower half and other for upper half power freq.

$$(\omega L-1/\omega C) = \pm R$$

then

$$\omega_1 L - \frac{1}{\omega_1 C} = -R$$

$$\omega_2 L - \frac{1}{\omega_2 C} = R$$

then adding these eqn

$$(\omega_1+\omega_2)L - \frac{1}{C}\left(\frac{1}{\omega_1}+\frac{1}{\omega_2}\right) = 0$$

$$(\omega_1+\omega_2)L - \frac{(\omega_1+\omega_2)}{C(\omega_1\omega_2)} = 0 \Rightarrow (\omega_1+\omega_2)L = \frac{\omega_1+\omega_2}{C(\omega_1\omega_2)}$$

$$\omega_1\omega_2 = \frac{1}{LC}$$

or $$\omega_1\omega_2 = \omega_r^2 = \frac{1}{LC}$$

Thus the resonant freq. f_r is the geometrical mean of upper half power freq.

subtracting the eqn

$$(\omega_2-\omega_1)L + \frac{1}{C}\left(\frac{1}{\omega_1}-\frac{1}{\omega_2}\right) = 2R$$

$$(\omega_2-\omega_1)L + \frac{1}{C}\frac{(\omega_2-\omega_1)}{(\omega_1\omega_2)} = 2R$$

$$(\omega_2-\omega_1)L + \frac{1}{1/LC}\frac{1}{C}(\omega_2-\omega_1) = 2R$$

$$2(\omega_2-\omega_1)L = 2R$$

$$\omega_2-\omega_1 = \frac{R}{L}$$

Therefore

$$\frac{\omega_r}{\omega_2-\omega_1} = \frac{f_r}{f_2-f_1} = \frac{\omega_r L}{R} = Q$$

Q in terms of band width is

$$Q = \frac{\omega_r}{\Delta\omega}$$

$\Delta\omega$ is called band width

Selectivity

If applied alternating voltage has a no. of freq. Components (signal received by a radio antenna with wide range of freq). then a series LCR circuit will give a max. responce (i.e. pass max. I and have a max. potential diff. across its inductance) for only that freq. components which has the freq.

$$f_r = \frac{1}{2\pi\sqrt{LC}}$$

Thus out of no. of available freq. it ~~selecti~~ selects one freq. for which the current is max. and for other freq. the current is comparatively very small, it shows the resonant circuit. is known as the selectivity. The resonant circuit and it practically accept only one freq. component of applied voltage and reject other. If Q is high circuit is highly selective

Voltage magnification

The Q is also a measure of voltage magnification in the series LCR circuit.

At resonance the potential diff. across inductance and capacitance are equal and 180° out of phase and hence cancel. Therefore the only potential diff. at resonance is across the resistance.

At the resonance, the current is max. and

$$(I_0)_{max} = \frac{E_0}{R}$$

The potential diff. across the $R = (I_0)_{max} R = E_0$

Thus the potential diff across R at resonance is equal to the applied e.m.f.

The voltage magnification in LCR circuit is defined as

$$\text{voltage magnification} = \frac{\text{potential diff. across the inductance or capacitance}}{\text{Applied voltage}}$$

$$m = \frac{(I_0)_{max}\, \omega_r L}{E_0}$$

$$= \frac{E_0}{R} \cdot \frac{\omega_r L}{E_0} = \frac{\omega_r L}{R} = Q = \frac{1}{R}\sqrt{L/C}$$

Similarly for capacitance

$$m = \frac{(I_0)_{max}}{E_0} \cdot \frac{1}{\omega_r C}$$

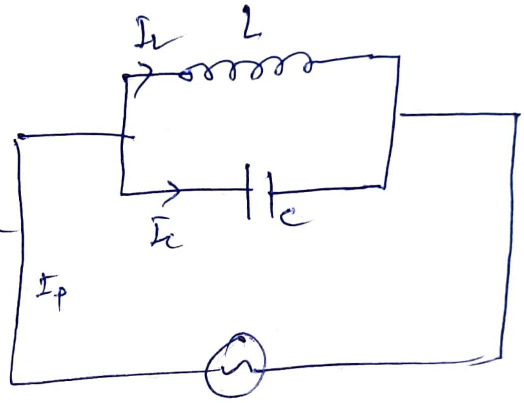
$$m = \frac{E_0}{R E_0} \cdot \frac{L}{\omega_r C}$$

$$= \frac{L}{\omega_r C R} = Q = \frac{1}{R}\sqrt{\frac{L}{C}}$$

Thus the voltage magnification of LCR circuit is equal to its quality factor Q at resonance.

A parallel (OR anti) resonance Circuit

Let an alternating emf $E = E_0 e^{j\omega t}$ is applied across the coil of inductance L of negligible resistance in parallel with capacitor of capacitance C.



