### Unit-II

>Non-equilibrium excess carriers in semiconductors >pn Junction Principles Metal-semiconductor junctions >Semiconductor-semiconductor junctions

# Non-equilibrium excess carriers in semiconductors

- Excess electrons in the conduction band and excess holes in the valence band may exist in addition to the thermalequilibrium concentrations if an external excitation is applied to the semiconductor.
- ✓ Excess electrons and excess holes do not move independently of each other.
- ✓ They diffuse, drift and recombine with the same effective diffusion coefficient, drift mobility and life time.

#### This phenomenon is called ambipolar transport.

#### **Carrier Generation & Recombination**

- Generation is the process whereby electrons and holes are created.
- Recombination is the process whereby electrons and holes are annihilated.
- In thermal equilibrium, electrons breaks out covalent bonds due to the acquisition of enough thermal energy and hop from the valence to the conduction band. This creates both a free electron and a free hole (generation).
- By the sake token there are free electrons that lose some of their thermal energy when they encounter a hole and fall back into covalent bond (recombination).

#### **Semiconductor in Equilibrium**



Thermally generated free electrons and holes must come in pairs

$$G_{no} = G_{po}$$

and they will also recombine in pairs so

 $R_{no}=R_{po}$ 

In thermal equilibrium the total number of free electrons and holes is constant so the rates at which they are being generated must be equal to the rates at which they are recombining.

$$G_{no} = G_{po} = R_{no} = R_{po}$$

#### **Excess Carrier Generation**

#### Photogeneration

Electron in VB may be excited into conduction band by incident of high energy photon on semiconductor



Fig. 5.2 Photogeneration



Fig. 5.3 (a) Experimental set-up for determining current vs frequency plot (b) Current vs frequency plot

#### **Excess Carrier Generation.....**

#### Phonon generation

- Phonon generation occurs when a semiconductor is under thermal excitation
- By increase of temperature, lattice vibration increases, which produces more phonons. Due to this covalent bonds break down and e-h pair generated



#### >Impact ionization

 One energetic charge carrier will create another charge carrier

#### Avalanche Breakdown



### **Excess Carrier Recombination**

#### **Radiative Recombination**

- It occurs in direct SC
- Electron from CB minimum falls to VB maximum without changing momentum and emitting one photon





#### Schockley Read Hall Recombination

Electron from CB minimum come to a defect level in between band gap by radiating energy by photon or phonon and turns from that defect level to VB.



### **Excess Carrier Recombination....**

#### Auger Recombination

- Three carriers are involved
- Electron and hole recombine but instead of emitting energy as photon or phonon, the energy is given to third electron in CB.
- The third excited electron comes back to the CB by emitting energy as heat.
- It occurs in heavily doped SC



### **Direct and Indirect Recombination**

- Direct recombination occurs in direct band gap SC where CB minimum and VB maximum occur for same momentum.
- Electron from CB directly falls to VB by emitting photon of energy equal to band gap of SC
- Indirect recombination occurs in indirect band gap SC
- Electron of CB has to change momentum to come upon the VB top and the recombination occurs



#### **Excess Carriers**

- External events, such as incident photons, increase in temperature etc., can disrupt the thermal equilibrium and create additional electron-hole pairs.
- These "excess" charge carriers are generated in pairs.
- The generation rates of excess electrons and excess holes, are equal i.e.



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#### Excess Carriers.....

 The number of actual excess electrons and holes generated by external event are δn and δp. Thus the total number of free electrons and holes in the semiconductor can now be given as:

$$n = n_o + \delta n$$
$$p = p_o + \delta p$$

 $n_o$  thermal equilibrium electron concentration  $P_o$  thermal equilibrium hole concentration

 In thermal equilibrium, the excess electrons and holes also recombine in pairs so recombination rates for electrons and holes must be equal to



#### **Excess Carriers.....**

- For the simple model of recombination i.e. direct band-to-band recombination, the recombination occurs spontaneously, thus the probability of an electron-hole pair recombining is constant with time.
- Moreover, the rate at which electrons recombine must be proportional to both the electron concentration in CB and hole concentration in VB.
- > The net rate of change in electron concentration is written as:

$$\frac{dn(t)}{dt} = \alpha_r \left[ n_i^2 - n(t) p(t) \right]$$

where

$$n(t) = n_o + \delta n(t)$$
$$p(t) = p_o + \delta p(t)$$

The term  $\alpha_r n_i^2$  is the thermal equilibrium generation rate.

Noted that the entire expression in the parentheses will be less than (or equal to) zero so that the derivative is negative. This indicates that the value of n(t) is decreasing due to recombination.

