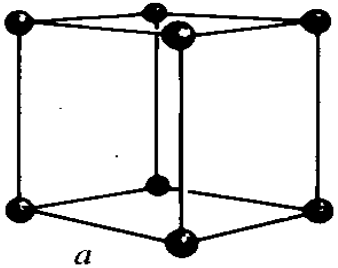
Unit-I

- Semiconductor Concepts and Energy Bands
- Carrier Transport Phenomena
- Graded impurity distribution

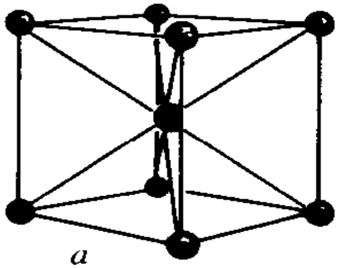
Solid state structures

Cubic lattices:

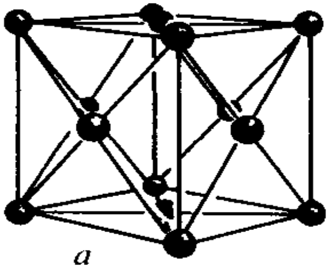
Unit cells for types of cubic lattice structure.



Simple cubic (sc)

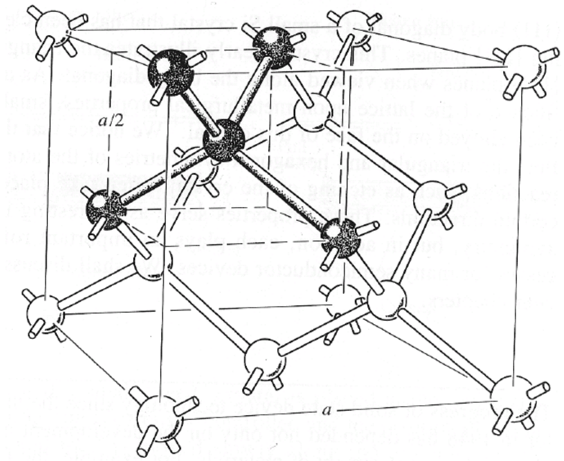


Body-centered cubic (bcc)



Face-centered cubic (fcc)

Diamond lattice unit cell, showing the four nearest neighbour structure



The basic lattice structure for many important semiconductors is the diamond lattice, which is characteristic of Si and Ge. In many compound semiconductors, atoms are arranged in a basic diamond structure but are different on alternating sites. This is called a *zincblende* lattice and is typical of the III-V compounds. The diamond lattice can be thought of as an *fcc* structure with an extra atom placed at **a**/4+**b**/4+**c**/4 from each of the *fcc* atoms.

The Silicon Atomic Structure

Solid → state of matter

Matter → a collection of atoms

Atoms comprise of a central positively charged nucleus, and negatively charged electrons.

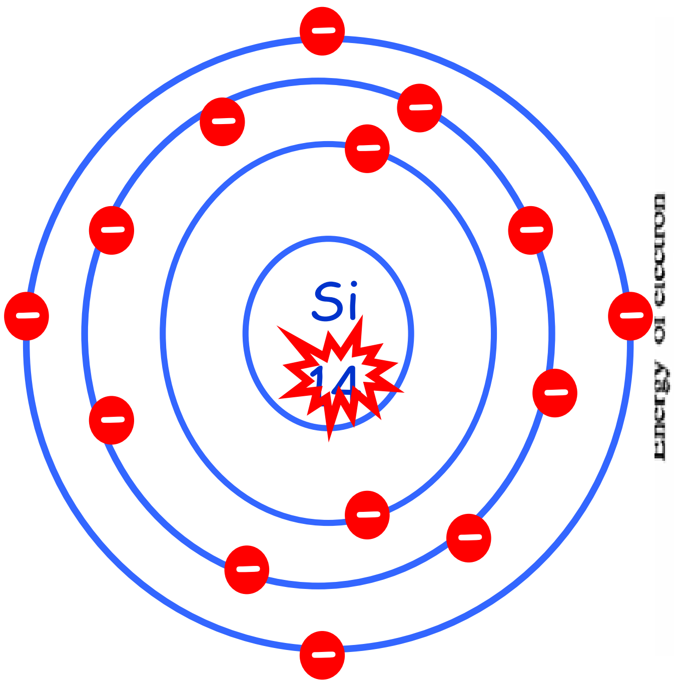
The motion of the constituents of an atom are described by the laws of quantum mechanics.

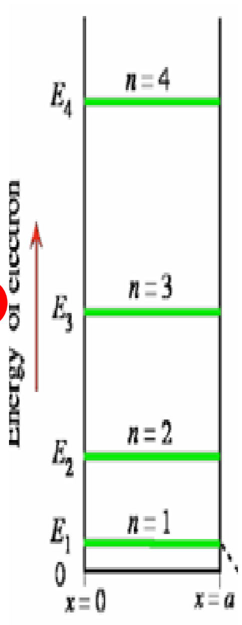
The motion of constituents are described by the Schrödinger equation.

Schrödinger equation is defined by

H ψ=E ψ ,

where H is called the Hamiltonian.





Silicon: our primary example and focus

Atomic no. 14

14 electrons in three shells: 2) 8) 4

i.e., 4 electrons in the outer "bonding" shell

Silicon forms strong covalent bonds with 4 neighbors

Energy Band Diagrams

- Quantization of the atom
- Lone atoms act like infinite potential wells in which bound electrons oscillate within allowed states at particular well defined energies
- The Schrödinger equation is used to define these allowed energy states

$$\frac{\partial^2 \Psi}{\partial x^2} + \frac{2m_e}{\hbar}\left[E - V(x)\right]\Psi = 0$$

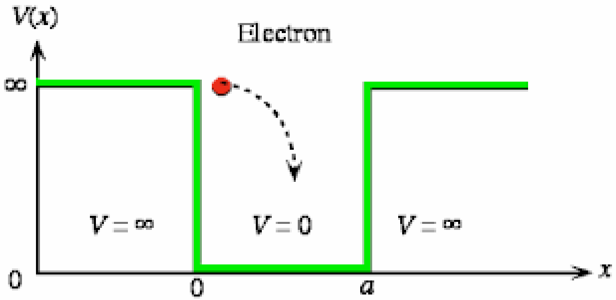
E = energy, V = potential energy

- Solutions are in the form of waves oscillating at quantized energies and related propagation constants defined by the differential equation

$$\Psi(x) = Ce^{ikx} + De^{-ikx}$$

$$\therefore \Psi_n(x) = \sqrt{\frac{2}{L}}\sin\left(\frac{n\pi}{L}x\right) \qquad k^2 = \frac{2mE}{\hbar^2}$$

$$E_n = \frac{n^2\pi^2\hbar^2}{2mL^2} = \frac{n^2h^2}{8mL^2} \qquad \text{since } \hbar = \frac{h}{2\pi}$$



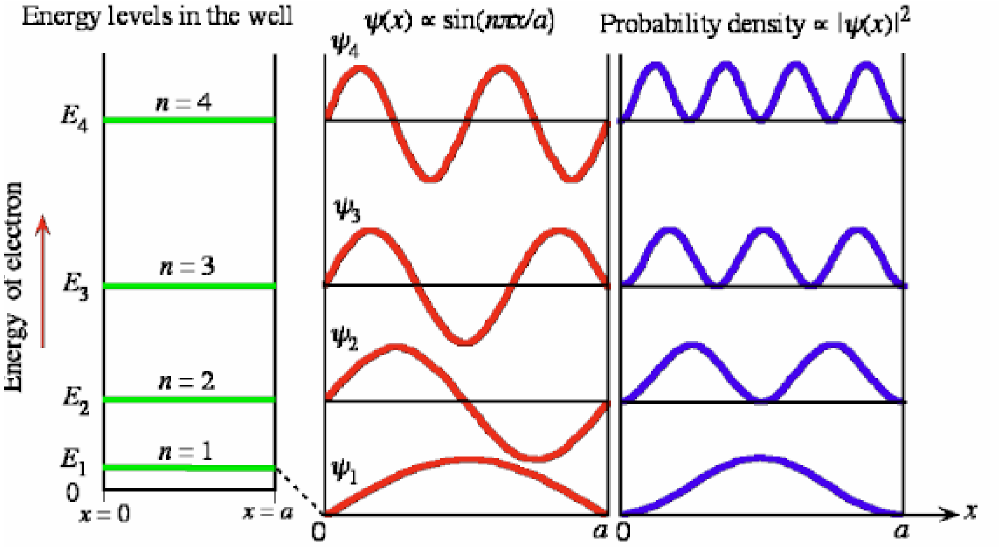


Fig. 3.15: Electron in a one-dimensional infinite PE well. The energy of the electron is quantized. Possible wavefunctions and the probability distributions for the electron are shown.

Quantum Mechanics and Molecular Binding

- All atoms have specific bound states
- Molecular binding requires filling of these allowed states in such a way as to reduce the amount of energy required to fill various states

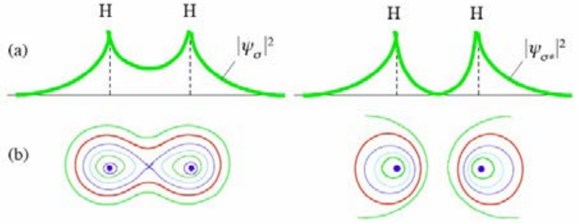


Fig. 4.2: (a) Electron probability distributions for bonding and antibonding orbitals, ψ_σ and ψ_{σ^*}. (b) Lines represent contors of constant probability.

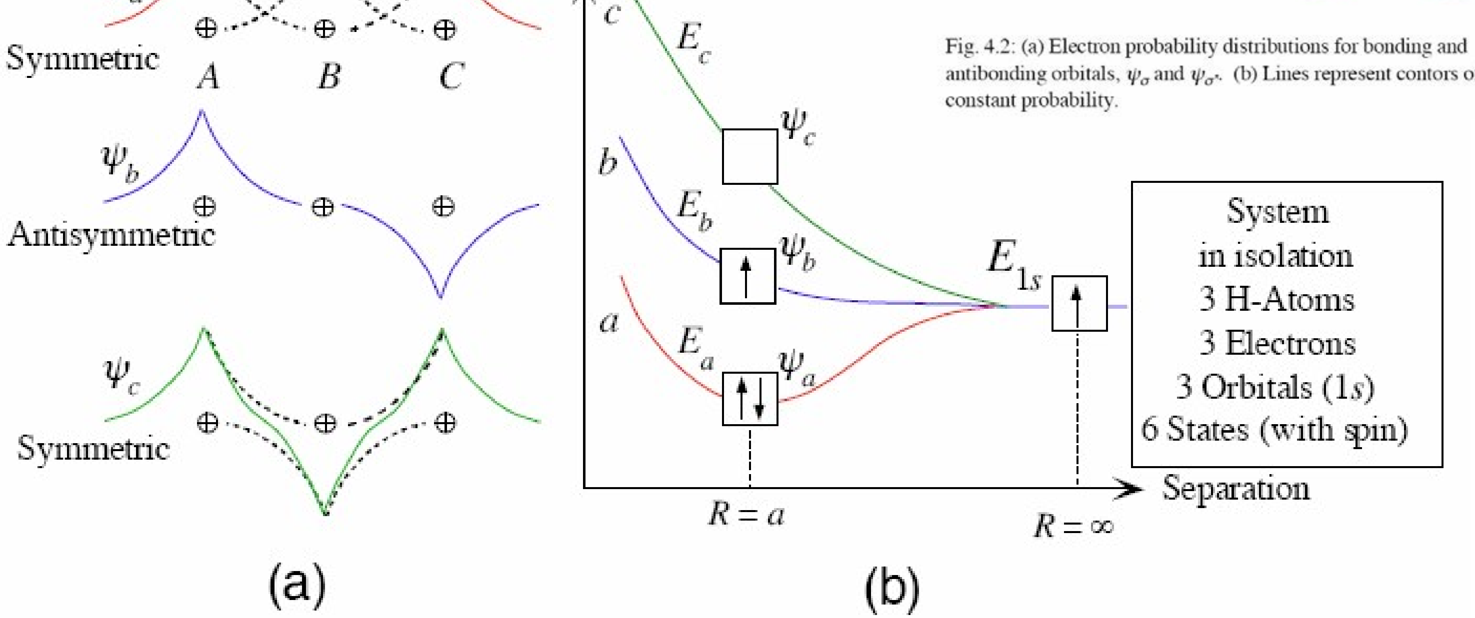
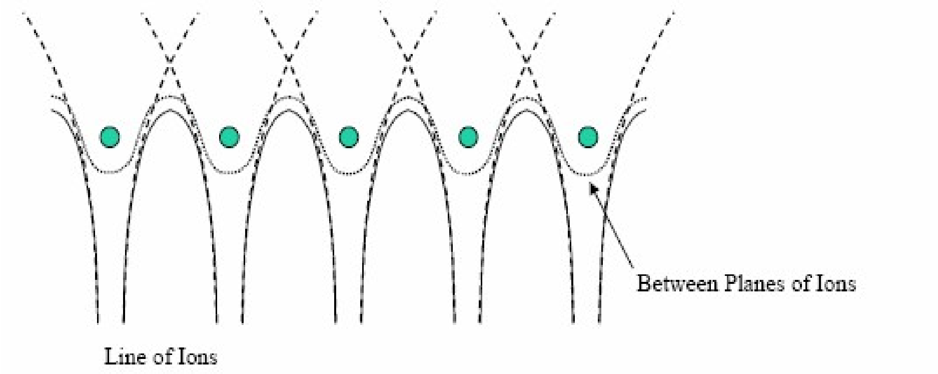
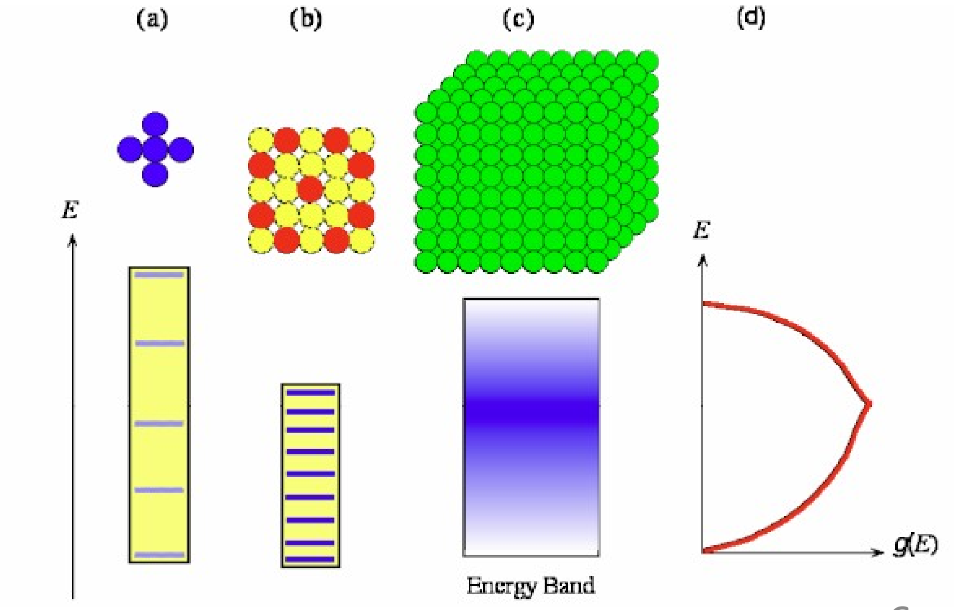


Fig. 4.7: (a) Three molecular orbitals from three ψ_{1s} atomic orbitals overlapping in three different ways. (b) The energies of the three molecular orbitals labeled as a, b and c in a system with 3 H atoms (highly simplified).