The current through L lags behind the e.m.f in phase by $\pi/2$ and given as

$$I_L = \frac{E_0}{j\omega L} e^{j\omega t} = \frac{E_0}{\omega L} e^{j(\omega t - \pi/2)} \quad -(1)$$

where $\frac{1}{j} = -j = e^{-j\pi/2}$

and current through C leads the e.m.f in phase by $\pi/2$ and given as

$$I_C = \frac{E_0}{1/j\omega C} = \frac{E_0}{1/\omega C} e^{j(\omega t + \pi/2)} \quad (2)$$

where $j = e^{j\pi/2}$ $\quad -(2)$

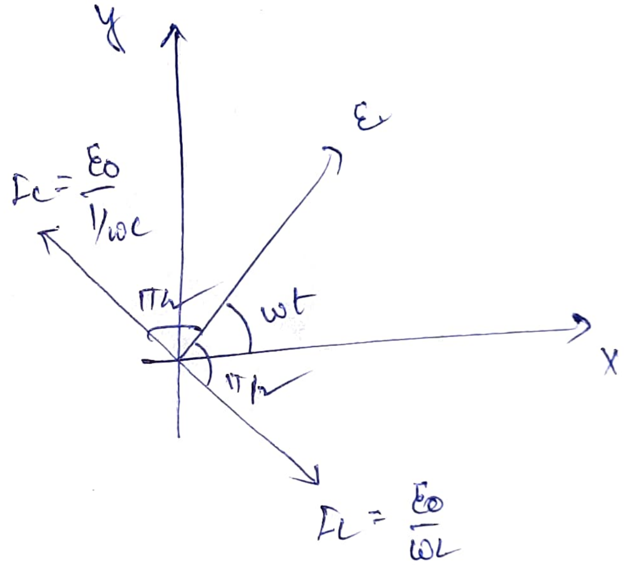
Therefore current in two branches differ in phase by π. The total current I_T will be vector sum of I_L and I_C and will be equal in their numerical diff. since they are 180° out of phase. Thus

$$I_T = I_L + I_C$$

$$= \frac{E_0}{j\omega L} e^{j\omega t} + j E_0 \omega C e^{j\omega t}$$

$$= j\left(\omega C - \frac{1}{\omega L}\right) E_0 e^{j\omega t} \quad \text{— (3)}$$

The phase relationship is shown in figure in complex plane



The total impedance of circuit is

$$\frac{1}{Z_T} = \frac{1}{Z_L} + \frac{1}{Z_C} = \frac{Z_L + Z_C}{Z_L Z_C}$$

$$Z_T = \frac{Z_L Z_C}{Z_L + Z_C}$$

$$Z_T = \frac{j\omega L * 1/j\omega C}{j(\omega L - 1/\omega C)}$$

$$= \frac{L/C}{j(\omega L - 1/\omega C)}$$

$$= \frac{L/C}{j\,L/C\,(\omega C - \frac{1}{\omega L})}$$

$$Z_T = \frac{1}{j(\omega C - 1/\omega L)} \quad \text{— (4)}$$

from eqn (3) and (4)

$$Z_f = \frac{E_0}{I_f} e^{j\omega t} \quad \text{——} (5)$$

The resonance is obtained when current in two branches is equal. Since current is out of phase in two branches. the total current will be zero.

at resonance

$$\omega_r C - \frac{L}{\omega_r L} = 0$$

or $\omega_r^2 = \frac{L}{LC}$

The freq. $f_r = \frac{L}{2\pi\sqrt{LC}}$

This is identical with the series resonance case.

when $I_f = 0$ then $Z_f = \infty$

Thus at freq f_r of supply voltage, the circuit offers infinite impedance to flow of current. so that no current is drawn from supply. such circuit is called parallel resonance and ~~fo~~ f_r is called resonant freq.

Thus $I = 0$ for parallel circuit and it works as a perfect choke for A.C. and circuit is called rejector circuit. such circuits are used in radios as filter circuit.

Further at resonance

$$Z_f = \frac{Z_L Z_C}{Z_L + Z_C}$$

$Z_L + Z_c$ is the total impedance when L and C are connected in ~~ourave~~ series

let $Z_s = Z_L + Z_c$

Then
$$Z_p = \frac{j\omega_r L \cdot \frac{1}{j\omega_r c}}{Z_s}$$

but at resonance
$$\omega_r L = \frac{1}{\omega_r c}$$

Then
$$Z_p = \frac{(\omega_r L)^2}{Z_s}$$

$$Z_p = \frac{(\omega_r L)^2}{E_0 e^{j\omega t}} \cdot \frac{E_0 e^{j\omega t}}{Z_s}$$

for given supply
$$\frac{\omega_r L}{E_0 e^{j\omega t}} \text{ is constant}$$

$$Z_p = \text{Constant } I_s$$

where $I_s = \dfrac{E_0 e^{j\omega t}}{Z_s}$ is current when L and C are connected in series with the same e.m.f applied.

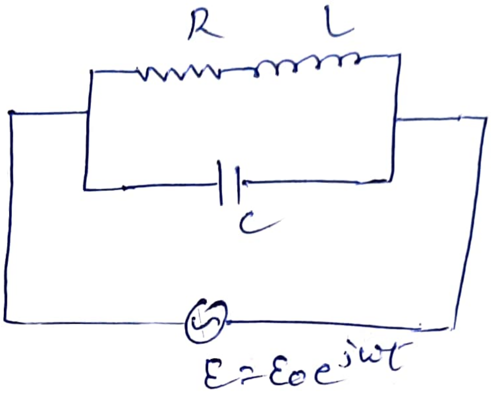
<u>parallel resonance circuit when inductance has some resistance</u>

let an A.C. voltage E is applied to C and L having a resistance R in parallel as shown in figure. Since R due to R, current through L will lag behind the applied e.m.f by an angle less than $\pi/2$ and so the phase diff. b/w I_L and I_c will be less than $180°$. Due to this resultant impedance will be max. at resonance not infinite there are two criteria for solving such Problem.

(i) the condition of max. impedance.