#### Excess Carriers.....

- o Since  $n_o$  is constant with respect to time the derivative can be taken with respect to δn instead of n.
- Also, since excess electrons and holes are generated and recombine in pairs so that  $\delta n(t) = \delta p(t)$ .
- Making these substitutions and expanding the terms out we find:

$$\frac{d(\delta n(t))}{dt} = \alpha_r \left[ n_i^2 - (n_o + \delta n(t))(p_o + \delta p(t)) \right]$$
  
$$\frac{d(\delta n(t))}{dt} = \alpha_r \left[ n_i^2 - \{n_o p_o + n_o \delta p(t) + \delta n(t) p_o + \delta n(t) \delta p(t) \} \right]$$
  
$$\frac{d(\delta n(t))}{dt} = -\alpha_r \left[ n_o \delta n(t) + \delta n(t) p_o + \delta n(t) \delta n(t) \right]$$

$$n_i = n_o p_o$$

#### **Low-Level Injection**

- The differential equation derived at this point isn't easy to solve at the moment. However, if restriction/condition is imposed on this equation i.e. "low-level injection" (a common situation) it can solve easily.
- Low-level injection simply means that the number of excess carriers is much smaller than the thermal equilibrium values of the majority carrier concentration. That is (for p-type material), δn(t) « p<sub>0</sub>.
- > For p-type material we also know that  $n_0 \ll p_0$ . Therefore, looking at that equation

$$\frac{d(\delta n(t))}{dt} = -\alpha_r [n_o \delta n(t) + \delta n(t) p_o + \delta n(t) \delta n(t)]$$

Noted that the  $\delta n(t)p_0$  term will dominate the other two terms on the right-hand side of the equation.

#### Low-Level Injection.....

We can thus approximate this equation as

$$\frac{d(\delta n(t))}{dt} = -\alpha_r [\delta n(t)p_o]$$

This is a simple first-order differential equation having a solution

$$\delta n(t) = \delta n(0) e^{-\alpha_r p_0 t} = \delta n(0) e^{-t/\tau_{n0}}$$

where  $\tau_{no}$ , the excess minority carrier lifetime, is given by  $\tau_{no} = (\alpha_r p_o)^{-1}$ .

Noted that the excess minority carrier lifetime depends on the majority carrier concentration.

The excess carrier recombination rate,  $R_n$ , is the change in the number of excess carriers,  $\delta n(t)$ , so we can write

$$R_{n}^{'} = \frac{-d(\delta n(t))}{dt} = \alpha_{r} p_{o} \delta n(t) = \frac{\delta n(t)}{\tau_{no}}$$

#### Low-Level Injection.....

- For direct band-to-band recombination, the excess majority carrier holes recombine at the same rate (if an electron has recombined, it obviously must have recombined with a hole therefore subtract BOTH one free electron and one free hole).
- Since the two rates are equal we can write, for p-type material,

$$R_{n}' = R_{p}' = \frac{\delta n(t)}{\tau_{no}}$$

A similar derivation can be done for low-level injection in n-type material to yield

$$R_{n}' = R_{p}' = \frac{\delta n(t)}{\tau_{po}}$$

#### **Continuity Equations**

- $\checkmark$  Let us consider the flux of particles into and out of a small box.
- ✓ Assume the flow of particles only occurs in 1-D along the x-axis and the box has a differential volume (dx·dy·dz).



✓ Assume particles are holes. Then using a first-order Taylor expansion we can relate the flux of particles into the box to the flux of particles out of the box as

$$F_{px}^{+}(x+dx) = F_{px}^{+}(x) + \frac{\partial F_{px}^{+}}{\partial x}dx$$

 $F^{+}_{px}$  is hole particle flux or flow

#### **Continuity Equations.....**

 The net increase in particles within the volume would be the difference of the two fluxes (multiplied by the surface area)

$$\left[F_{px}^{+}(x)-F_{px}^{+}(x+dx)\right]dydz = \left[\frac{-\partial F_{px}^{+}}{\partial x}dx\right]dydz$$

✓ But the net increase in particles inside the box could also be written



So

$$\frac{\partial p}{\partial t} dx dy dz = -\frac{\partial F_{px}^+}{\partial x} dx dy dz$$

 This only represents the buildup or decrease of particles in the box due to different flow rates.

#### **Continuity Equations.....**

✓ We also need to account for the effects of generation and recombination that may be occurring within the box. These two phenomena would also contribute to an increase or decrease of the particle concentration within the box. Including these terms we have:

$$\frac{\partial p}{\partial t}dxdydz = -\frac{\partial F_{px}^{+}}{\partial x}dxdydz + g_{p}dxdydz - \frac{p}{\tau_{pt}}dxdydz$$

p is the density of holes and  $\tau_{pt}$  is the combined hole lifetime (it includes both the thermal equilibrium carrier lifetime and the excess carrier lifetime).

 $\checkmark$  Dividing through by dx-dy-dz we reach the Continuity Equations.

$$\frac{\partial p}{\partial t} = -\frac{\partial F_p^+}{\partial x} + g_p - \frac{p}{\tau_{pt}} \qquad (Holes)$$
$$\frac{\partial n}{\partial t} = -\frac{\partial F_n^-}{\partial x} + g_n - \frac{n}{\tau_{nt}} \qquad (Electrons)$$

#### **Time-Dependent Diffusion Equations**

Current densities are given by

$$J_{p} = e\mu_{p}pE - eD_{p}\frac{\partial p}{\partial x}$$
$$J_{n} = e\mu_{n}nE + eD_{n}\frac{\partial n}{\partial x}$$