Effect of Periodicity in Solid Systems

- The compression of quantum states into periodic structures results overlap of available quantum states
- Pauli exclusion principle states that:
  - electrons must fill available quantum states from lowest to highest potential
  - allowed states are defined by orbital solutions obtained from quantum mechanics
- As atoms come closer together, orbitals become shared allowing electrons to fill exchange orbitals between the materials
- As the number of atoms brought into proximity increases degeneracies occur resulting in allowable energy bands
- These bands can be modeled using the Kronig Penning model with solutions that give an electron Density of States





Quantized States in Solid Li

Allowable Quantum States in Li

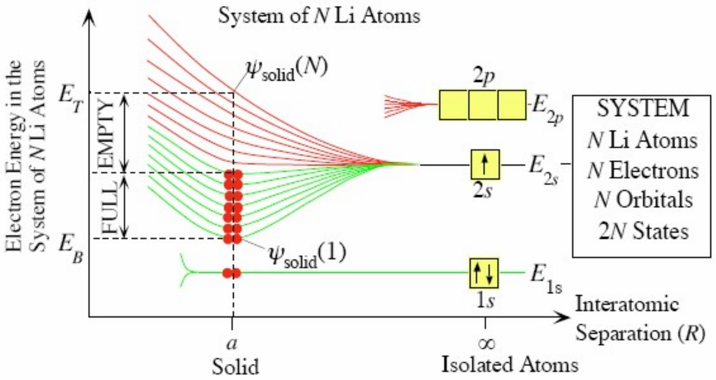


Fig. 4.8: The formation of a 2s-energy band from the 2s-orbitals when N Li atoms come together to form the Li solid. The are N 2s-electrons but 2N states in the band. The 2s-band therefore is only half full. The atomic 1s orbital is close to the Li nucleus and remains undisturbed in the solid. Thus each Li atom has a closed K-shell (full 1s orbital).

Overlapping Orbitals in 1 mol of Li

1 mol = 10^{23} atoms

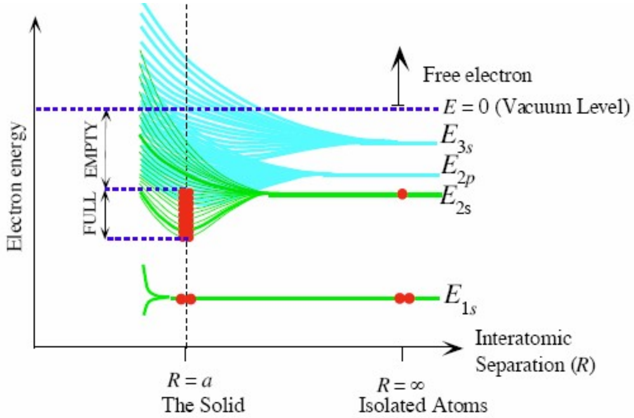


Fig. 4.9: As solid atoms are brought together from infinity, the atomic orbitals overlap and give rise to bands. Outer orbitals overlap first. The 3s orbitals give rise to the 3s band, 2p orbitals to the 2p band and so on. The various bands overlap to produce a single band in which the energy is nearly continuous.

Metal Energy Bands

- Overlapping energy degeneracies in metals
- Lead to continuous energy bands
- Statistically stable energy for electrons lies within these overlapping bands and only slight excitations lead to conduction b/c the variation in allowable quantum states is nearly continuous

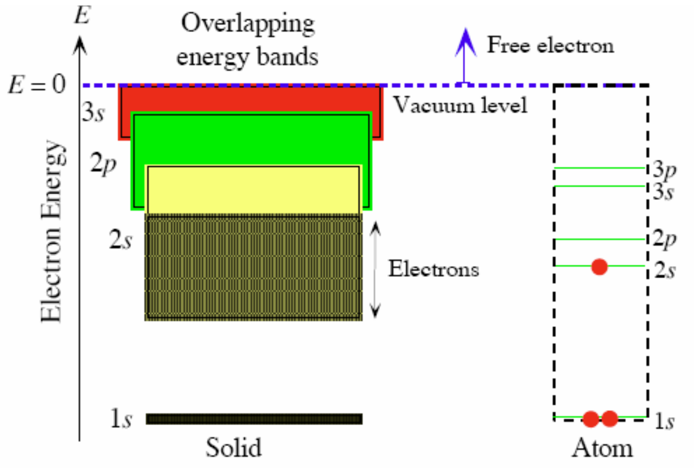


Fig. 4.10: In a metal the various energy bands overlap to give a single band of energies that is only partially full of electrons. There are states with energies up to the vacuum level where the electron is free.

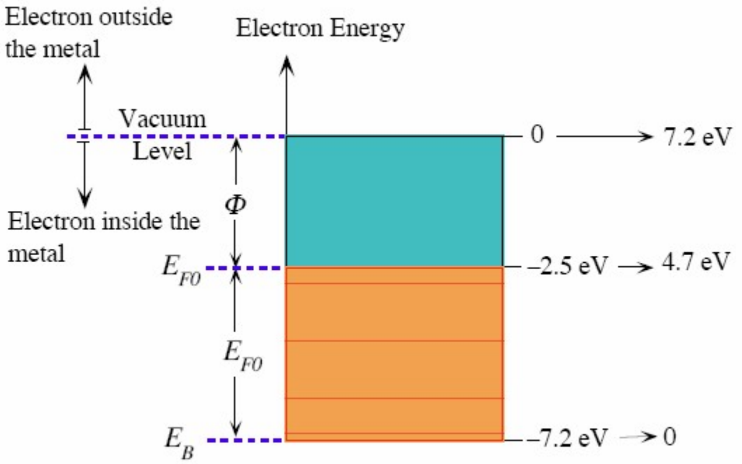


Fig. 4.11: Typical electron energy band diagram for a metal All the valence electrons are in an energy band which they only partially fill. The top of the band is the vacuum level where the electron is free from the solid ($PE = 0$).

Semiconductors

- Semiconductors are distinctly different
- In Semiconductors there is no overlapping degeneracy between conduction and valence bands
- The result is a bandgap, E_g, that is present between bound and conducting electron states $E_g = E_c - E_v$
- The width of the conduction band is called the electron affinity, χ
- At energies above the $E_c + \chi$ electrons can be ejected from the material
- In silicon for example, all of the valence electrons are used to fill the binding orbitals located in the valance band

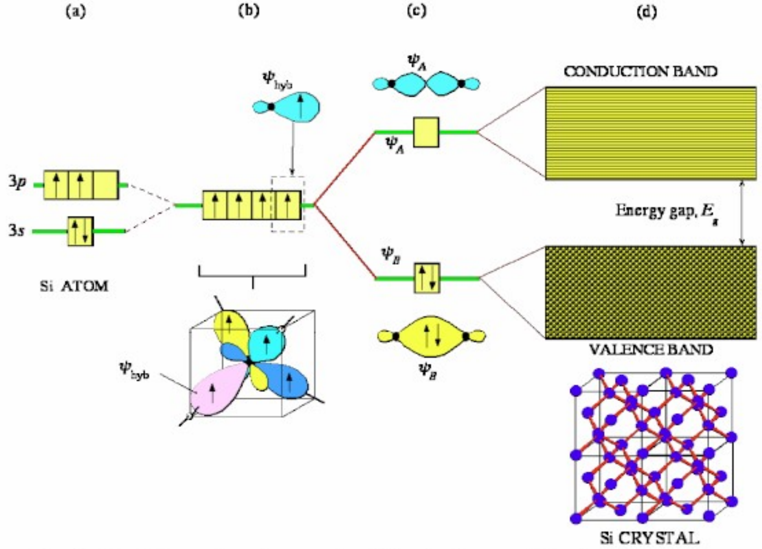


Fig. 4.17: (a) Formation of energy bands in the Si crystal first involves hybridization of 3s and 3p orbitals to four identical ψ_{hyb} orbitals which make 109.5° with each other as shown in (b). (c) ψ_{hyb} orbitals on two neighboring Si atoms can overlap to form ψ_B or ψ_A. The first is a bonding orbital (full) and the second is an antibondiong orbital (empty). In the crystal ψ_B overlap to give the valence band (full) and ψ_A overlap to give the conduction band (empty).

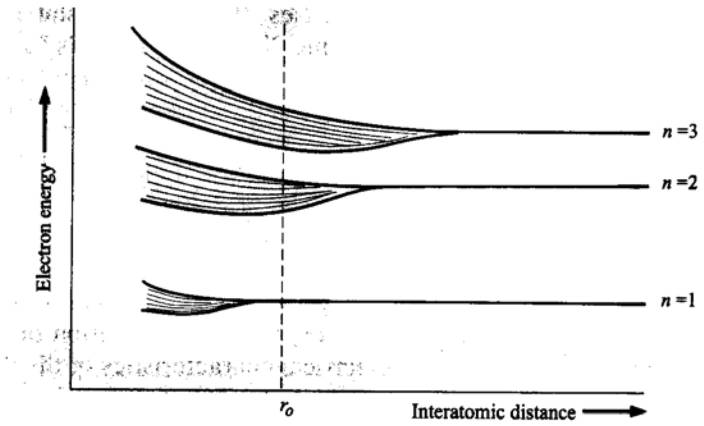


Figure 3–3 Schematic showing the splitting of three energy states into allowed bands of energies.

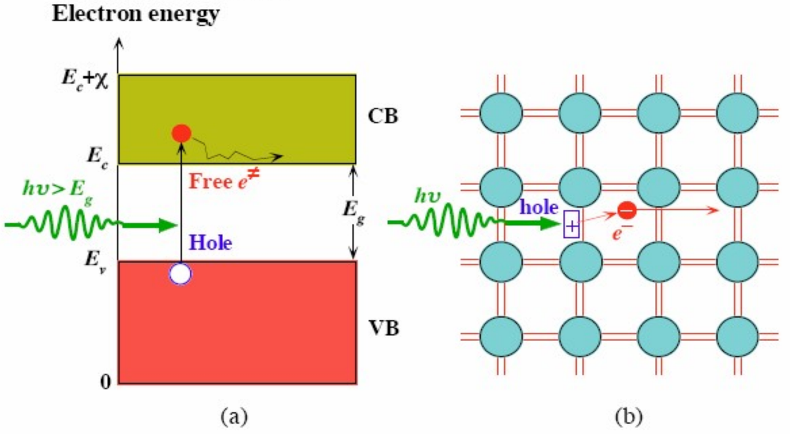


Fig. 5.3: (a) A photon with an energy greater than E_g can excite an electron from the VB to the CB. (b) When a photon breaks a Si-Si bond, a free electron and a hole in the Si-Si bond is created.

Bandgap Basics

- The application of excess energy (light, thermal, electrical) or the addition of extra electrons into the system results in conduction by moving electrons into the conduction band
- In thermal equilibrium electrons can be excited into the conduction band leaving a hole in the valance band
- Holes and electrons propagate in throughout the material via quantum mechanical tunneling from site to site randomly
- The application of a driving potential forces electrons and holes to migrate in opposite directions based on charge density
- The effective mass of holes ,m_h*, and electrons m_e* is a quantum mechanical quantity relating the inertial resistance to acceleration of each under a driving force due to electric fields within the periodic structure

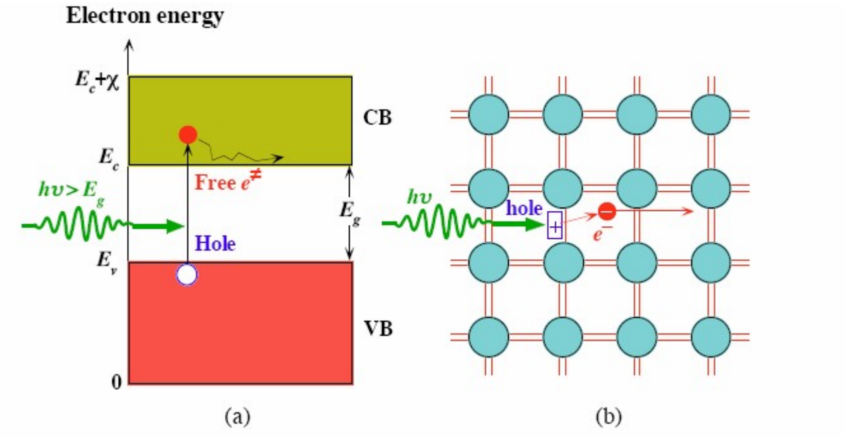
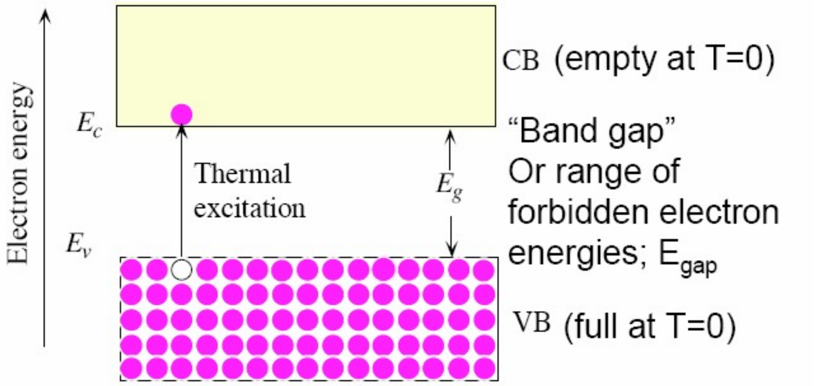


Fig. 5.3: (a) A photon with an energy greater than E_g can excite an electron from the VB to the CB. (b) When a photon breaks a Si-Si bond, a free electron and a hole in the Si-Si bond is created.