(ii) the condition of unity Power factor.

the condition for maximum impedance is called anti-resonance



the e.m.f of circuit is

$$E = E_0 e^{j\omega t}$$

the total impedance of circuit is

$$\frac{1}{Z_T} = \frac{1}{Z_L} + \frac{1}{Z_C}$$

$$= \frac{1}{R+j\omega L} + j\omega C$$

$$\frac{1}{Z_T} = \frac{1+j\omega CR - \omega^2 LC}{R+j\omega L}$$

$$Z_P = \frac{R+j\omega L}{(1-\omega^2 LC) + j\omega CR}$$

$$Z_P = \frac{R+j\omega L}{(1-\omega^2 LC) + j\omega CR} \times \frac{(1-\omega^2 LC) - j\omega CR}{(1-\omega^2 LC) - j\omega CR}$$

$$= \frac{R - \omega^2 LCR - j\omega CR^2 + j\omega L - j\omega^3 L^2 C + \omega^2 LCR}{(1-\omega^2 LC)^2 + \omega^2 C^2 R^2}$$

$$Z_T = \frac{R + j\omega\left(L - C(\omega^2 L^2 + R^2)\right)}{(1-\omega^2 LC)^2 + \omega^2 C^2 R^2} \quad -(1)$$

$$Z_T = R_0 + j\omega L_0 \quad (\text{let}) \quad -(2)$$

then $R_0 = \dfrac{R}{(1-\omega^2 LC)^2 + \omega^2 C^2 R^2}$ —(3) (this is effective resistance of coil) and $L_0 = \dfrac{L - C(\omega^2 L^2 + R^2)}{(1-\omega^2 LC)^2 + \omega^2 C^2 R^2}$ —— (4)

(This is effective inductance of coil)

The magnitude of Z_P is

$$Z = |Z_P| = \sqrt{R_0^2 + \omega^2 L_0^2}$$

or $$Z^2 = R_0^2 + \omega^2 L_0^2$$

$$Z^2 [(1-\omega^2 LC)^2 + \omega^2 C^2 R^2]^2 = R^2 + \omega^2 [L - C(\omega^2 L^2 + R^2)]^2$$

$$Z^2 [(1-\omega^2 LC)^2 + \omega^2 C^2 R^2]^2 = R^2 + \omega^2 L^2 - 2\omega^2 LC(\omega^2 L^2 + R^2) + \omega^2 C^2 (\omega^2 L^2 + R^2)^2$$

$$= R^2 [(1-\omega^2 LC)^2 + \omega^2 C^2 R^2] + \omega^2 L^2 [(1-\omega^2 LC)^2 + \omega^2 C^2 R^2]$$

$$= (R^2 + \omega^2 L^2) [(1-\omega^2 LC)^2 + \omega^2 C^2 R^2]$$

$$Z^2 = \frac{R^2 + \omega^2 L^2 [(1-\omega^2 LC)^2 + \omega^2 C^2 R^2]}{[(1-\omega^2 LC)^2 + \omega^2 C^2 R^2]^2}$$

$$Z = \left[\frac{R^2 + \omega^2 L^2}{(1-\omega^2 LC)^2 + \omega^2 C^2 R^2}\right]^{1/2} \quad \text{—— (5)}$$

Condition for unity power factor

The current in the circuit will be in phase with applied voltage when the impedance of circuit is purely resistive, which corresponds when power factor is unity. Then effective reactance of circuit (coil) must be zero from eqn (4)

$$L_0 = \frac{L - C(\omega^2L^2+R^2)}{(1-\omega^2LC)^2 + \omega^2C^2R^2} = 0$$

$$L - C(\omega^2L^2+R^2) = 0$$

$$R^2+\omega^2L^2 = L/C \quad -(6)$$

freq. at which the effective reactance is zero, is denoted by ω_r

$$R^2+\omega_r^2L^2 = (L/C)$$

$$L^2\omega_r^2 = L/C - R^2$$

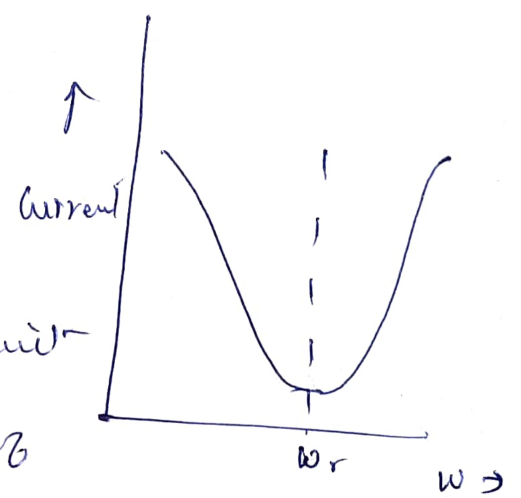
$$\omega_r = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}} \quad -(7)$$

$$f_r = \frac{\omega_r}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}} \quad -(8)$$

The freq f_r for which power factor is unity is called the resonance freq, for given parallel circuit, at this freq. impedance is max. (but not infinite) The current will be min. The circuit works as rejector.