By dividing the current density by the unit of charge we obtain particle flux:

$$\frac{J_p}{e} = F_p^+ = \mu_p p E - D_p \frac{\partial p}{\partial x}$$
$$\frac{J_n}{(-e)} = F_n^- = -\mu_n n E - D_n \frac{\partial n}{\partial x}$$

#### **Time-Dependent Diffusion Equations....**

By substituting these expressions into the Continuity Equations we get:

$$\frac{\partial p}{\partial t} = -\frac{\partial F_p^+}{\partial x} + g_p - \frac{p}{\tau_{pt}} = -\mu_p \frac{\partial (pE)}{\partial x} + D_p \frac{\partial^2 p}{\partial x^2} + g_p - \frac{p}{\tau_{pt}}$$
$$\frac{\partial n}{\partial t} = -\frac{\partial F_n^-}{\partial x} + g_p - \frac{n}{\tau_{nt}} = +\mu_n \frac{\partial (nE)}{\partial x} + D_n \frac{\partial^2 n}{\partial x^2} + g_n - \frac{n}{\tau_{nt}}$$

Since both p (or n) and E can be functions of position therefore

$$\frac{\partial(pE)}{\partial x} = E\frac{\partial p}{\partial x} + p\frac{\partial E}{\partial x}$$
$$\frac{\partial(nE)}{\partial x} = E\frac{\partial n}{\partial x} + n\frac{\partial E}{\partial x}$$

#### **Time-Dependent Diffusion Equations....**

Thus

$$\frac{\partial p}{\partial t} = -\mu_p \left( E \frac{\partial p}{\partial x} + p \frac{\partial E}{\partial x} \right) + D_p \frac{\partial^2 p}{\partial x^2} + g_p - \frac{p}{\tau_{pt}}$$
$$\frac{\partial n}{\partial t} = +\mu_n \left( E \frac{\partial n}{\partial x} + n \frac{\partial E}{\partial x} \right) + D_n \frac{\partial^2 n}{\partial x^2} + g_n - \frac{n}{\tau_{pt}}$$

If we assume we have a homogeneous semiconductor (the doping concentration of electrons and holes is uniform throughout the semiconductor), then and partial derivatives of p(x) and n(x) just become partial derivatives of  $\delta p(x)$  and  $\delta n(x)$ .

$$p(x) = p_o + \delta p(x)$$
$$n(x) = n_o + \delta n(x)$$

Thus we get the time-dependent diffusion equations for electrons and holes in a homogeneous semiconductor:

$$\frac{\partial(\delta p)}{\partial t} = D_p \frac{\partial^2(\delta p)}{\partial x^2} - \mu_p \left( E \frac{\partial(\delta p)}{\partial x} + p \frac{\partial E}{\partial x} \right) + g_p - \frac{p}{\tau_{pt}}$$
$$\frac{\partial(\delta n)}{\partial t} = D_n \frac{\partial^2(\delta n)}{\partial x^2} + \mu_n \left( E \frac{\partial(\delta n)}{\partial x} + n \frac{\partial E}{\partial x} \right) + g_n - \frac{n}{\tau_{nt}}$$

#### **Ambipolar Transport**

- If the pulse of excess electrons and holes are created in semiconductor then pulse of these excess electrons and holes will trend to drift in opposite direction by applying electric filed.
- > Any separation between these charge carriers induces an internal electric filed.



- Since the internal E-field creates a force attracting the electrons and holes, this E-field will hold the pulses of excess electrons and holes together.
- The negatively charged electrons and positively charged holes then will drift or diffuse together with a single effective mobility or diffusion coefficient.
- > This phenomenon is called Ambipolar diffusion or Ambipolar transport.

Besides the time dependent diffusion equation for excess electrons and holes, a third equation is required to relates the excess electron and holes concentration to the internal electric filed.

This relation is Poisson equation as given

$$\nabla . E_{\rm int} = \frac{-e(\delta p - \delta n)}{\varepsilon_{\rm s}} = \frac{\partial E_{\rm int}}{\partial x}$$

To make approximation that a relatively small internal electric filed is sufficient to keen excess electron and holes drifting and diffusing together i.e.

$$E_{\rm int} | \prec \prec | E_{app} |$$

Only a small difference in the excess electron concentration and excess hole concentration will set up an internal electric field to keep the particles drifting and diffusing together.

This will results in non-negligible values of div  $E = div E_{int}$  term in the time dependent diffusion equation.