Thermal Considerations and Recombination

- ✓ The presence of a finite band gap requires that at T=0K, there is no electrical conduction within the material
- ✓ As temperature increases, more and more free energy present in the semiconductor allows for the population of conduction bands with electrons.
  - ✓ Due to atomic vibrations that increase with temperature allowing for excitation of conduction band energy states
  - ✓ Production of electrons in the conduction band due to increased free energy generates an equal number of holes in the valance band
  - ✓ This is referred to as thermal generation
- ✓ When a wondering electron crosses a site within the lattice where a hole is present, the electron releases its free energy and binds to the atoms valence band. This process is called recombination
- ✓ Electron concentration, n, within the conduction band
- ✓ Hole concentration, p, within the valance band

Semiconductor Statistics: Density of States

- Many important properties of semiconductors are described by considering electrons in the conduction band and holes in the valance band.
- Density of States (DOS), g(E), represents the number of electronics states in a band per unit energy per unit volume of the crystal
- We use quantum mechanics (QM) to calculate the DOS by considering how many electron wave functions there are within a given energy range per unit volume
- According to QM

$$E = \frac{h^2}{8mL^2}\left(n_x^2 + n_y^2 + n_z^2\right) = \frac{h^2\mathbf{n}^2}{8mL^2}$$

$$\mathbf{n} = \sqrt{\frac{8\mathrm{mL}^2 n^2}{h^2}}$$

$$g(E) = (8\pi 2^{1/2}(\frac{m_e}{h^2})^{3/2} E^{1/2}$$

where E = E – E_c for electrons in the conduction band

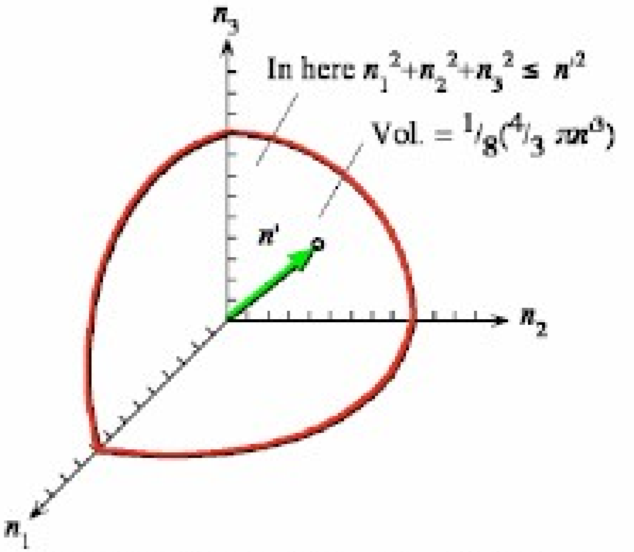


Fig. 4.22: In three dimensions, the volume defined by a sphere of radius n' and the positive axes n_1, n_2 and n_3, is all the possible combinations of poisitive n_1, n_2 and n_3, values which satisfy $n_1^2+n_2^2+n_3^2 \le n'^2$.

DOS (Cont…)

Due to spin

Positive integers only

Volume of sphere

$$from\_E = \frac{h^2}{8mL^2} n^2$$

$$\# \, states = \square \times \blacksquare \times \boxed{\frac{4}{3}\pi n^3} = \frac{2\pi}{6}\left(\frac{8mL^2E}{h^2}\right)^{3/2} = \frac{2\pi}{6}\left(\frac{8mL^2}{h^2}\right)^{3/2} E^{3/2}$$

$$\frac{d}{dE}(\# \, states) = \frac{d}{dE}\frac{2\pi}{6}\left(\frac{8mL^2}{h^2}\right)^{3/2} E^{3/2}$$

$$\longrightarrow \longrightarrow = \frac{2\pi}{6}\left(\frac{8mL^2}{h^2}\right)^{3/2} \frac{3}{2} E^{1/2} dE$$

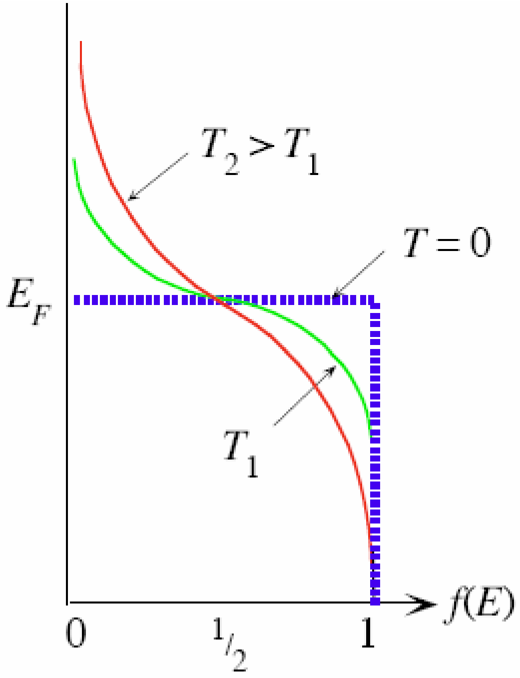
$$g(E) = \frac{4\pi L^3 (2m)^{3/2}}{h^3} E^{1/2} dE = (constants) \times E^{1/2} dE$$

Semiconductor Statistics: Fermi Dirac Function

- The Fermi Dirac Function, f(E), is the probability of finding an electron in a quantum state with energy E. This function is a fundamental property of a collection of interacting electrons in thermal equilibrium

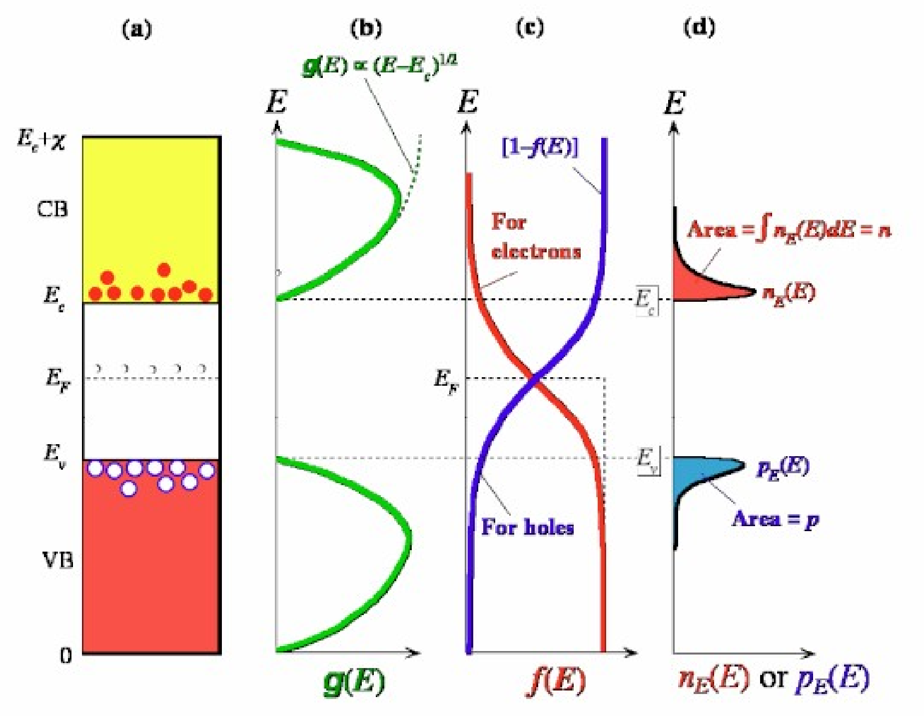
$$f(E) = \frac{1}{1 + \exp\left(\frac{E - E_F}{kT}\right)}$$

- Where k_b is the Boltzmann constant, T is the temperature in Kelvin, E_f is the Fermi energy
- Fermi Energy = energy required to fill all states at T=0K
- The Fermi energy is the chemical potential (or Gibbs free energy) per electron in the material
- Changes in the Fermi energy across the material represent the electrical work input or output per electron
- In the equilibrium state of a semiconductor with no light or applied voltage, the change in Fermi energy, $\Delta E_f = 0$, AND E_f must be uniform throughout the system
- Note: The probability of a finding a hole is 1-F(E)



Conduction Band Concentrations in a Semiconductor

- Carrier concentration applet for variable temperature
- Note: DOS, g(E), remains constant. The carrier concentration varies as a function of F(E)



At large energies where $E-E_F \gg kT$

(note: kT=0.025eV at 293K)

$$F(E) = \frac{1}{\exp\left(\frac{E-E_F}{kT}\right)+1} \cong \exp\left(\frac{-(E-E_F)}{kT}\right)$$

Electron concentration in the conduction band

$$n = \int_{E_C}^{E_C+\chi} n_e(E)dE = \int_{E_C}^{E_C+\chi} g_{CB}(E)f(E)dE$$

$$n = \frac{\pi 8\sqrt{2}}{h^3} m_e^{*3/2} \int_{E_C}^{\infty} (E-E_C)^{1/2} \exp - \left[\frac{(E-E_F)}{kT}\right] dE$$

$$n = N_c \exp\left[-\frac{(E_c-E_F)}{kT}\right]$$

where $$N_c = 2\left(\frac{2\pi m_e^* kT}{h^2}\right)^{3/2}$$

is the effective density of states at the conduction band edge

Valance Band Solution

At energies where E_F-E >>kT (i.e. E is below E_F)

$$F(E) = \frac{1}{\exp\left(\frac{E-E_F}{kT}\right)+1} \cong \frac{1}{0+1} \cong 1$$

Large negative exponential

So, the probability of occupancy far below E_F is 100%.

Note that the probability of non-occupancy is 1-F(E) (or the probability of a state being empty)

Important note: The only assumptions specific to these derivations for n and p is that the Fermi energy is only a few k_BT away from the band edges

hole concentration in the valence band

$$p = \int_0^{E_V} p_e(E)dE = \int_0^{E_V} g_{VB}(E)[1-f(E)]dE$$

$$p = N_v \exp\left[-\frac{(E_F - E_v)}{kT}\right]$$

where $N_v = 2\left(\frac{2\pi m_h^* kT}{h^2}\right)^{3/2}$

is the effective density of states at the valance band edge

Intrinsic Semiconductor

- Intrinsic semiconductors are pure crystals where n = p
- It can be shown that in an intrinsic semiconductor that the Fermi level, E_{fi}, is above E_v and located in the bandgap at

$$E_{fi} = E_v + \frac{1}{2}E_g - \frac{1}{2}k_B T \ln\left(\frac{N_c}{N_v}\right)$$

- Typically N_c and N_v values are comparable and both occur in the logarithmic term so that Efi is approximately in the middle of the bandgap as shown in previous slides
- The product of n and p in an intrinsic semiconductor provides the mass action law

$$np = N_c N_v e^{\left(-\frac{E_g}{k_B T}\right)} = n_i^2$$

- Where $E_g = E_c - E_v$ is the bandgap energy, n_i^2 is the constant that depends on temperature and material properties, and not the Fermi energy.
- Thermal velocity of electrons in an intrinsic semiconductor at room temperature

$$E = \left\langle \frac{m_e^* v^2}{2} \right\rangle = \frac{3}{2}k_B T$$

$$\sqrt{\langle v^2 \rangle} \approx 10^5 \frac{m}{s}$$

Extrinsic Semiconductors

- Semiconductors with small amounts of impurities
- These impurities increase/decrease the probability of obtaining an electron in the conduction band
- N-type semiconductors
  - extrinsic semiconductors with excess electrons
  - Arsenic added to silicon to which have one more valence (available electron) than silicon
  - Arsenic is called a donor it donates electrons to the system
  - For $N_d >> n_i$, at room temperature, the electron concentration inside the conduction band will be nearly equal to N_d such that $N_d=n$
  - Number of holes,

    $p =$

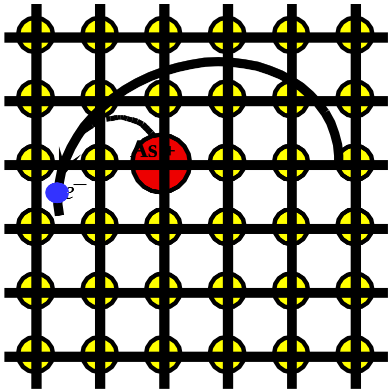
    n_i^2/N_dConductivity,
  - σ, depends on drift mobilities, μ, of electrons and holes

$$\sigma = en\mu_e + ep\mu_h$$

$$\sigma = eN_d\mu_e + e\frac{n_i^2}{N_d}\mu_h$$

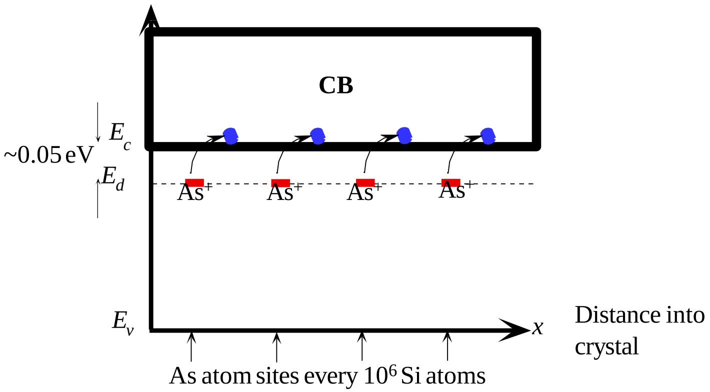
$$\sigma \approx eN_d\mu_e$$

(a)



(a) The four valence electrons of As allow it to bond just like Si but the fifth electron is left orbiting the As site. The energy required to release to free fifth-electron into the CB is very small.