When R is very small then R^2/L^2 is negligible then

$f_r = \frac{1}{2\pi}\sqrt{\frac{1}{LC}}$ is equal to the resonance freq. of series circuit



At resonance the impedance of circuit is from eqn (5) and (7)

(19)

$$Z_{Tr} = Z_r = \left[\frac{R^2 + \left(\frac{1}{LC} - \frac{R^2}{L^2}\right)L^2}{\left\{1 - \left(\frac{1}{LC} - \frac{R^2}{L^2}\right)LC\right\}^2 + \left(\frac{1}{LC} - \frac{R^2}{L^2}\right)C^2R^2}\right]^{1/2}$$

$$= \left[\frac{L/C}{\left(1 - 1 + \frac{R^2C}{L}\right)^2 + \left(\frac{1}{LC} - \frac{R^2}{L^2}\right)C^2R^2}\right]^{1/2}$$

$$= \left[\frac{L/C}{\cancel{\frac{R^4C^2}{L^2}} + \frac{C^2R^2}{LC} - \cancel{\frac{R^4C^2}{L^2}}}\right]^{1/2}$$

$$= \left[\frac{L/C}{\frac{C^2R^2}{LC}}\right]^{1/2}$$

$$= \left[\frac{L^2}{R^2C^2}\right]^{1/2}$$

$$Z_{Tr} = \frac{L}{CR} \quad \text{— (9)}$$

Since $\frac{L}{R}$ is $= \tau$ and $CR = \tau$ there

then

$$\frac{L}{\tau R} = R$$

Therefore Z_{Tr} is purely resistive and known as dynamic resistance of combination

for parallel resonance to occur

$\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$ should be real

$\frac{1}{LC} - \frac{R^2}{L^2} > 0$ then $R^2 < \frac{L}{C}$ or $R < \sqrt{L/C}$ Hence R should be kept as low as pos

Current magnification

The A.C. supply current at resonance is

$$I_r = \frac{E}{Z_{Tr}} = \frac{E_0 e^{j\omega t}}{Z_{Tr}}$$

$$= \frac{E_0 e^{j\omega t}}{L/CR} = \frac{E_0 CR}{L} e^{j\omega t}$$

$$I_r = I_{r0} e^{j\omega t} \quad — (10)$$

where $I_{r0} = \dfrac{E_0 CR}{L}$

The oscillating current I_{cr} in the capacitor is

$$I_{cr} = \frac{E_0 e^{j\omega t}}{\text{capacitive reactance}}$$

$$= \frac{E_0 e^{j\omega t}}{1/j\omega_r c} = j E_0 \omega_r c\, e^{j\omega t}$$

$$= E_0 \omega_r c\, e^{j(\omega t + \pi/2)} \qquad \text{since } j = e^{j\pi/2}$$

$$I_{cr} = I_{cr0}\, e^{j(\omega t + \pi/2)} \quad — (11)$$

where $I_{cr0} = E_0 \omega_r c$

The current magnification is defined as

$$m = \frac{\text{amplitude of current across capacitor}}{\text{amplitude of the supply current}}$$

$$= \frac{I_{cr0}}{I_{r0}}$$

$$= \frac{E_0 \omega_r c}{E_0 CR/L}$$

$$m = \frac{\omega_r L}{R} = Q \quad \text{--- (12)}$$

Similarly oscillating current I_{Lr} in inducter is

$$I_{Lr} = \frac{E_0 e^{j\omega t}}{\text{Inductive reactance}}$$

$$= \frac{E_0 e^{j\omega t}}{j\omega_r L} = -j \frac{E_0}{\omega_r L} e^{j\omega t}$$

$$= \frac{E_0}{\omega_r L} e^{j(\omega t - \pi/2)} \qquad \left\{ \text{Since } \tfrac{1}{j} = -j = e^{-j\pi/2} \right.$$

$$I_{Lr} = I_{Lr0}\, e^{j(\omega t - \pi/2)} \quad \text{--- (13)}$$

where $I_{Lr0} = \dfrac{E_0}{\omega_r L}$

magnification is

$$m = \frac{I_{Lr0}}{I_{r0}} = \frac{\frac{E_0}{\omega_r L}}{E_0 CR/L}$$

$$m = \frac{L}{\omega_r CR} = Q \quad \text{--- (14)}$$

Thus the parallel circuit gives a current magnification equal to the Q at the resonance similar to the voltage magnification by series circuit

Selectivity of a parallel resonance Circuit

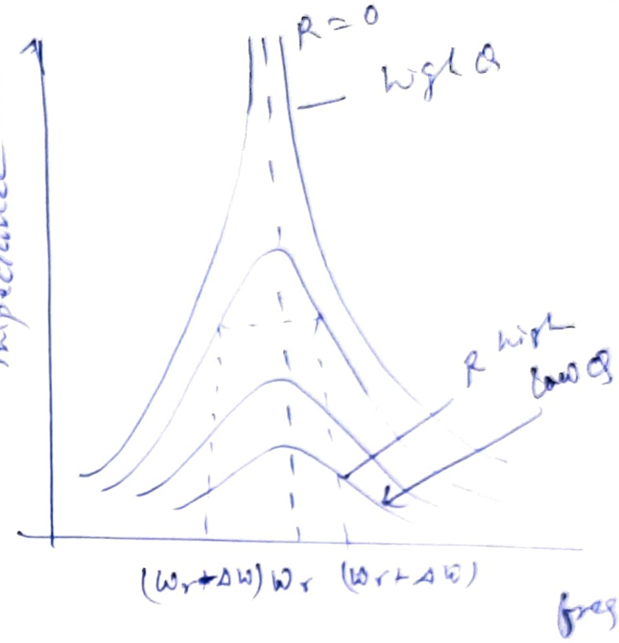
The variation of impedance with freq. in parallel circuit is same as the variation of current with freq. in series circuit.

At resonance, impedance is ~~eq~~ given by

$$Z_{Tr} = \frac{L}{CR}$$

selectivity of circuit means its ability to present a high impedance at resonance freq. and much lower impedance at other freq.