Since electrons and holes are generated in pairs

$$g_n = g_p \cong g$$

Similarly, they also recombine in pairs

$$R_n = \frac{n}{\tau_{nt}} = R_p = \frac{p}{\tau_{pt}} \equiv R$$

And since they are generated in pairs (assuming charge neutrality),  $\delta n = \delta p$ . Thus we can write the time-dependent diffusion equations as:

$$\frac{\partial(\delta p)}{\partial t} = \frac{\partial(\delta n)}{\partial t} = D_p \frac{\partial^2(\delta n)}{\partial x^2} - \mu_p \left( E \frac{\partial(\delta n)}{\partial x} + p \frac{\partial E}{\partial x} \right) + g - R$$
$$\frac{\partial(\delta n)}{\partial t} = D_n \frac{\partial^2(\delta n)}{\partial x^2} + \mu_n \left( E \frac{\partial(\delta n)}{\partial x} + n \frac{\partial E}{\partial x} \right) + g - R$$

Multiplying the first equation by  $\mu_n n$  and the second equation by  $\mu_p p$  we get:

$$\mu_{n}n\frac{\partial(\delta n)}{\partial t} = \mu_{n}nD_{p}\frac{\partial^{2}(\delta n)}{\partial x^{2}} - \mu_{n}\mu_{p}n\left[E\frac{\partial(\delta n)}{\partial x} + p\frac{\partial E}{\partial x}\right] + \mu_{n}n(g-R)$$
$$\mu_{p}p\frac{\partial(\delta n)}{\partial t} = \mu_{p}pD_{n}\frac{\partial^{2}(\delta n)}{\partial x^{2}} + \mu_{n}\mu_{p}p\left[E\frac{\partial(\delta n)}{\partial x} + n\frac{\partial E}{\partial x}\right] + \mu_{p}p(g-R)$$

Adding these we eliminate the 
$$\frac{\partial E}{\partial x}$$
 term  
 $\left(\mu_n n + \mu_p p\right) \frac{\partial(\delta n)}{\partial t} = \left(\mu_n n D_p + \mu_p p D_n\right) \frac{\partial^2(\delta n)}{\partial x^2} + \mu_n \mu_p (p-n) E \frac{\partial(\delta n)}{\partial x} + \left(\mu_n n + \mu_p p\right) (g-R)$ 

On simplifying this by dividing through by  $(\mu_n n + \mu_p p)$  then final we get Ambipolar Transport Equation

$$\frac{\partial(\delta n)}{\partial t} = D' \frac{\partial^2(\delta n)}{\partial x^2} + \mu' E \frac{\partial(\delta n)}{\partial x} + g - R$$
$$D' = \frac{\mu_n n D_p + \mu_p p D_n}{\mu_n n + \mu_p p} \qquad \qquad \mu' = \frac{\mu_n \mu_p (p - n)}{\mu_n n + \mu_p p}$$

Ambipolar diffusion coefficient

Ambipolar mobility coefficient

Using Einstein relation 
$$\frac{D_n}{\mu_n} = \frac{D_p}{\mu_p} = \frac{kT}{q}$$
$$D' = \frac{D_n D_p (n+p)}{D_n n + D_p p} \longrightarrow D' = \frac{D_n D_p [(n_o + \delta n) + (p_o + \delta p)]}{D_n (n_o + \delta n) + D_p (p_o + \delta p)}$$
$$\mu' = \frac{\mu_n \mu_p (p-n)}{\mu_n n + \mu_p p} \longrightarrow \mu' = \frac{\mu_n \mu_p [(n_o + \delta n) - (p_o + \delta p)]}{\mu_n (n_o + \delta n) + \mu_p (p_o + \delta p)}$$

For "strongly" p-type (or n-type) material under low level injection above equations considerably reduce to;

For p-type, low-level injection

For n-type, low-level injection

 $p_{0} \gg n_{0} \qquad n_{0} \gg p_{0}$   $p_{0} \gg \delta n \qquad n_{0} \gg \delta n$   $D' \cong D_{n} \qquad D' \cong D_{p}$   $\mu' \cong \mu_{n} \qquad \mu' \cong -\mu_{p}$ 

- Now consider the another terms in ambipolar transport equations i.e. generation and recombination rates
- Since the electron and hole recombination rates are equal i.e.

$$R_n = R_p = \frac{n}{\tau_{nt}} = \frac{p}{\tau_{pt}} = R$$

 $\tau_{nt}$  and  $\tau_{pt}$  are mean electron and hole lifetimes.

- > The inverse of  $\tau_{nt}$  i.e. 1/  $\tau_{nt}$  is probability per unit time that an electron will encounter a hole and recombine.
- > The inverse of  $\tau_{pt}$  i.e. 1/  $\tau_{pt}$  is probability per unit time that an hole will encounter an electron and recombine.
- For low-level injection (excess carriers are small) the probability per unit time of minority carriers encountering the majority carriers will be almost constant (the chance of hitting a majority carrier won't change much), so  $\tau_{nt} = \tau_n$  for p-type semiconductor and  $\tau_{pt} = \tau_p$  for n-type semiconductor.
- The probability per unit time of majority carriers encountering minority carriers may change drastically.