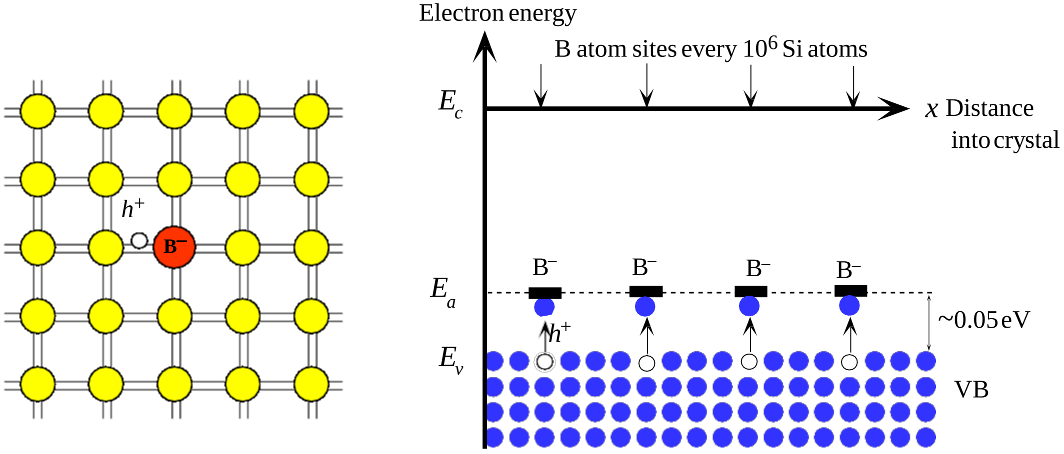
(b)



(b) Energy band diagram for an *n*-type Si doped with 1 ppm As. There are donor energy levels just below E_c around As+ sites.

Extrinsic Semiconductors…..

- Semiconductors with small amounts of impurities
- These impurities increase/decrease the probability of obtaining an electron in the conduction band
- P-type semiconductor
  - Extrinsic with less electrons
  - Adding Boron (+3) metal which has one fewer electron and yields an increased hole per doped atom
  - Boron is called an acceptor
  - For $N_a >> n_i$, at room temperature, the hole concentration inside the valence band will be nearly equal to N_a such that N_a=p
  - Electron carrier concentration is determined by the mass action law as n = n_i^2/N_a
  - This value is much smaller than p and thus the conductivity is given by $\sigma = eN_a\mu_h$

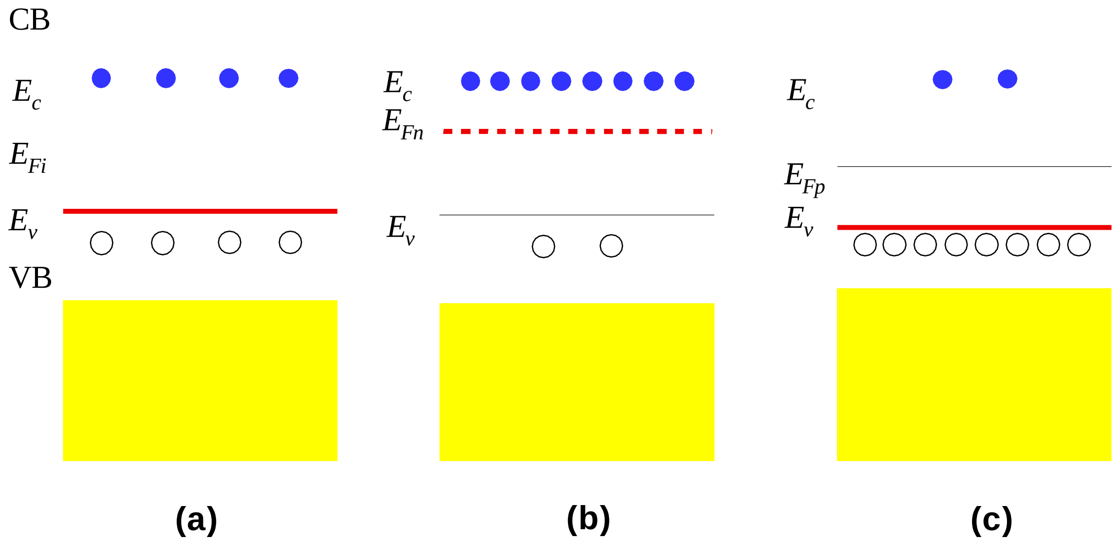


(a) **(b)**

(a) Boron doped Si crystal. B has only three valence electrons. When it substitutes for a Si atom one of its bonds has an electron missing and therefore a hole. (b) Energy band diagram for a *p*-type Si doped with 1 ppm B. There are acceptor energy levels just above E_v around B⁻ sites. These acceptor levels accept electrons from the VB and therefore create holes in the VB.

Simplified Band Diagrams for Semiconductors

- Notice in the chart below that the Fermi level changes as a function of doping
- Notice also that carrier concentration (holes or electrons) also changes as a function of doping
- N-type: majority carriers are electrons and minority carriers are holes
- P-type: majority carriers are holes and minority carriers are electrons
- Mass action law still valid: $n_{no}p_{no}=n_i^2$ where no is the doped equilibrium carrier concentration



Energy band diagrams for (a) intrinsic (b) *n*-type and (c) *p*-type semiconductors. In all cases, $np = n_i^2$. Note that donor and acceptor energy levels are not shown.

Ionization Energy

- Ionization energy is the energy required to elevate the donor electron into the conduction band and hole into the valence band.

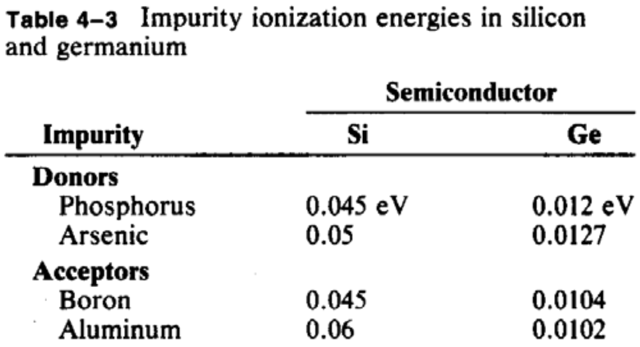


Table 4–3 Impurity ionization energies in silicon and germanium

Impurity	Semiconductor	
	Si	Ge
Donors		
Phosphorus	0.045 eV	0.012 eV
Arsenic	0.05	0.0127
Acceptors		
Boron	0.045	0.0104
Aluminum	0.06	0.0102

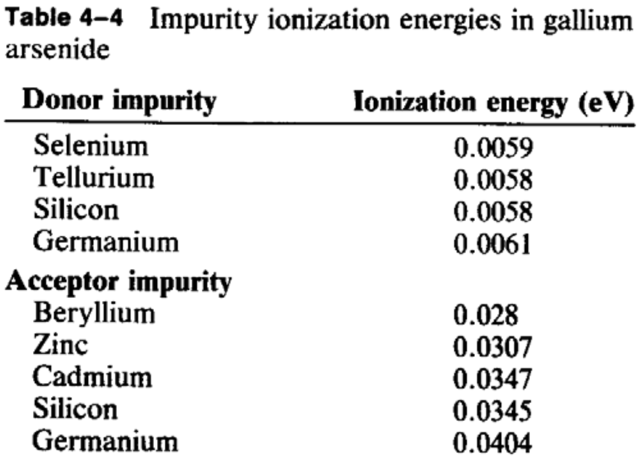


Table 4–4 Impurity ionization energies in gallium arsenide

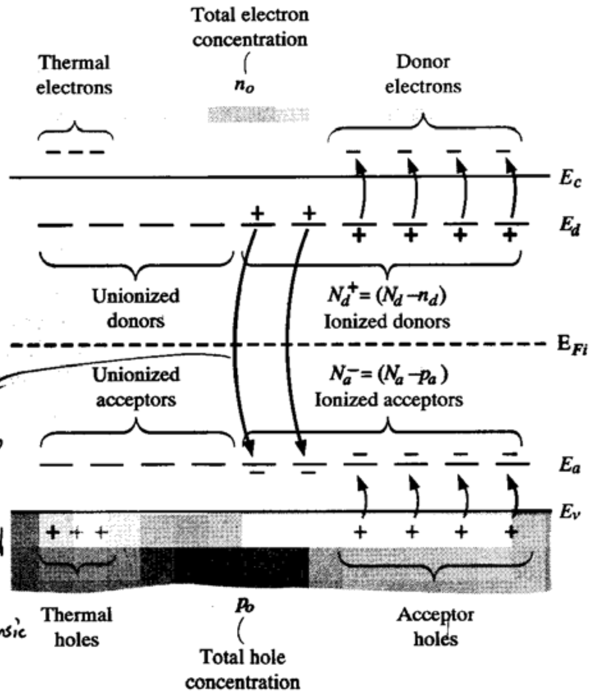
Donor impurity	Ionization energy (eV)
Selenium	0.0059
Tellurium	0.0058
Silicon	0.0058
Germanium	0.0061
Acceptor impurity	
Beryllium	0.028
Zinc	0.0307
Cadmium	0.0347
Silicon	0.0345
Germanium	0.0404

Compensation Doping

- Doping of a semiconductor with both donors and acceptors to control properties
- Provides precise control of carrier concentrations

For p-type $p_o = N_a - N_d$, *For n-type* $n_o = N_d - N_a$

- Utilized extensively where p and n type doping of different regions meet.



A few donor electrons fall into the empty states in the valence band and annihilate some of the intrinsic holes

Figure 4–14 Energy-band diagram of a compensated semiconductor showing ionized and un-ionized donors and acceptors.

Compensated Semiconductor…….

- If we assume complete ionization, $N_d^+ = N_d$ and $N_a^- = N_a$, then

$$n_o + N_a = p_o + N_d \text{ , recall } p_o = {n_i^2}/{n_o}$$

$$n_o + N_a = \frac{n_i^2}{n_n} + N_d \Rightarrow n_o^2 - (N_d - N_a)n_o - n_i^2 = 0$$

$$\Rightarrow n_o = \frac{(N_d - N_a)}{2} + \sqrt{\left(\frac{N_d - N_a}{2}\right)^2 + n_i^2}$$

- If $N_a = N_d = 0$, (for the intrinsic case), $\Rightarrow n_o = p_o$
- If $N_d >> N_a$, $\Rightarrow n_o = N_d$
- If $N_a > N_d$, $\Rightarrow p_o = \frac{(N_a - N_d)}{2} + \sqrt{\left(\frac{N_a - N_d}{2}\right)^2 + n_i^2}$ is used to calculate the conc. of holes in valence band

Compensated Semiconductor……

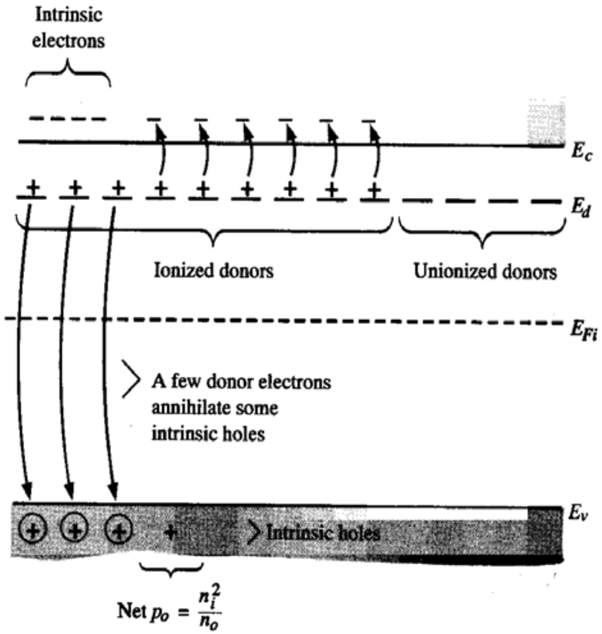
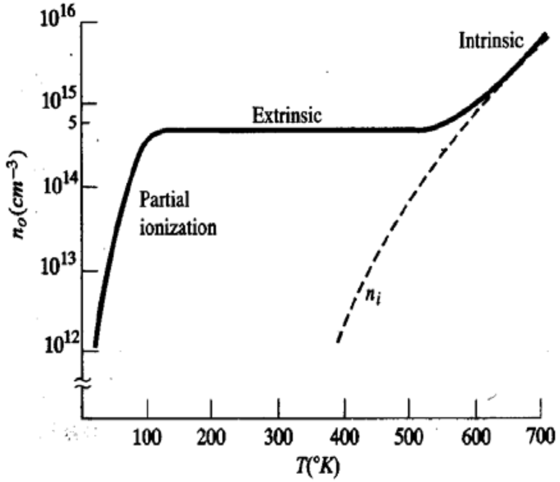


Figure 4–15 Energy-band diagram showing the redistribution of electrons when donors are added.



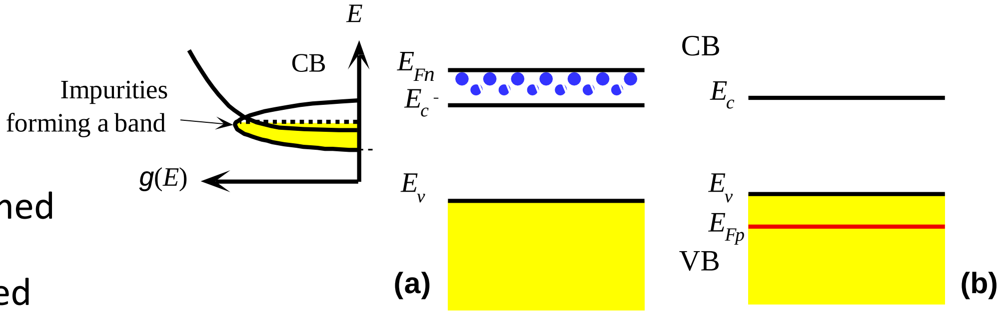
–16 Electron concentration versus temperature showing the three regions: partial on, extrinsic, and intrinsic.