Thus greater and sharpness of curve (i.e. high Q) the greater is selectivity



The impedance of circuit is

$$\frac{1}{Z_T} = \frac{1}{Z_L} + \frac{1}{Z_C}$$

$$Z_T = \frac{R + j\omega L}{(1-\omega^2 LC) + j\omega CR}$$

In practice $R \to 0$ then

$$Z_T = \frac{j\omega L}{(1-\omega^2 LC) + j\omega CR}$$

its magnitude is

$$Z_T = \frac{\omega L}{\sqrt{(1-\omega^2 LC)^2 + \omega^2 C^2 R^2}}$$

at $\omega = \omega_r$, $Z_T = Z_{Tr}$

Now if $\omega = \omega_r \pm \Delta\omega$ the term ωL and $\omega^2 C^2 R^2$ will not differ appreciably from $\omega_r L$ and $\omega_r^2 C^2 R^2$ but term $(1-\omega^2 LC)$ will give a considerable change.

Thus when $\omega = \omega_r$ and $R \to 0$, we get

$$Z_P = \frac{\omega_r L}{\sqrt{(1-\omega_r^2 LC) + \omega_r^2 C^2 R^2}}$$

If at $w = w_r + \Delta w$

the impedance Z_T is $\frac{1}{\sqrt{2}}$ time it's max. value

then

$$Z_T = \frac{1}{\sqrt{2}} Z_{Tr} = \frac{1}{\sqrt{2}} \frac{L}{CR}$$

At resonance and $R \to 0$ then

$$w_r = \frac{1}{\sqrt{LC}}$$

$$\frac{1}{\sqrt{2}} \frac{L}{CR} = \frac{w_r L}{\sqrt{(1-(w_r+\Delta w)^2 LC)^2 + w_r^2 C^2 R^2}}$$

on squaring

$$[1-(w_r+\Delta w)^2 LC]^2 + w_r^2 C^2 R^2 = 2 w_r^2 C^2 R^2$$

$$[1-(w_r+\Delta w)^2 LC]^2 = w_r^2 C^2 R^2$$

$$[1-(w_r^2+\Delta w^2+2\Delta w . w_r) LC]^2 = w_r^2 C^2 R^2$$

Δw^2 is neglected since Δw is very small

$$[1-w_r^2 LC - 2 w_r \Delta w LC]^2 = w_r^2 C^2 R^2$$

but $w_r^2 = \frac{1}{LC}$ then

$$[1-1-2\Delta w LC]^2 = w_r^2 C^2 R^2$$

$$\left(-\frac{2\Delta w}{w_r^2}\right)^2 = w_r^2 C^2 R^2$$

$$-\frac{2\Delta w}{w_r} = \pm w_r CR$$

then

$$\frac{-2\Delta w}{w_r} = -w_r CR$$

$$\frac{w_r}{2\Delta w} = \frac{1}{w_r CR} = Q$$

$$\frac{w_r}{2\Delta w} = Q = \frac{w_r L}{R} = \frac{1}{R}\sqrt{L/C}$$

$2\Delta\omega$ is band width of resonance curve.

thus greater value of Q corresponds to small value of band width and sharper the resonance and greater selectivity.

Power of A.C. Circuit

Power is rate of doing work. As in A.C. circuit e.m.f. and current both vary continuously with time. hence power is equal to the product of instantaneous current and instantaneous e.m.f average over a complete cycle.

(i) Purely resistance circuit

e.m.f. and current is given by

~~$E = E_0 e^{j\omega t}$~~

$$\mathcal{E} = E_0 \sin\omega t$$

$$I = I_0 \sin\omega t$$

The power $= EI = E_0 I_0 \sin^2\omega t$

Average value of doing work or power in one complete cycle

$$= \frac{E_0 I_0 \int_0^{2\pi} \sin^2 x \, dx}{\int_0^{2\pi} dx} \quad \text{where } x = \omega t$$

$$= \frac{E_0 I_0}{2\pi} \cdot \frac{1}{2} \int_0^{2\pi} (1 - \cos 2x)\, dx$$

$$= \frac{E_0 I_0}{2\pi} \cdot \frac{1}{2} \cdot 2\pi = \frac{E_0}{\sqrt{2}} \cdot \frac{I_0}{\sqrt{2}}$$

$$= E_{rms} \times I_{rms}$$

(ii) Power in Inductive circuit

For inductive circuit current through pure inductance lags behind the e.m.f. in phase by $\pi/2$

$$E = E_0 \sin \omega t$$

$$I = I_0 \sin(\omega t - \pi/2)$$

Instantaneous power $= EI$

$$= E_0 I_0 \sin \omega t \sin(\omega t - \pi/2)$$

$$= -E_0 I_0 \sin \omega t \cos \omega t$$

$$= -\frac{E_0 I_0}{2} \sin 2\omega t$$

Average power over one cycle.

$$= -E_0 I_0 \frac{\int_0^{2\pi} \sin 2x \, dx}{\int_0^{2\pi} dx}$$

$$= -\frac{E_0 I_0}{2\pi} \left[-\frac{\cos 2x}{2} \right]_0^{2\pi}$$

$$= 0$$

Thus power dissipation in pure inductance is zero, and current in such circuit is known as wattless current.

The choke coil works on this principle.