For generation and recombination we have a combination of thermalequilibrium generation/recombination and excess carrier generation/ recombination. Looking at just electrons we have:

$$g - R = g_n - R_n = (G_{no} + g'_n) - (R_{no} + R'_n)$$

However, for thermal equilibrium we know  $G_{no} = R_{no}$  therefore

$$g - R = g'_n - R'_n = g'_n - \frac{\delta n}{\tau_n}$$

and similarly for the holes.

$$g - R = g_{p} - R_{p} = (G_{po} + g_{p}) - (R_{po} + R_{p})$$

Again for thermal equilibrium  $G_{po} = R_{po}$  therefore

$$g - R = g'_p - R'_p = g'_p - \frac{\delta p}{\tau_p}$$

The excess carrier generation rates for electron and holes are equal i.e.

$$g'_n = g'_p = g'$$

Combining all these final Ambipolar Transport Equations are:

$$D_{n} \frac{\partial^{2}(\delta n)}{\partial x^{2}} + \mu_{n} E \frac{\partial(\delta n)}{\partial x} + g' - \frac{\delta n}{\tau_{n0}} = \frac{\partial(\delta n)}{\partial t} \qquad p-type$$
$$D_{p} \frac{\partial^{2}(\delta p)}{\partial x^{2}} - \mu_{p} E \frac{\partial(\delta p)}{\partial x} + g' - \frac{\delta p}{\tau_{p0}} = \frac{\partial(\delta p)}{\partial t} \qquad n-type$$

These equations simply say that there can be a change in excess carrier concentration over time because:

- 1. They diffuse away.
- 2. They drift away.
- 3. More are generated.
- 4. They recombine.

#### **Quasi-Fermi Energy Levels**

The thermal-equilibrium electron and hole concentrations are functions of the Fermi energy level as given below:



**Figure 6.14** | Thermal-equilibrium energy-band diagrams for (a) n-type semiconductor and (b) p-type semiconductor.

### Quasi-Fermi Energy Levels.....

- The Fermi Level is constant in a semiconductor under equilibrium. However, with generation or current flowing (even in steady-state) It is in non-equilibrium so E<sub>F</sub> is no longer a constant.
- ❑ However, we still want to know (or relate) total electron and hole concentrations. This can be done by considering a "shift" in E<sub>F</sub> that represent the "More" or "Less" p or n nature of the semiconductor. These new levels are called E<sub>Fn</sub> and E<sub>Fp</sub>, the Quasi-Fermi Levels.

$$n = n_o + \delta n = n_i e^{(E_{Fn} - E_{Fi})/kT}$$
$$p = p_o + \delta p = n_i e^{(E_{Fi} - E_{Fp})/kT}$$

If excess carriers are created in a semiconductor, we are no longer in thermal equilibrium and the Fermi energy is strictly no longer defined. If δn and δp are excess electron and hole concentrations respectively, we write

$$n_o + \delta n = n_i \exp\left[\frac{E_{Fn} - E_{Fi}}{kT}\right] \qquad p_o + \delta p = n_i \exp\left[\frac{-(E_{Fp} - E_{Fi})}{kT}\right]$$

#### **Shockley-Read-Hall Theory of Recombination**

- Allowed energy state, i.e. trap, with in the forbidden band gap may act as a recombination center capturing both electron and hole with almost equal probability.
- A single recombination center or trap exists at energy E<sub>t</sub> with in the band gap.
- > There are four basic process that may occur at this single traps.
- > There are two types of traps; acceptor and donor types.

Consider acceptor type trap;

- It is negatively charged when it contains an electron and
- neutral when it does not contain an electron

#### **Shockley-Read-Hall Theory of Recombination....**

#### Process-1

The capture of electron from the CB by an initial neutral empty trap.

#### Process-2

The inverse of process-1; the emission of an electron that is initially occupying a trap level back into the CB.

#### **Process-3**

The capture of a hole from VB by a trap containing an electron.

#### OR

Emission of an electron from the trap into the VB.

#### **Process-4**

The inverse of process-3; the emission of a hole from neutral trap into VB.

#### OR

Capture of an electron from VB.



#### In process-4, recombination rate is $R=\delta n/\tau$

#### **Surface State**

- The perfect periodic nature ends abruptly at the surface due to abrupt termination of semiconductor.
- > The disruption of this results in allowed energy states with in band gap.
- The abrupt termination of periodic potential at surface results in a distribution of allowed energy states within band gap



#### Surface State.....

- ✓ According to Shockley-Read-Hall recombination theory, Excess minority carrier lifetime is inversely proportional to density of trap states.
- $\checkmark\,$  The density of traps at surface is larger than in the bulk
- ✓ The excess minority carrier lifetime at the surface will be smaller than the corresponding lifetime in the bulk material

The recombination rate of excess carriers in bulk for n-type semiconductor is

$$R = \frac{\delta p}{\tau_{po}} = \frac{\delta p_B}{\tau_{po}}$$

 $\delta p_B$  is concentration of excess minority carrier holes

Similarly the recombination rate of excess carriers at surface is

$$R = \frac{\delta p}{\tau_{po}} = \frac{\delta p_s}{\tau_{pos}}$$

 $\delta p_s$  is concentration of excess minority carrier holes at surface and  $\tau_{pos}$  excess minority carrier hole lifetime at surface.



semiconductor surface

#### **Surface Recombination Velocity**

- A gradient in the excess carrier concentration exists near the surface, then excess carriers from the bulk region diffuse towards the surface where they recombine.
- The diffusion towards the surface is given as

$$-D_{p}\left[\widehat{n}.\frac{d(\delta p)}{dx}\right]_{surf} = s\,\delta p_{surf}$$

n is unit outward vector normal to the surface."s" is called surface recombination velocity.