Degenerate and Nondegenerate Semiconductors

- In nondegenerate semiconductors the number of states in the carrier band far exceeds the number of electrons
- Thus the probability of two electrons trying to occupy the same allowed state is virtually zero
- This means that the Pauli exclusion principle can be neglected and the DOS is represented only by Boltzmann statistics
- In this case,

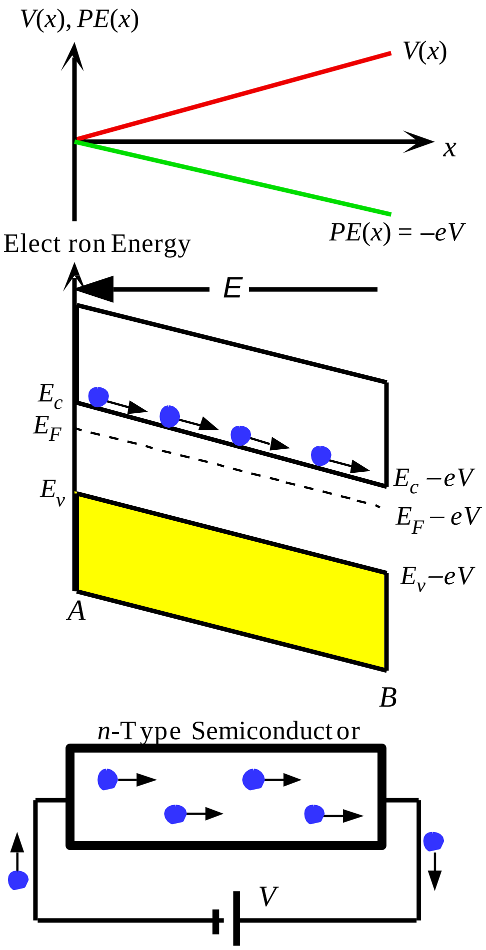
$$f(E) = \frac{1}{1 + \exp\left(\frac{E - E_F}{kT}\right)}$$

  is only valid when n << N_c
- Semiconductors where n<< N_c and p << N_v are termed nondegenerate
- When a semiconductors has been excessively doped with donors
  - then n may be $10^{19} – 10^{20}$ /cm^3 n > N_p or P>N_v
  - N is then comparable to N_c and the Pauli exclusion principle comes into play
  - In this case Fermi Dirac statistics are required
  - Degenerate semiconductor



(a) Degenerate *n*-type semiconductor. Large number of donors form a band that overlaps the CB. (b) Degenerate *p*-type semiconductor.

Energy Band Diagram in an Applied Field



Energy band diagram of an *n*-type semiconductor connected to a voltage supply of *V* volts. The whole energy diagram tilts because the electron now has an electrostatic potential energy as well.

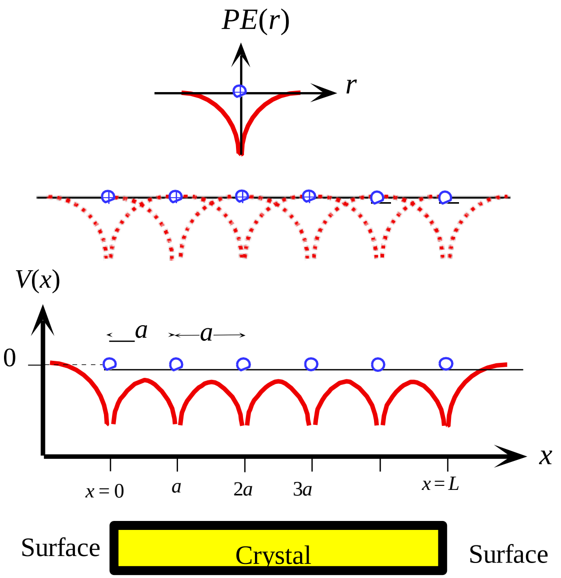
Potential Theory: A More Precise Band Diagram

- The time independent Schrödinger equation for a given potential function is written as

$$\nabla\Psi + \frac{2m_e}{\hbar^2}\left[E - V(x)\right] = 0$$ with a general solution of $$\Psi(x) = Ce^{ikx} + De^{-ikx}$$

If the potential energy, V, is periodic in nature as that shown below, then one can write it as

$$V(x) = V(x + ma) \qquad m = 1,2,3,\ldots$$



PE of the electron around an isolated atom

When *N* atoms are arranged to form the crystal then there is an overlap of individual electron *PE* functions.

PE of the electron, *V*(*x*), inside the crystal is periodic with a period *a*.

The electron potential energy (*PE*), *V*(*x*), inside the crystal is periodic with the same periodicity as that of the crystal, *a*. Far away outside the crystal, by choice, *V* = 0 (the electron is free and *PE* = 0).

E-K Bandgap Diagram (Bloch Wavefunction)

- The solution is called a Bloch Wavefunction

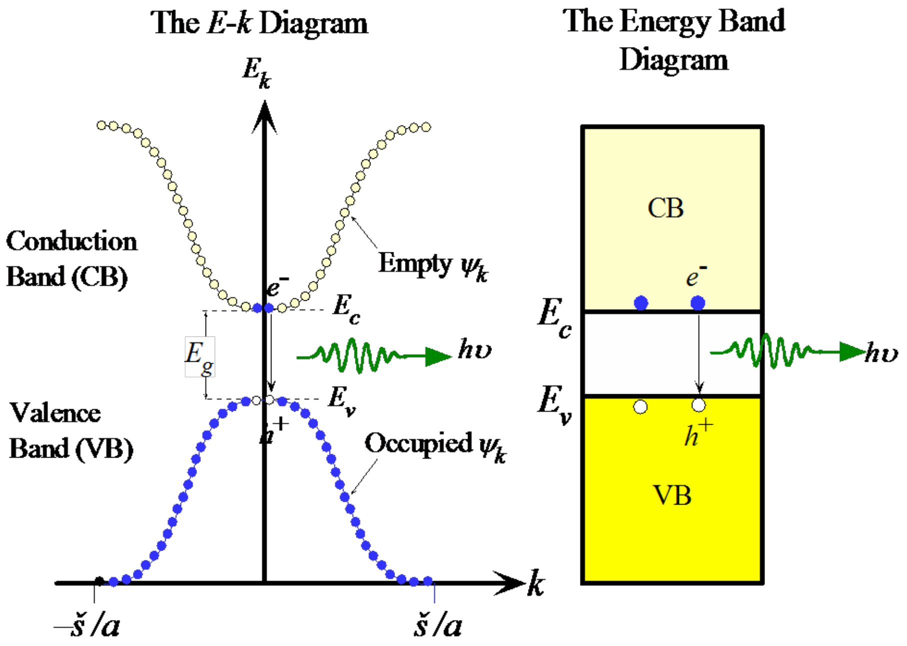
$$\Psi_k(x) = U(x)_k e^{jkx}$$

  - where U(x) is a periodic function that depends on V(x). The two share the same periodicity
- The wavevector, k, in this solution acts like a quantum number and has values from –π/a to π /a

$$k^2 = \frac{2mE}{\hbar^2}$$

- Momentum, p, in the crystal is ħk
- External forces:

$$F = qE = \frac{dp}{dt} = \frac{d(\hbar k)}{dt}$$



The *E-k* diagram of a direct bandgap semiconductor such as GaAs. The *E-k* curve consists of many discrete points with each point corresponding to a possible state, wavefunction $\psi_k(x)$, that is allowed to exist in the crystal. The points are so close that we normally draw the *E-k* relationship as a continuous curve. In the energy range E_v to E_c there are no points ($\psi_k(x)$ solutions).

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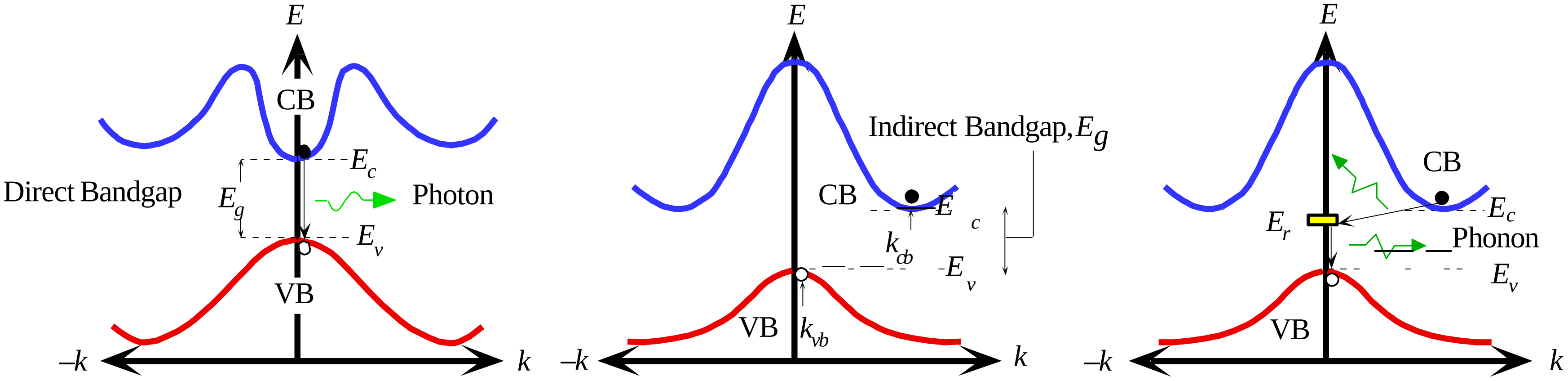
Direct vs. Indirect Bandgap

- Direct Bandgap
  - Base of the conduction band is matched to the max height of the valence band
  - Recombination through the emission of a photon (Light!!!!!!)
- Indirect Bandgap
  - direct recombination would require a momentum change (not allowed)
  - Recombination centers (lattice defects) are required to recombine CB to VB bands
  - The result is a phonon emission (lattice vibration) that propagates across the lattice



(a) GaAs　(b) Si　(c) Si with a recombination center

(a) In GaAs the minimum of the CB is directly above the maximum of the VB. GaAs is therefore a direct bandgap semiconductor. (b) In Si, the minimum of the CB is displaced from the maximum of the VB and Si is an indirect bandgap semiconductor. (c) Recombination of an electron and a hole in Si involves a recombination center .

- Semiconductor Concepts and Energy Bands
- Carrier Transport Phenomena
- Graded impurity distribution

- The net flow of the electron and holes in a semiconductor will generate current.

- The process which these charged particles move is call transport.

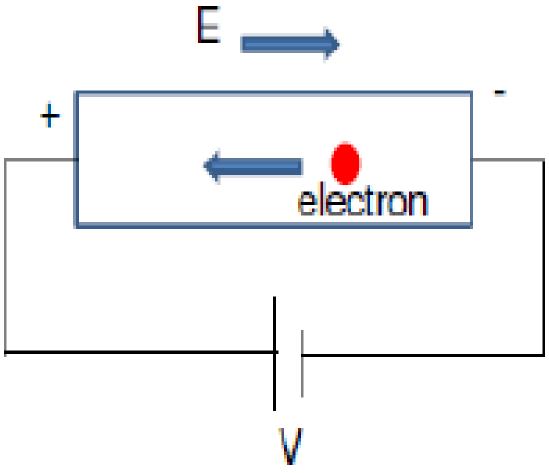
- Two basic transport : Drift & Diffusion.

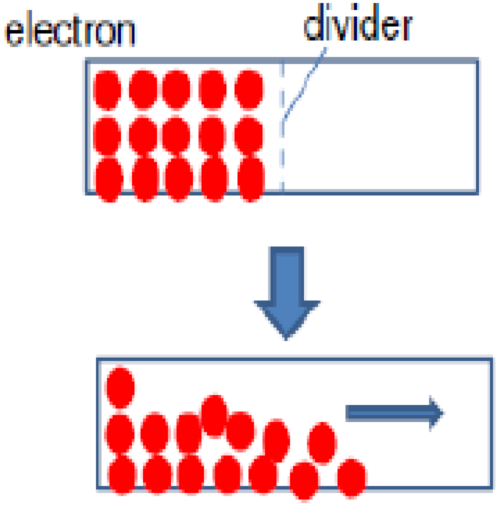
- The carrier transport phenomena are the foundation for finally determining the current-voltage characteristics of semiconductor devices.

Carrier Transport

"Drift"
The movement of carrier due to electric field (E)

"Diffusion"
The flow of carrier due to density gradients (dn/dx)





Carrier Drift

- Transport : The process which charged particles (holes or electrons) are move.
- Understanding of the electrical properties (I-V properties)
- Basic current Equation :

$$I \propto e \cdot \mu \cdot n \cdot E$$

e; electronic charged (constant, 1.6 x 10-19 C)
u; mobility (figure of merit that reflect the speed)
n; carrier concentration
E; Electric field

Carrier Drift....

- When an E-field (force) applied to a semiconductor, electrons and holes will experience a net acceleration and net movement, if there are available energy states in the conduction band and valence band. The net movement of charge due to an electric field (force) is called "*drift*".
- Mobility: the acceleration of a hole due to an E-field is related by

  $$F = m_p^* \frac{dv}{dt} = qE$$

  If we assume the effective mass and E-field are constants, the we can obtain the drift velocity of the hole by

  $$v_d = \frac{qEt}{m_p^*} + v_i \propto t,\ E$$

  where v_i is the initial velocity (e.g. thermal velocity) of the hole and t is the acceleration time.