(iii) Power in capacitive circuit

(iv) Power in circuit containing L, C and R

Current and e.m.f. of circuit containing L, C and R are not in phase and given by

$$E = E_0 \sin \omega t$$

and $I = I_0 \sin(\omega t + \theta)$

where phase angle θ is

$$\theta = \tan^{-1}\left(\frac{\omega L - 1/\omega C}{R}\right)$$

Power at any instant

$$= EI$$

$$= E_0 I_0 \sin\omega t \sin(\omega t + \theta)$$

$$= E_0 I_0 \sin\alpha \sin(\alpha + \theta) \quad \text{where } \alpha = \omega t$$

$$= E_0 I_0 [\sin^2\alpha \cos\theta + \sin\alpha \cos\alpha \sin\theta]$$

$$= E_0 I_0 [\sin^2\alpha \cos\theta + \tfrac{1}{2}\sin 2\alpha \sin\theta]$$

The average power over one cycle

$$= \frac{E_0 I_0 \cos\theta \int_0^{2\pi} \sin^2\alpha \, d\alpha}{\int_0^{2\pi} d\alpha} + \frac{\frac{1}{2} E_0 I_0 \int_0^{2\pi} \sin 2\alpha \, d\alpha \cdot \sin\theta}{\int_0^{2\pi} d\alpha}$$

The average value of $\sin^2\alpha$ is $1/2$
and $\sin 2\alpha$ is 0

Then
Average power over one cycle is

$$= \frac{1}{2} E_0 I_0 \cos\theta$$

$$= \frac{E_0}{\sqrt{2}} \cdot \frac{I_0}{\sqrt{2}} \cos\theta$$

$$= E_{rms} \cdot I_{rms} \cos\theta \quad \text{—— (1)}$$

The term $E_{rms} \cdot I_{rms}$ is called apparent power
and $\cos\theta$ is called power factor
Thus power factor is defined as

$$\text{power factor } (\cos\theta) = \frac{\text{True power}}{\text{Apparent power}} \quad (2)$$

(a) Power factor in L-C-R circuit

$$\cos\theta = \frac{R}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}$$

(b) power factor in inductive circuit

$$\cos\theta = \frac{R}{\sqrt{R^2 + \omega^2 L^2}}$$

(c) Power factor in capacitive circuit

$$\cos\theta = \frac{R}{\sqrt{R^2 + 1/\omega^2 C^2}}$$

Couple circuit

One of the ~~po~~ most important feature of A.C. is that the energy (e.m.f or current) can be transfered from one circuit or coil to another circuit or coil easily and in an efficient manner.

~~Co~~ Couple circuit is defined as the circuit so arranged that energy can be transfered from one circuit to another circuit

Example - transformer.

The circuit are said to be coupled together if they have a common impedance. This common impedance may be a resistance, a inductance, or a capacitance or a combination of these. and is refered as a coupling element.

Coupled circuit are two types

(i) Simple Couple circuit

when coupling element is only resistance, inductance or Capacitance.

(ii) Complex Couple circuit

when coupling element is combination of resistance, inductance or capacitance

Induction Couple circuit

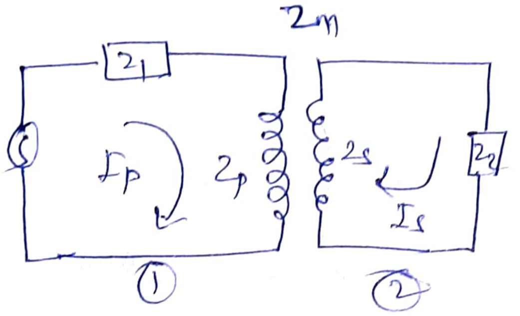


Figure shows two coils inductively coupled.

Z_1 is complex vector impedance connected in series with primary. and Z_2 (load impedance) connected across the secondary

Z_p and Z_s are complex impedance of primary and secondary.

Z_m is mutual impedance of two coils. I_p and I_s are currents in primary secondary.

By applying Kirchhoff's second law

we get

$$E = I_p Z_1 + I_p Z_p + I_s Z_m \quad \text{—} (1)$$

and $$0 = I_s Z_2 + I_s Z_s + I_p Z_m \quad \text{—} (2)$$

then

$$I_s = -\frac{Z_m}{Z_2 + Z_s} I_p \quad \text{———} (3)$$

from eqn (1) and (3)

$$E = I_p (Z_1 + Z_p) - \frac{Z_m^2}{Z_2 + Z_s} I_p$$

$$E = \frac{I_p\left[(Z_1+Z_p)(Z_2+Z_s) - Z_M^2\right]}{(Z_2+Z_s)}$$

$$I_p = \frac{(Z_2+Z_s)E}{(Z_1+Z_p)(Z_2+Z_s) - Z_m^2} \quad \text{--- (4)}$$

and from eqn (3) and (4)

$$I_s = -\frac{Z_m E}{(Z_1+Z_p)(Z_2+Z_s) - Z_M^2} \quad \text{--- (5)}$$

here negative sign indicates the fact that secondary current is ~~is~~ 180 out of phase with primary current.

The impedance at primary terminal is

$$Z = \frac{E}{I_p} = \frac{(Z_1+Z_p)(Z_2+Z_s) - Z_m^2}{(Z_2+Z_s)}$$

$$= (Z_1+Z_p) - \frac{Z_m^2}{(Z_2+Z_s)} \quad \text{--- (6)}$$

Due to the coupling action primary impedance has been modified

~~From~~ eqn (6) shows that presence of secondary i.e. effect of coupling of secondary circuit is to increase the primary impedance by $\left(-\frac{Z_m^2}{Z_2+Z_s}\right)$

If there is no coupling ~~then~~ so $Z_m = 0$ then impedance of primary ~~is~~ was $Z_1 + Z_p$.