- As the excess concentration at the surface is small, the gradient term becomes larger and the surface recombination velocity increases.
- If excess concentration at the surface and in the bulk were equal then gradient would be zero and surface recombination velocity would be zero.

We have that  $\dot{n} = -1$  then

$$D_p \left[ \frac{d(\delta p)}{dx} \right]_{surf} = s \, \delta p_{surf}$$

An infinite surface recombination velocity implies that the excess minority carrier concentration and lifetime at the surface are zero.

### Unit-II

Non-equilibrium excess carriers in semiconductors

### >pn Junction Principles

- Metal-semiconductor junctions
- Semiconductor-semiconductor

junctions

#### **pn Junction Principles**

A. Open Circuit

- Consider what happens when one side of a semiconductor such as Si is doped n-type and the other is doped p-type.
- The abrupt discontinuity between the two sides is called a metallurgical junction, M
- The M region contains a depletion region of carriers of width  $W = W_p + W_n$ where  $W_n$  is the space charge region of the n-doped Si and vice versa.
- The net space charge density  $\rho_{net}$  is equal to  $-eN_a$  in the SCL (from  $x = -W_p$ to 0) and  $+eN_d$  in the SCL (from x = 0 to  $W_n$ )



### A. Open Circuit.....

- The internal electric filed E<sub>o</sub> from position ions to negative ions tries to drift hole and electron back into p and n regions respectively.
- The rates of hole and electron diffusing towards right and left respectively are just balanced by hole and electron drifting towards p and n region respectively.
- An electric field generated in the M region is minimized the free energy at the boundary and satisfy the mass action law n<sub>i</sub><sup>2</sup> = pn under equilibrium condition.
- The result is a bias voltage generated across the junction.



### A. Open Circuit.....

#### **Equations for pn Junction**

- ✓ Conservation of charge  $N_a W_p = N_d W_n$
- ✓ The potential established across the boundary on the n-side is derived by integrating the Electric field established by the change in charge density across the boundary.

$$\rho = \begin{cases} -qN_A & \text{for } -x_p < x < 0\\ qN_D & \text{for } 0 < x < x_n \end{cases} \text{ Charge Density} \\ F = \begin{cases} -\frac{qN_A}{\varepsilon_S} \cdot (x_p + x) & \text{for } -x_p < x < 0\\ -\frac{qN_D}{\varepsilon_S} \cdot (x_n - x) & \text{for } 0 < x < x_n \end{cases} \text{ Force, } F = qE = q \int \frac{\rho}{\varepsilon} dx \\ \varphi = \begin{cases} -\varphi(-x_p) + \frac{qN_A(x + x_p)^2}{2\varepsilon_S} & \text{for } -x_p < x < 0\\ \varphi(x_n) - \frac{qN_D(x - x_n)^2}{2\varepsilon_S} & \text{for } 0 < x < x_n \end{aligned}$$



- ➢ If  $N_d < N_a$  then  $W_n > W_n$ i.e. the SCL penetrates the lightly doped n-side more than heavily doped p-side.
- If the p-side is heavily doped (p+) then SCL is almost entirely on the nside.
- The relation between electric filed and charge density

$$\frac{dE}{dx} = \frac{\rho_{net}}{\varepsilon_s}$$

### **Equations for pn Junctions....**

✓ The Maximum value of the electric field and built in potential generated at the edge of the n- side of the M region are

$$E_o = -\frac{eN_dW_n}{\varepsilon} = -\frac{eN_aW_p}{\varepsilon} \qquad \qquad V_o = -\frac{1}{2}E_oW_o = \frac{eN_dN_dW_o^2}{2\varepsilon(N_a + N_d)}$$

✓ One can relate  $V_0$  to doping parameters using the ratios of the carriers  $n_2$  and  $n_1$ 

$$\frac{n_2}{n_1} = \exp\left[\frac{-\left(E_2 - E_1\right)}{k_B T}\right] \rightarrow \frac{n_{po}}{n_{no}} = \exp\left[\frac{-eV_o}{k_B T}\right] \text{ and } \frac{p_{no}}{p_{po}} = \exp\left[\frac{-eV_o}{k_B T}\right]$$

$$\Rightarrow \qquad \frac{N_d}{n_i} = \exp\left[\frac{eV_o}{k_B T}\right] \qquad \frac{N_a}{n_i} = \exp\left[\frac{eV_o}{k_B T}\right]$$

$$\Rightarrow \quad V_o = \frac{k_B T}{e} \ln \left( \frac{N_a N_d}{n_i^2} \right)$$

- Use of Boltzmann statistics.
- E is potential energy = qV.
- q is charge, for electron (q=-e)
- Considering E=0 on p-side and n=n<sub>po</sub> and E=-eV<sub>o</sub> and on n-side n= n<sub>no</sub>

### **B. Forward Bias**

- When a battery with voltage V is connected across the pn junction. The negative polarity of supply voltage reduces V<sub>o</sub> by V.
- The applied voltage drops mostly across the depletion layer and V directly opposes  $V_0$ .
- $V_{o}$  against diffusion reduces to ( $V_{o}$ -V) and consequently probability of hole diffusion from p-side to n-side is proportional to exp  $[-(V_0-V)/k_BT]$ .
- Many hole can now diffuse across the diffusion layer and enter the n-side resulting in the injection excess minority carriers, holes into n-region