Drift Current Density

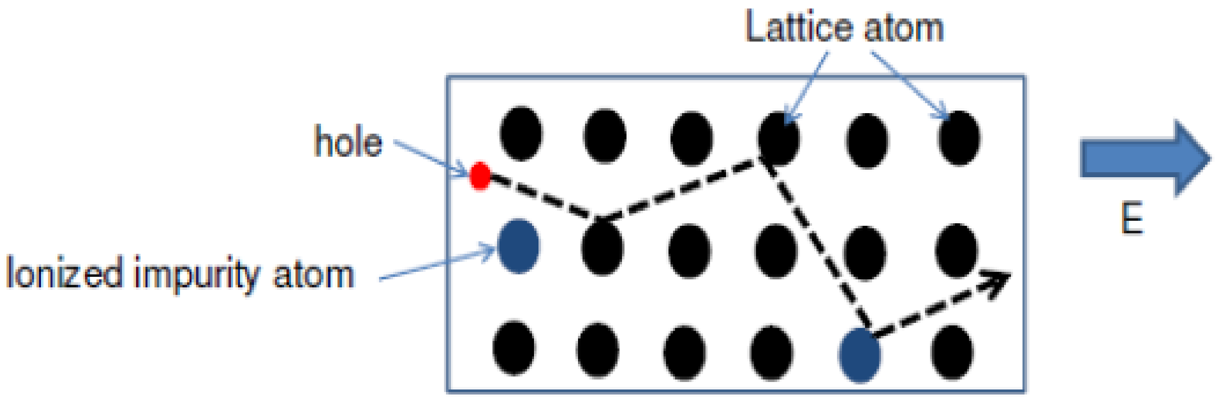
Consider a **positively charged hole,**

- When electric field, E, is applied, the hole accelerates

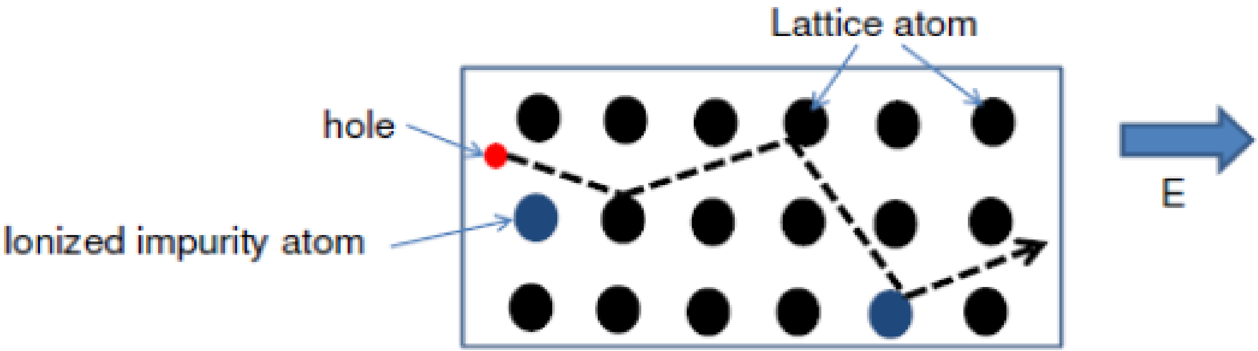
$$F = m_p^* a = eE$$

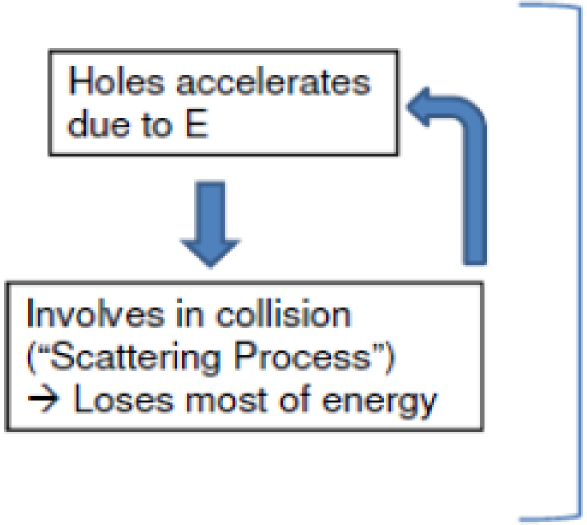
m*p; effective mass of hole, a; acceleration, e; electronic charge

- However, hole collides with ionized impurity atoms and with thermally vibrating lattice atom



Drift Current Density……





Holes accelerates due to E

Involves in collision ("Scattering Process") → Loses most of energy

Gain average drift velocity, v_{dp}

$$v_{dp} = \mu_p E$$

μ_p; Hole mobility (unit; cm^2/Vs)
Describes how well a carrier move due to E

Drift Current Density……

The drift current due to the motion of electrons is $J = -e\sum_{i=1}^{n} v_i$

where n is the number of electrons per volume and v_i is the electron velocity in the crystal.

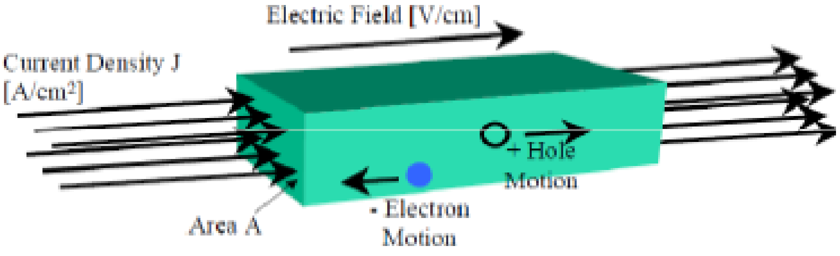
Drift current density, J_{drf} (unit; A/cm^2) due to hole

$$J_{p|drf} = epv_{dp}$$

$$\boxed{J_{p|drf} = e\mu_p pE}$$

Drift current density due to electron

$$\boxed{J_{n|drf} = e\mu_n nE}$$

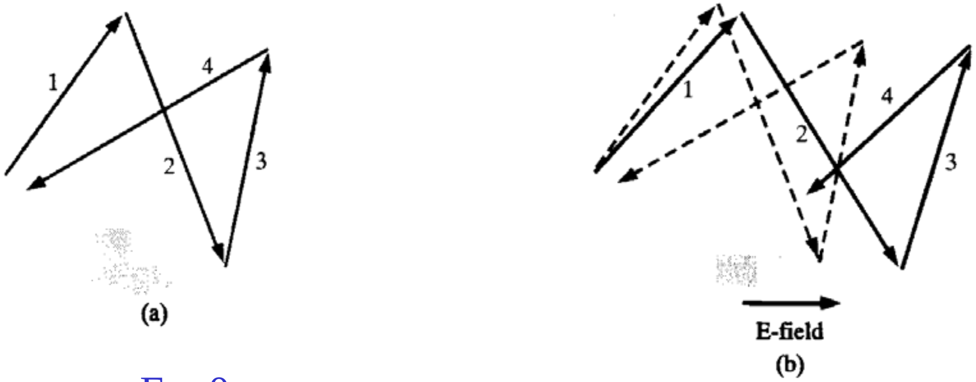


Total drift current;

$$\boxed{J_{drf} = e(\mu_n n + \mu_p p)E}$$

The sum of the individual electron and hole drift current densities

Mobility



E = 0

- In semiconductors, holes/electrons are involved in collisions with ionized impurity atoms and with thermally vibration lattice atoms. As the hole accelerates in a crystal due to the E-field, the velocity/kinetic energy increases. When it collides with an atom in the crystal, it lose s most of its energy. The hole will again accelerate/gain energy until is again involved in a scattering process.

Mobility Effects…..

- The electron in s/c have 3 degree of freedom – they can move in a 3-D space. The K.E of electron is given by

$$K_e = \frac{1}{2} m_n v_{th}^2 = \frac{3}{2} kT$$

From the theorem for equipartition of energy, ½ kT unit energy per degree of freedom.

- m_n – effective mass of electron,
- v_{th} – average thermal velocity (~ 10^7cm/s at T=300K)

Mobility Effects……

- Electron in s/c moving rapidly in all direction, where thermal motion of an individual electron may be visualized as a succession of random scattering from collisions with lattice atoms, impurity atoms, and other scattering centers

- Average distance between collisions – mean free path.
- Average time between collisions – mean free time τ_C.
- For typical mean free path ~ 10^{-5}cm, $\tau_C = 10^{-15}/v_{th} \sim 10^{-12}$s (or in 1ps).

Mobility Effects……..

- When small electric field, E, is applied to s/c sample, each electron will experience a force –qE from the field and accelerated along the field (in opposite direction) during the time between collisions – additional thermal velocity component.
- This additional component called *drift velocity*.
- Combination displacement of an electron (due to random thermal motion) & drift component illustrated in Fig. 3.1(b).
- Note that: net displacement of the electron is in the opposite direction of applied field.

Mobility Effects……

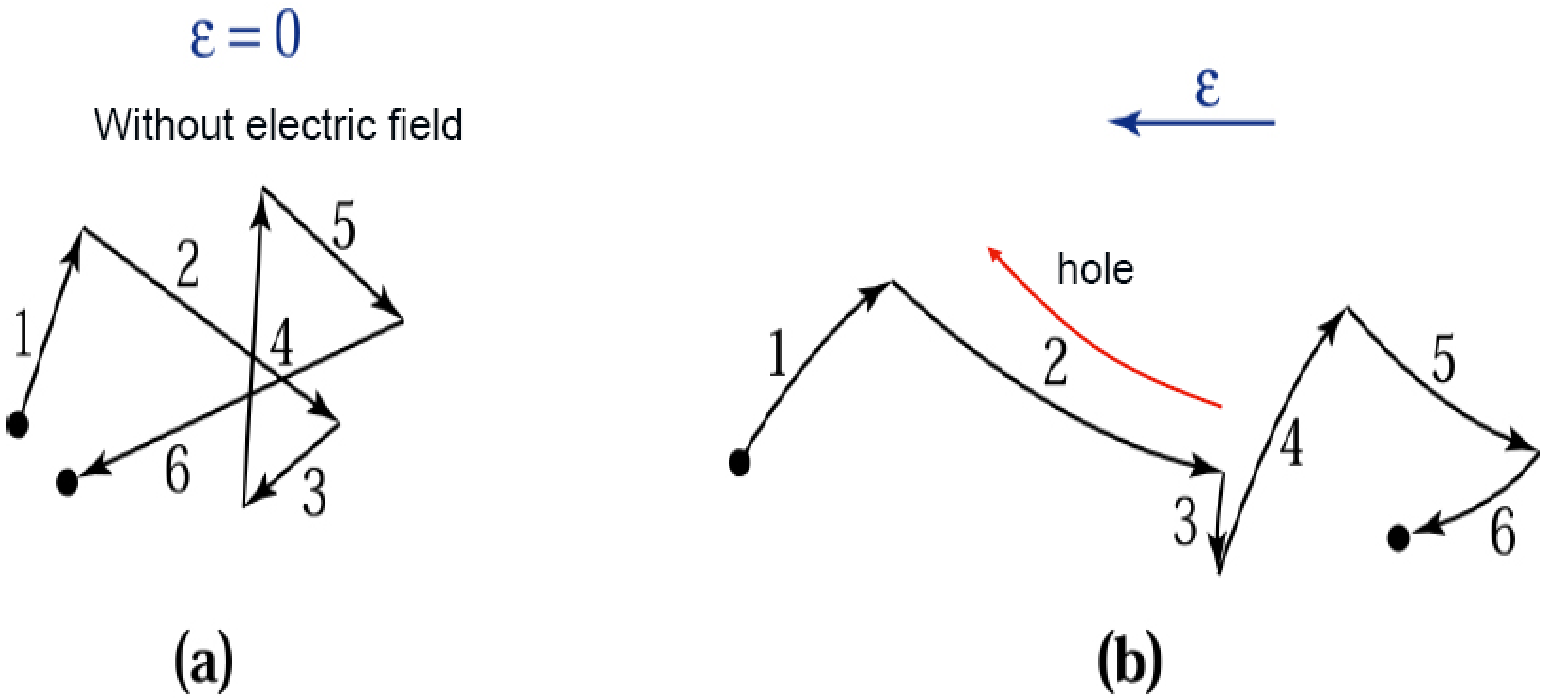


Figure 3.1. Schematic path of an **electron** in a semiconductor. (*a*) Random thermal motion. (*b*) Combined motion due to random thermal motion and an applied electric field.

Mobility Effects….

- The momentum applied to an electron is given by -qEτ_C, and momentum gained is $m_n v_n$. Thus, using physics conservation of energy, electron drift velocity:

$$v_n = -\mu_n E \qquad (2)$$

- Note that: v_n is proportional to E
- The proportionality factor may be written as

$$\mu_n = \frac{q \tau_C}{m_n} \qquad (3)$$

- The proportionality factor also called *electron mobility*.
- A similar expression may be written for holes in valence band may be written as: $v_p = \mu_p E$

- Mobility is very important parameter for carrier transport – it describes how strongly the motion of an electron is influenced by an applied electric field.

Mobility Effects……

- From eq. (3), mobility is related directly to mean free time between collisions determined by the various scattering mechanism.
- Two MOST important mechanisms: lattice scattering and impurity scattering.
- Lattice scattering – results from thermal vibrations of the lattice atoms at any temperature, T>0K (it becomes dominant at high temp. – mobility decreases with increasing temp.) – theoretically mobility due to lattice scattering μ_L decrease in proportion to $T^{-3/2}$
- Impurity scattering – results when charge carrier travels past am ionized dopant impurity (donor or acceptor). It depend on Coulomb force interaction.
- Impurity scattering depends on total concentration of ionization impurities (sum of +ve and –ve charge ions). It becomes less significant at higher temperatures.

Mobility Effects….

The probability of a collision taking place in unit time, $1/\tau_C$, - the sum of the probabilities of collision due to the various scattering mechanism:

$$\frac{1}{\tau_C} = \frac{1}{\tau_{C,\,\mathrm{lattice}}} + \frac{1}{\tau_{C,\,\mathrm{impurity}}} \qquad (4)$$

or

$$\frac{1}{\mu} = \frac{1}{\mu_L} + \frac{1}{\mu_I} \qquad (4a)$$

μ_L – lattice scattering mobility

μ_I – impurity scattering mobility

Mobility Effects…..

- Electron mobility as a function of temp. for Si with 5 different donor concentration is given by Fig. 3.2.
- For lightly doping (i.e $10^{14}cm^{-3}$) – lattice scattering dominates and mobility decreases as the temp. increases.
- For heavily doped (i.e $10^{19}cm^{-3}$) – at low temp. impurity scattering is most pronounced. Mobility is increases as temp. increases.
- For a given temp., mobility decreases with increasing impurity concentration (due to enhanced impurity scattering).

Mobility Effects…..