This additional term $\left(-\frac{Z_M^2}{Z_2+Z_S}\right)$ is the impedance reflected into the ~~Primpo~~ primary from the secondary.

or Impedance coupled into primary by secondary and hence known as coupled impedance of secondary in the primary

$$Z_{ref} = -\frac{Z_M^2}{Z_2+Z_S} \quad \text{--- (7)}$$

<u>Transformer</u>

It is an electrical device based on the principle of mutual inductance for converting large current at low voltage to low current at high voltage and vice versa with very little loss in energy.

It consists of two coils known as primary and secondary wound on a core. The coil to which energy is supply is called primary and that from energy is delivered to ~~output~~ outer circuit is called secondary.

A.C. current in primary coil set up an A.C. ~~mang~~ magnetic flux in the core. This change in flux by faraday law's induces an A.C. e.m.f in secondary coil. Thus power is transfered from one coil to other.

Let N_p and N_s are no. of turns in primary and secondary coils.

Then magnetic flux linked with primary coil is $N_p \phi_B A$ and with secondary coil $N_s \phi_B A$

where A is area and ϕ_B is magnetic flux linked with each ~~coil~~ turn of ~~co~~ both coils.

Induced e.m.f E_p in primary and E_s in secondary are given

$$E_p = -\frac{d}{dt}(N_p \phi_B A)$$

$$= -N_p A \frac{d\phi_B}{dt} \quad \text{--- (1)}$$

and $$E_s = -\frac{d}{dt}(N_s \phi_B A)$$

$$= -N_s A \frac{d\phi_B}{dt} \quad \text{--- (2)}$$

from eqn (1) and (2)

$$\frac{E_s}{E_p} = \frac{N_s}{N_p}$$

Since the resistance of primary is very low so that there is no energy loss then induced e.m.f. E_p in primary will be equal to applied voltage V_p across primary.

~~Similarly~~ If secondary may be considered to open then V_s across the secondary will be equal induced e.m.f E_s

thus

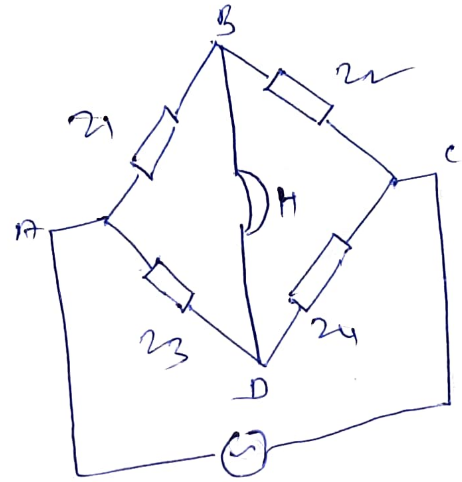
$$\frac{V_s}{V_p} = \frac{E_s}{E_p} = \frac{N_s}{N_p} = K$$

where K is called transformer ratio

A.C. Bridges

The wheatstone bridge principle is also application for A.C. ~~bri~~ network with modification

Here we have to consider the impedance instead of resistance and null point is determing by headphone or vibration galvanometer



At the balance condition for bridge

$$\frac{Z_1}{Z_2} = \frac{Z_3}{Z_4}$$

The phase/balance condition is adjusted by changing/ reactances (L or C) in ~~term~~ arms.

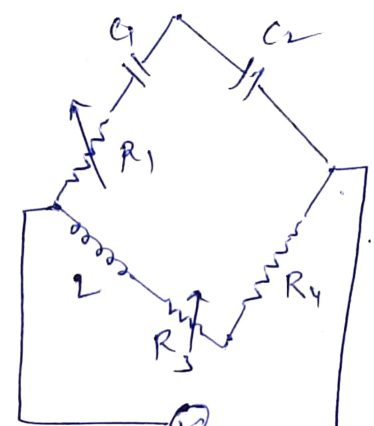
A.C. Bridge for the measurement of inductance

(i) Maxwell's bridge

(ii) Owen bridge

(iii) Anderson's bridge

Owen Bridge



This is a sensitive bridge for measurement of inductance in term of resistance and a fixed/standard capacitance.

The inductor of which self inductance to be measured is connected in series with a non-inductive variable resistance R_3, C_1 and C_2 are fixed capacitors. R_4 is fixed resistance at the balance point

$$\frac{Z_1}{Z_2} = \frac{Z_3}{Z_4}$$

But $Z_1 = R_1 + \frac{1}{j\omega C_1}$

$$Z_2 = \frac{1}{j\omega C_2}$$

$Z_3 = R_3 + j\omega L$, where R_3 also includes the resistance of inductor

$$Z_4 = R_4$$

then

$$\frac{R_1 + \frac{1}{j\omega C_1}}{1/j\omega C_2} = \frac{R_3 + j\omega L}{R_4}$$

$$\left(R_1 + \frac{1}{j\omega C_1}\right) j\omega C_2 = \frac{R_3 + j\omega L}{R_4}$$

$$j\omega C_2 R_1 + \frac{C_2}{C_1} = \frac{R_3}{R_4} + \frac{j\omega L}{R_4}$$

Equating real and imaginary part

$$\frac{C_2}{C_1} = \frac{R_3}{R_4} \quad -(1)$$

and

$$C_2 R_1 = \frac{L}{R_4} \quad -(2)$$

then $L = C_2 R_1 R_4$ - (3)

The condition (1) is satisfied by varying R_3 and (2) by varying R_1 till the sound is minimum in both the case. The small inductance of connecting leads and terminal referred as residual inductance can be eliminated using this bridge as follows.