Hole injected into n-region draw some electron form the bulk of n-region (and hence from the battery) to maintain charge neutrality in the n-side resulting in a small increase in electron concentration. 43

#### **B. Forward Bias....**

The hole concentration p<sub>n</sub> (0) = p<sub>n</sub>(x'=0), just outside the depletion region at x'=0 (at W<sub>n</sub>) is due to the excess hole diffusing because of reduction in V<sub>o</sub>.as given below

$$p_n(0) = p_{po} \exp\left[\frac{-e(V_o - V)}{k_B T}\right] = p_{no} \exp\left[\frac{eV}{k_B T}\right] = \frac{n_i^2}{N_d} \exp\left[\frac{eV}{k_B T}\right]$$

 Similarly electron concentration n<sub>n</sub> (0), just outside the depletion region at W<sub>p</sub> is due to the excess hole diffusing because of reduction in V<sub>o</sub>.as given below

$$n_p(0) = n_{no} \exp\left[\frac{-e(V-V)}{k_B T}\right] = n_{po} \exp\left[\frac{eV}{k_B T}\right] = \frac{n_i^2}{N_a} \exp\left[\frac{eV}{k_B T}\right]$$

- The resultant equations for carrier concentrations are referred to as the Law of the Junction
- Injected holes diffuse in the n-region and recombine with electron in this region. Those electron lost by recombination are readily replenished by negative terminal of battery.
- The current due to hole diffusing scan be sustained since more holes can be supplied by p region (positive terminal of battery).
- Similarly injected electron diffuse in the p-region and recombine with holes in this region.

#### B. Forward Bias..... Hole Diffusion

- If the length of both neutral region are longer than the minority carrier diffusion length, the hole concentration profile on n-side falls exponentially towards the thermal equilibrium value p<sub>no</sub>.
- Under an applied forward bias, excess minority carrier concentrations and a hole diffusion length (L) are related as

$$\Delta p_n(x') = p_n(x') - p_{no} = \Delta p_n(0) \exp\left(\frac{-x'}{L_h}\right) \qquad L_h = \sqrt{(D_h \tau_h)}$$

where  $D_h$  is the diffusion coefficient of holes in the lattice and  $\tau_h$  is the hole recombination lifetime

Current density due to carrier diffusion i.e. holes and electron diffusion is

$$J_{d,hole} = -eD_{h} \frac{d\Delta p_{n}(x')}{dx} = \frac{-eD_{h}}{L_{h}} \Delta p_{n}(0) \exp\left[\left(\frac{-x'}{L_{h}}\right)\right] = \frac{eD_{h}n_{i}^{2}}{L_{h}N_{d}} \left[\exp\left(\left(\frac{eV}{k_{B}T}\right)\right) - 1\right]$$
$$J_{d,elec} = \frac{eD_{e}n_{i}^{2}}{L_{e}N_{a}} \left[\exp\left(\left(\frac{eV}{k_{B}T}\right)\right) - 1\right]$$

### **B. Forward Bias....**

Total current density neglecting the recombination in the SCL is



- This is Shockley equation
- It represents the diffusion of minority carriers in neutral region
- J<sub>so</sub> is known as reverse saturation current density since applying the a reverse bias V=-V<sub>r</sub> greater than thermal voltage (kT/e) it becomes J =-J<sub>so</sub>
- The decrease in the minority carrier diffusion current with x' is made up by the increase in the current due to the drift of majority carriers since the field in the neutral region is not totally zero but a small value, just sufficient to drift the huge majority carriers.
- The current density through out the junction remains constant.

#### B. Forward Bias..... Recombination Times

 Consider under applied forward bias minority carriers diffusing and recombine in the SCL, the external current must also supply the carriers lost in the recombination process
 Log (carrier concentration)





Forward biased *pn* junction and the injection of carriers and their recombination in the SCL.

Where  $\eta$  is the diode ideality factor and is valued between 1 and 2



Reverse *I-V* characteristics of a *pn* junction (the positive and negative current axes have different scales)

Where  $\eta$  is the diode ideality factor and is valued between 1 and 2

#### **C. Reverse Bias**

- The current in the reverse bias is very small.
- The applied voltage mainly drops across the resistive depletion region, which becomes wider since negative terminal will cause holes in p-side to move away from SCL and similarly electrons n-side move away from SCL.
- > The applied voltage increases  $V_o$  by  $V_r$  i.e.  $V = V_o + V_r$
- The electric field in SCL is larger than E<sub>o</sub>, which extracts and sweeps the small no. of holes on n-side near SCL across p-side. This small current can be maintained by diffusion of holes form n-side bulk to SCL boundary.

From the law of junction

$$p_n(0) = p_{no} \exp\left[\frac{-e(V_o + V)}{k_B T}\right]$$

The hole concentration  $p_n$  (0) just outside the depletion region is zero.

There is a small concentration gradient and hence a small hole diffusion current towards SCL.