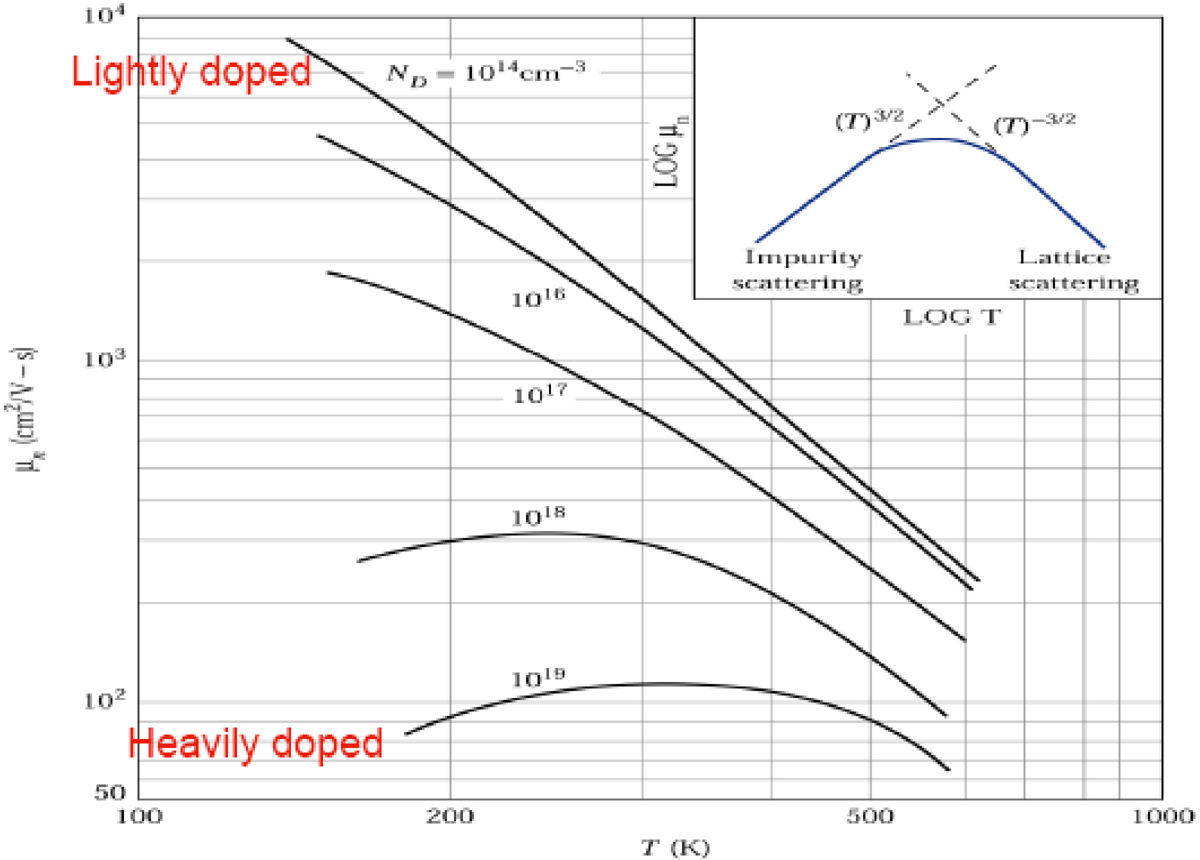


Figure 3.2.

Electron mobility in silicon versus temperature for various donor concentrations. Insert shows the theoretical temperature dependence of electron mobility.

Mobility Effects

- Mobility reaches a maximum value at low impurity concentrations corresponds to the lattice scattering limitation.
- Both electron & hole mobilities decrease with increasing impurity concentration.
- Mobility of electrons is greater than holes due to the smaller effective mass of electrons.

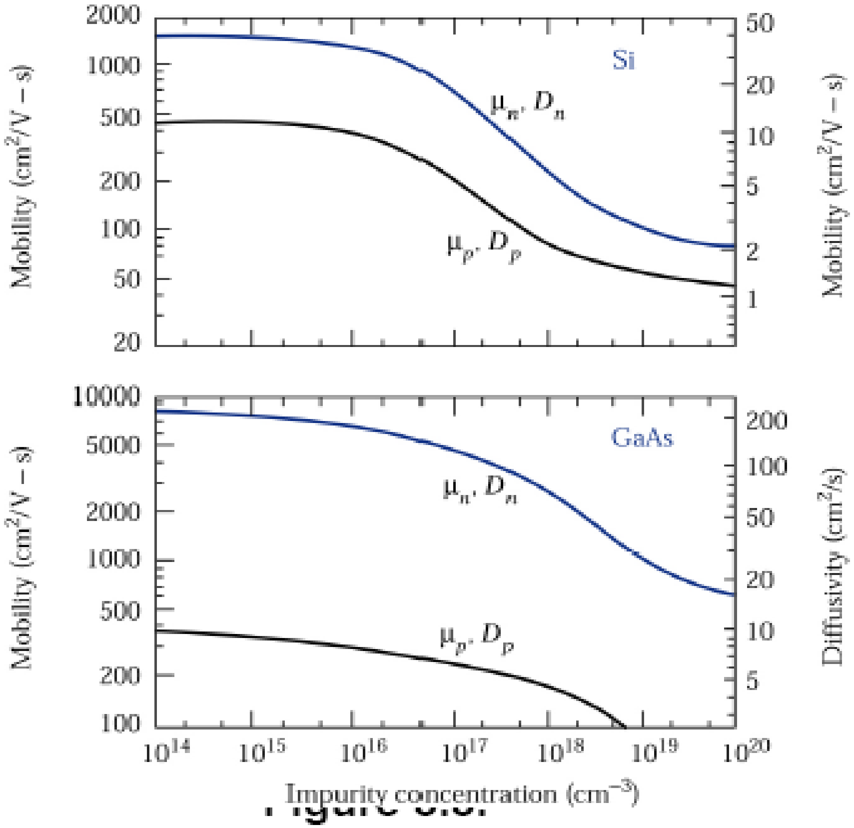


Figure [illegible]

Mobilities and diffusivities in Si and GaAs at 300 K as a function of impurity concentration.

Conductivity

Conductivity : the ability of a material to conduct electric current

Drift current

$$J_{drf} = e(\overset{\text{electron}}{\boxed{\mu_n n}} + \underset{\text{hole}}{\boxed{\mu_p p}})E = \sigma E$$

σ; conductivity [(Ω.cm)$^{-1}$]

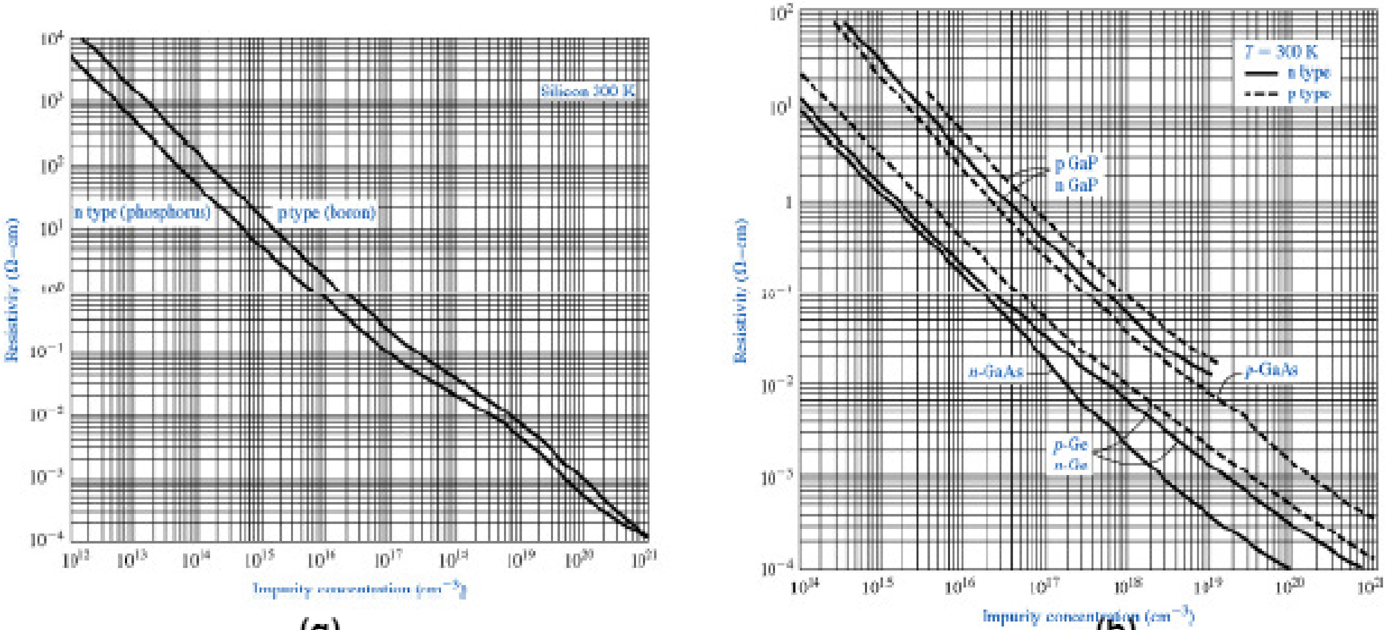
$$\sigma = e(\mu_n n + \mu_p p)$$

Function of electron and hole concentrations and mobilities

P; resistivity [Ω.cm]

$$\rho = \frac{1}{\sigma} = \frac{1}{e(\mu_n n + \mu_p p)}$$

Conductivity……

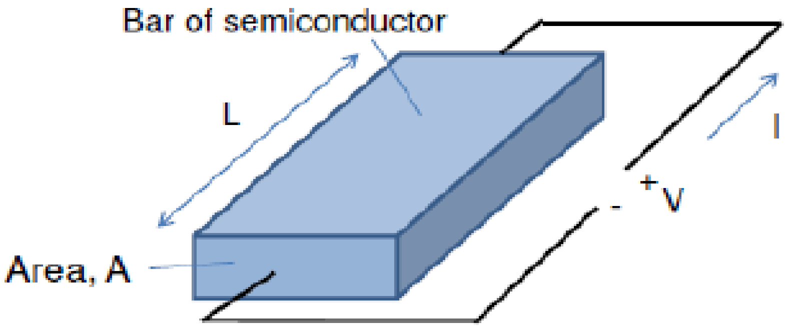


(a)

(b)

Fig 6: Resistivity vs impurity concentration at T=300K in (a) silicon and (b) germanium, GaAs and GaP

Conductivity.....



Current density, $J = \frac{I}{A}$

Electric field, $E = \frac{V}{L}$

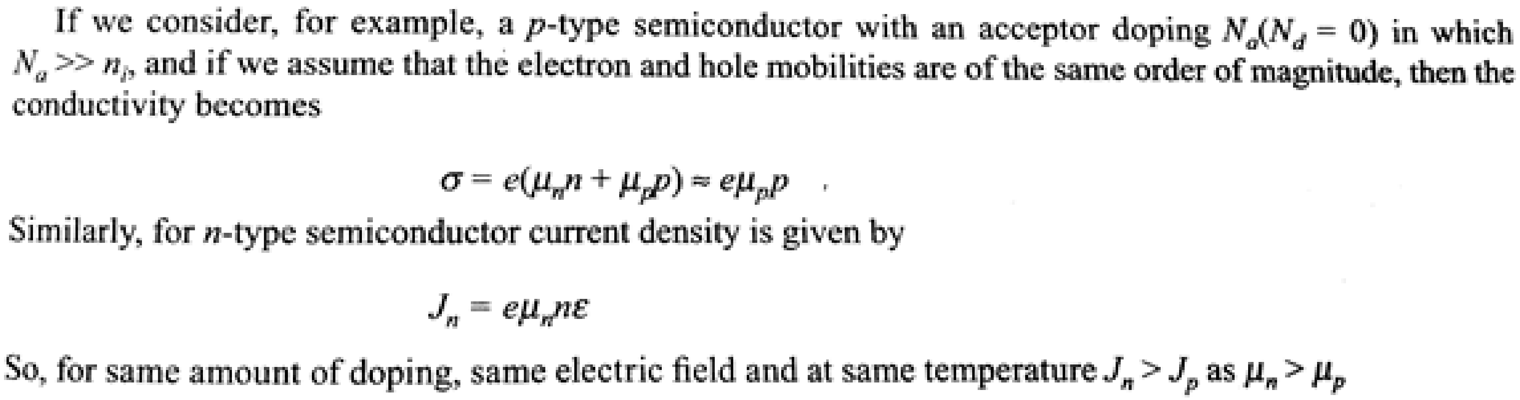
$$J = \sigma E$$

$$\frac{I}{A} = \sigma \frac{V}{L}$$

⇨ $$V = \left(\frac{L}{\sigma A}\right) I = \left(\frac{\rho L}{A}\right) I = IR$$

Resistance, R is a function of resistivity, or conductivity, as well as the geometry of the semiconductor

Conductivity / resistivity….



If we consider, for example, a *p*-type semiconductor with an acceptor doping $N_a(N_d = 0)$ in which $N_a >> n_i$, and if we assume that the electron and hole mobilities are of the same order of magnitude, then the conductivity becomes

$$\sigma = e(\mu_n n + \mu_p p) \approx e\mu_p p$$

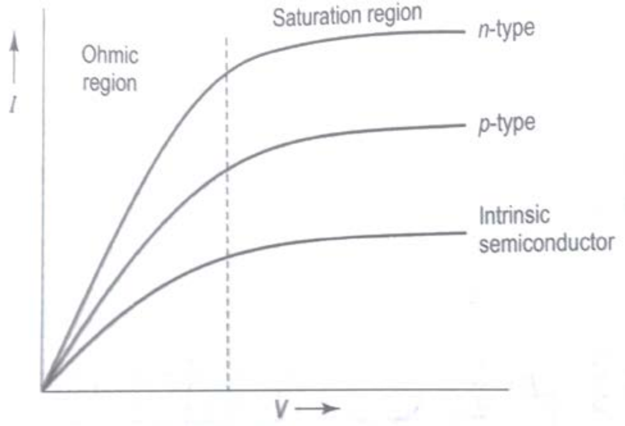
Similarly, for *n*-type semiconductor current density is given by

$$J_n = e\mu_n n\varepsilon$$

So, for same amount of doping, same electric field and at same temperature $J_n > J_p$ as $\mu_n > \mu_p$

Conductivity first increases with the increase of temperature as both mobility and concentration increase. After that conductivity decrease as concentration remains constant (extrinsic region) but mobility decreases due to lattice scattering. Then the conductivity again starts rising in this region)

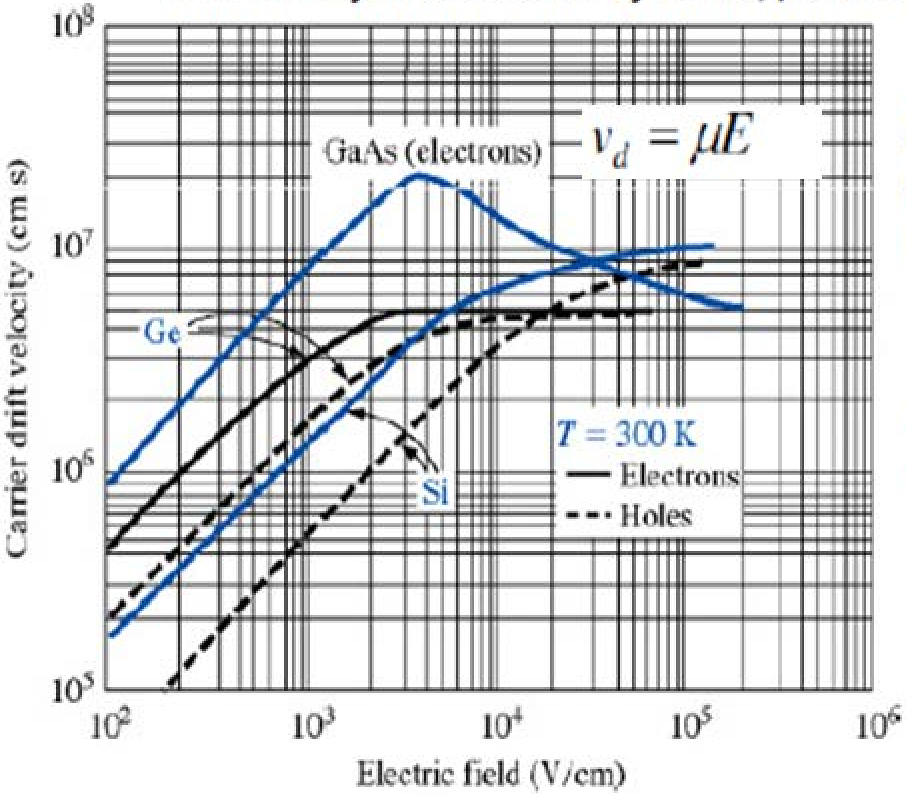
Upto certain region current increases linearly with the increase of voltage. After that limit of voltage current saturates because of velocity saturation.