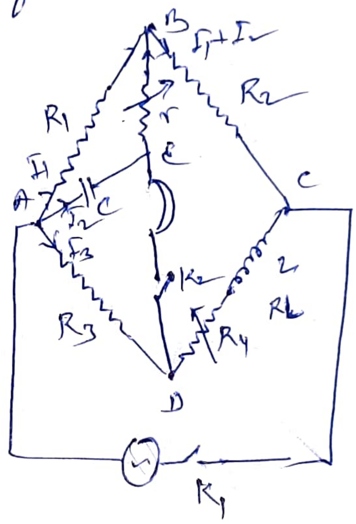
obtain two balances, 1st with L in circuit and then with L short circuit, if R_1 and R_1' are respective value of R_1 for two balance time

$$L = C_2 R_4 (R - R_1')$$

~~Ander~~

Anderson's Bridge

This is one of the most accurate bridge which is commonly used for measurement of self inductance.



At first a steady balance is obtained with cell and galvanometer connected ~~with~~ by K_1 and K_2 respectively and adjusting the resistance in the four arms of the bridge.

At ~~Bla~~ Balance

$$\frac{R_1}{R_2} = \frac{R_3}{R_4 + R_L} \quad -(1)$$

Next an A.C. source and a headphone are connected and an inductive balance is obtained by adjusting the resistance r

When current in the headphone is zero i.e. Bridge is balanced the potential drop b/w ~~A~~ and E will be the same as that b/w A and D thus

$$\frac{I_2}{j\omega C} = I_3 R_3 \quad (2)$$

At balance the potential drop along ABC is same as along ADC.

By Kirchoff's 2nd law

$$I_1 R_1 + (I_1 + I_2) R_2 = I_3 (R_3 + R_4 + R_L + j\omega L)$$

or $$I_1 (R_1 + R_2) + I_2 R_2 = I_3 (R_3 + R_4 + R_L + j\omega L) \quad \text{--- (3)}$$

when R_4 is resistance of inductance

There is no source of e.m.f in AQBA then

from Kirchhoff's law

$$I_2 \left(\frac{1}{j\omega C} + r\right) = I_1 R_1 \quad \text{--- (4)}$$

from eq^n (2) and (3)

$$I_1 (R_1 + R_2) + I_2 R_2 = \frac{I_2}{j\omega C R_3} (R_3 + R_4 + R_L + j\omega L)$$

$$I_1 (R_1 + R_2) = I_2 \left(\frac{R_3 + R_4 + R_L + j\omega L}{j\omega C R_3} - R_2 \right) \quad \text{--- (5)}$$

from eq^n (4) and (5)

$$\frac{I_2}{R_1} \left(\frac{1}{j\omega C} + r\right)(R_1 + R_2) = I_2 \left(\frac{R_3 + R_4 + R_L + j\omega L}{j\omega C R_3} - R_2 \right)$$

$$\left(\frac{R_1 + R_2}{R_1}\right)\left(\frac{1}{j\omega C} + r\right) = \left(\frac{R_3 + R_4 + R_4 + j\omega L}{j\omega C R_3} - R_2\right) \quad \text{--- (6)}$$

Comparing real and imaginary parts

$$\left(1 + \frac{R_2}{R_1}\right) r = \frac{L}{C R_3} - R_2$$

or

$$L = C R_3 \left[R_2 + \left(1 + \frac{R_2}{R_1}\right) r \right] \quad \text{--- (7)}$$

and $$\left(1 + \frac{R_2}{R_1}\right) \frac{L}{\omega C} = \frac{R_3 + R_4 + R_L}{\omega C R_3}$$

$$\frac{R_2}{R_1} = \frac{R_4 + R_L}{R_3} \quad \text{--- (8)}$$

Comparison of eq^n (8) and eq^n (1) shows that is is the condition for DC balance. Eq^n (7) for AC balance.

The inductance balance is obtained by adjusting R_4 and r. In practice $R_1 = R_2$ then $L = C R_3 (R_2 + 2r)$ and $R_L = R_3 - R_4$. Thus L and R_L are determined

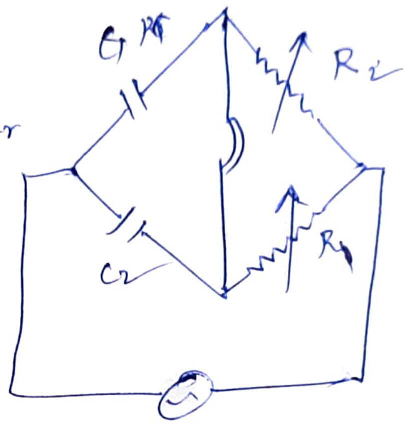
A.C. Bridges for the measurement of capacitance

(i) Desauty's A.C. Bridge

(ii) Wien bridge

(iii) Schering bridge

De-Sauty's A.C. Bridge



It is used to determine the capacity of an unknown capacitor in terms of the capacitance of a standard known capacitor.

The capacitor and two known non-inductive variable resistances are connected in a wheatstone bridge. One resistance is fixed value and other one is variable

at Balance

$$\frac{Z_1}{Z_2} = \frac{Z_3}{Z_4}$$

$$\frac{1/j\omega C_1}{R_2} = \frac{\frac{1}{j\omega C_2}}{R_y} \Rightarrow \frac{1}{C_1 R_1} = \frac{1}{C_2 R_y}$$

$$\frac{C_2}{C_1} = \frac{R_1}{R_y}$$

$$C_2 = C_1 \frac{R_2}{R_y}$$

This is also used for comparing two capacitor