Velocity Saturation

So far we assumed that mobility is indep. of E-field, that is the drift velocity is in proportion with the E-field. This holds for low E-filed. In reality, the drift velocity saturates at ~10^7 cm/sec at an E-field ~30 kV/cm. So the drift current density will also saturate and becomes indep. of the applied E-field.

Drift velocity increase linearly with applied electric field.



✓At low electric field, vd increase linearly with applied E. slope=mobility

✓At high electric field, vd saturates → Constant value

For GaAs, the electron drift velocity reaches a peak and then decreases as the E-field increases. ⇒negative differential mobility/resistivity, which could be used in the design of oscillators.

This could be understood by considering the E-k diagram of GaAs.

Velocity Saturation of GaAs….

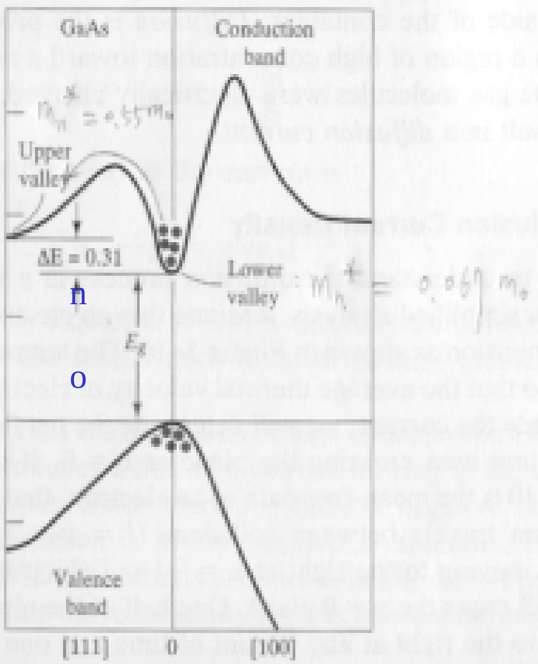


Figure 5–8 Energy-band structure for gallium arsenide showing the upper valley and lower valley in the conduction band. (From Sze [8]).

In the lower valley, the effective mass of the electron $m^* = 0.067m_o$. The small effective mass leads to a large mobility. As the E-field increases, the energy of the electron increases and can be scattered into the upper valley, where the effective mass is $0.55m_o$. The large effective mass yields a smaller mobility.

The intervalley transfer mechanism results in a decreasing average drift velocity of electrons with E-field, or the negative differential mobility characteristic.

Carrier Diffusion

Diffusion is the process whereby particles flow from a region of high concentration toward a region of low concentration. The net flow of charge would result in a diffusion current.

current density may explain by mathematical formalism below:

LHS $F_1 = \dfrac{n(-l)v_{th}}{2}$

RHS $F_2 = \dfrac{n(l)v_{th}}{2}$

$$F = F_1 - F_2 = \frac{1}{2}\left\{\left[n(0) - l\frac{dn}{dx}\right] - \left[n(0) + l\frac{dn}{dx}\right]\right\}$$

$$= -v_{th} l \frac{dn}{dx} = -D_n \frac{dn}{dx}$$

F ~ average electron flow per unit area.

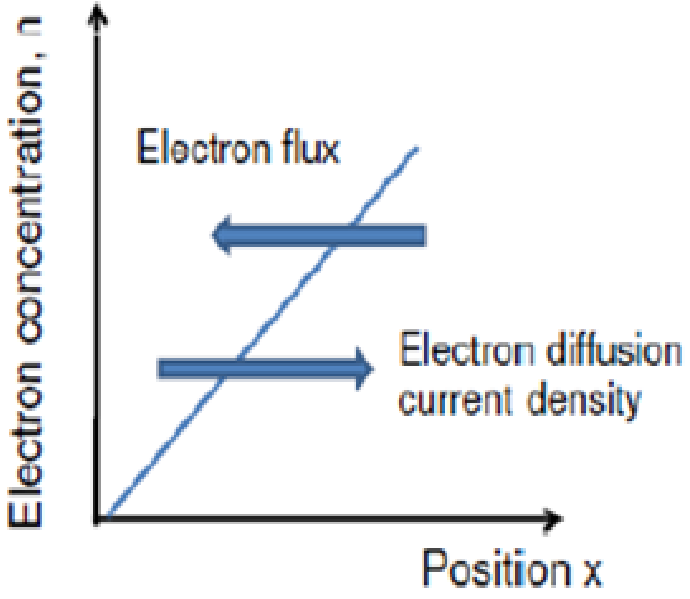
l ~ mean free path

D_n ~ diffusion coefficient

$$\therefore \quad J_n = qD_n \frac{dn}{dx}$$

Carrier Diffusion…..

Diffusion; process whereby particles from a region of high concentration toward a region of low concentration.



$$J_{n|dif} = (-e)D_n\left(-\frac{dn}{dx}\right)$$

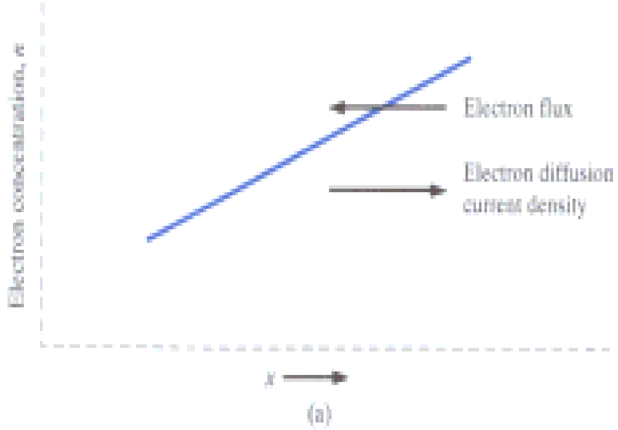
$$\boxed{J_{n|dif} = eD_n\frac{dn}{dx}}$$

D_n: electron diffusion coefficient

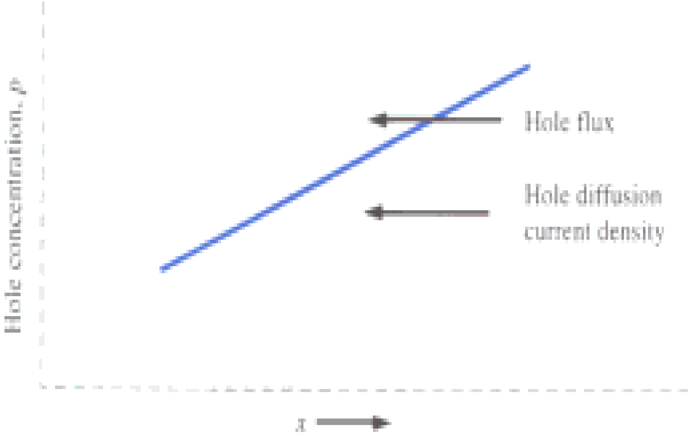
NOTE: Things diffuse (spontaneously rearrange) from regions of high concentration to low concentration

Diffusion moves things downhill => Flow is proportional to NEGATIVE of concentration slope!!!

Diffusion Current Density



(a)



- The electron diffusion current density is given by $J_{ndif} = eD_n dn/dx$, where D_n is called the electron diffusion coefficient, has units of cm^2/s.
- The hole diffusion current density is given by $J_{pdif} = -eD_p dp/dx$, where D_p is called the hole diffusion coefficient, has units of cm^2/s.
- The total current density composed of the drift and the diffusion current density.

1-D

$$J = en\mu_n E_x + ep\mu_p E_x + eD_n \frac{dn}{dx} - eD_p \frac{dp}{dx}$$

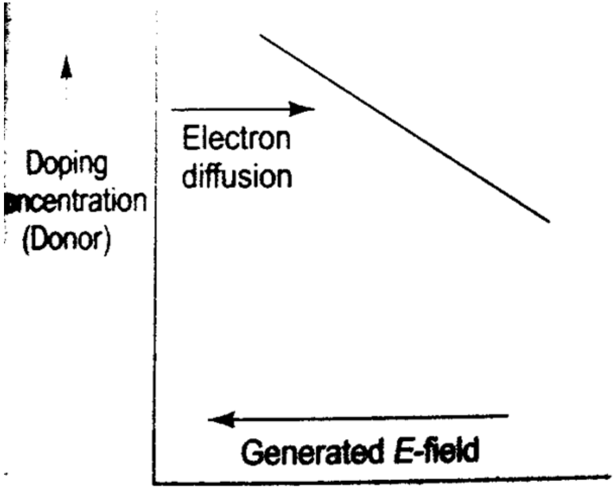
or 3-D

$$J = en\mu_n E_x + ep\mu_p E_x + eD_n \nabla n - eD_p \nabla p$$

- Semiconductor Concepts and Energy Bands
- Carrier Transport Phenomena
- Graded impurity distribution

Graded Impurity Distribution

- In some cases, a semiconductors is not doped uniformly. If the semiconductor reaches thermal equilibrium, the Fermi level is constant through the crystal.



4.14a *Variation of doping concentration with distance*

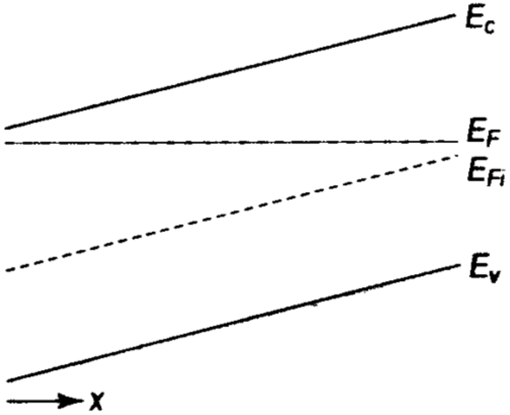


Fig. 4.14b *Energy-band diagram for a semiconductor in thermal equilibrium with a nonuniform donor impurity concentration*

- Since the doping concentration decreases as x increases, there will be a diffusion of majority carrier electrons in the +x direction.
- The flow of electrons leave behind positive donor ions. The separation of positive ions and negative electrons induces an E-field in +x direction to oppose the diffusion process.

Induced E-Field

- The Electric potential ϕ is related to electron potential energy by charge (-e)

$$\phi = +\frac{1}{e}(E_F - E_{Fi})$$

- The induced E-field is defined as $E_x = -\frac{d\phi}{dx} = -\frac{d\,(E_{Fi}/(-e))}{dx} = \frac{1}{e}\frac{dE_{Fi}}{dx}$ that is, if the intrinsic Fermi level changes as a function of distance through a semiconductor in thermal equilibrium, an E-field exists.
- If we assume a quasi-neutrality condition in which the electron concentration is almost equal to the donor impurity concentration, then

$$n_o \approx n_i \exp\left[\frac{E_F - E_i}{kT}\right] \approx N_d(x) \Rightarrow E_F - E_i = kT\ln\left(\frac{N_d(x)}{n_i}\right)$$

$$\Rightarrow \frac{d\,(E_F - E_i)}{dx} = \frac{d(-E_i)}{dx} = \frac{kT}{N_d(x)}\frac{dN_d(x)}{dx}$$

$$\Rightarrow E_x = -\left(\frac{kT}{e}\right)\frac{1}{N_d(x)}\frac{dN_d(x)}{dx}$$

- So an E-field is induced due to the non-uniform doping.

Einstein Relation

- Assuming there are no electrical connections between the nonuniformly doped semiconducotr, so that the semiconductor is in thermal equilibrium, then the individual electron and hole currents must be zero.

$$\Rightarrow J_n = 0 = en\mu_n E_x + eD_n \frac{dn}{dx}$$

- Assuming quasi-neutrality so that $n \approx N_d(x)$ and

$$J_n = 0 = eN_d(x)\mu_n E_x + eD_n \frac{dN_d(x)}{dx}$$

$$\Rightarrow 0 = -e_n\mu_n N_d(x)\left(\frac{kT}{e}\right)\frac{1}{N_d(x)}\frac{dN_d(x)}{dx} + eD_n \frac{dN_d(x)}{dx}$$

$$\Rightarrow \frac{D_n}{\mu_n} = \frac{kT}{e}\text{----Einstein relation}$$

- Similarly, the hole current $J_p = 0$ $\Rightarrow \frac{D_p}{\mu_p} = \frac{kT}{e}$

Einstein Relation……..

- Einstein relation says that the diffusion coefficient and mobility are not independent parameters.

Typical mobility and diffusion coefficient values at T=300K
(μ = cm^2/V-sec and D = cm^2/sec)

	μ_n	D_n	μ_p	D_p
Silicon	1350	35	480	12.4
GaAs	8500	220	400	10.4
Germaium	3900	101	1900	49.2