Similarly a small electron diffusion current with in SCL



#### C. Reverse Bias.....

 The reverse current density with negative voltage leads to the diode current density of –J<sub>so</sub> called reverse saturation current density

$$\mathbf{J}_{\text{rev}} = -\mathbf{J}_{\text{so}} = \frac{eD_h n_i^2}{L_h N_d} + \frac{eD_e n_i^2}{L_e N_a}$$

 $J_{so}$  depends on materials i.e.  $\mu_n$ ,  $\mu_p$ ,  $n_i$ ) and dopand concentration.

Hence n<sub>i</sub> is strongly temperature dependent.

## Thermal generation of minority carriers in the neutral region within a diffusion length to SCL, the diffusion of these carriers to SCL and their subsequent drift through SCL is cause the reverse current.

- Thermal generation of electron-hole pairs (EHPs) in depletion region can also contribute to the observed reverse current.
- This drift will result in an external current in addition to the reverse saturation current due to diffusion of minority carriers.

Let the mean time to generate the EHPs by thermal vibration of lattice,  $\tau_{\rm g}$  (i.e. mean thermal generation time)

- ✓ The rate of thermal generation of EHPs per unit volume is  $n_i / \tau_g$
- ✓ The rate of thermal generation within depletion region is WAn<sub>i</sub>/ $\tau_g$
- $\checkmark$  The current density due to thermal generation of EHPs

Total reverse current density

$$J_{rev} = \frac{eD_h n_i^2}{L_h N_d} + \frac{eD_e n_i^2}{L_e N_a} + \frac{eWn_i}{\tau_g} \quad \text{where} \quad n_i \propto \exp\left[\frac{-E_g}{2k_B T}\right]$$

 $J_{gen} = \frac{eWn_i}{\tau_{\sigma}}$ 

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#### **C. Reverse Bias.**

#### **Reverse current in Ge pn junction**

A pn junction, which emits a photon and is referred to as a photodiode

Reverse diode current (A) at V = -5 V



Reverse diode current in a Ge pn junction as a function of temperature in a  $\ln(I_{rev})$  vs. 1/T plot. Above 238 K,  $I_{rev}$  is controlled by  $n_i^2$  and below 238 K it is controlled by  $n_i$ . The vertical axis is a logarithmic scale with actual current values.

$$J_{gen} = \left(\frac{eD_h}{L_h N_d} + \frac{eD_e}{L_e N_a}\right)n_i^2 + \frac{eW}{\tau_g}n_i$$
  
I/Temperature (1/K)  
Bias driven Driven by thermal generation

#### **D. Depletion Layer Capacitance**

The depletion region of a pn junction has positive and negative charge separated by a distance W similar to a parallel plate capacitor.



- ➢ In pn junction Q does not depend linearly on V as depend to parallel plate capacitor.
- If V across the junction changes by dV to V+dV, then W also changes accordingly resulting Q+dQ charge in depletion region.
- $\succ$  The depletion layer capacitance  $C_{dep}$  is

$$C_{dep} = \left| \frac{dQ}{dV} \right|$$

➢ If applied voltage is V the voltage across the depletion layer W is V₀-V

#### **D. Depletion Layer Capacitance...**

Width of the Depletion layer

$$W = \sqrt{\frac{2\varepsilon (N_d + N_a)(V_o - V)}{eN_d N_a}}$$

Depletion Layer Capacitance

$$C_{dep} = \begin{vmatrix} dQ \\ dV \end{vmatrix} = \frac{\varepsilon A}{W} = A \sqrt{\frac{\varepsilon e N_d N_a}{2(N_d + N_a)(V_o - V)}}$$

$$C_j = A \sqrt{\frac{\varepsilon e N_d N_a}{2(N_d + N_a)(V_o)}} \quad \text{Junction capacitance}$$

$$C_{dep} = \frac{C_j}{(1 - V/V_o)^m}; m \approx 1/2$$

Voltage (V)

#### E. Recombination Lifetime

Consider recombination in direct band gap S.C. like GaAs (p type). Let  $\Delta n_p$  and  $\Delta P_p$ are excess electrons and holes and they would be  $\Delta n_p = \Delta P_p$  then

- Instantaneous minority carrier concentration •
- Instantaneous majority carrier concentration  $p_p = p_{po} + \Delta n_p$ ۲
- Thermal generation rate, G<sub>thermal</sub> ۲
- Net rate of change of holes in the • semiconductor
- Where B is the direct recombination capture coefficient ۰
- In equilibrium  $\partial \Delta n_p / \partial t = 0$ , using  $n_p = n_{po}$  and  $p_p = p_{po}$ •
- The generation is  $G_{thermal} = Bn_{po}p_{po}$ ۲
- Excess minority carrier recombination lifetime,  $\tau_e$  is • defined by
- Weak injections  $\Delta n_p \ll p_{po}$ •

ak injections
$$\Delta n_p << p_{po}$$
 $\frac{\partial \Delta n_p}{\partial t} = -BN_a \Delta n_p$  $n_p \approx \Delta n_p$  $\frac{\partial \Delta n_p}{\partial t} = -BN_a \Delta n_p$  $p_p \approx p_{po} \approx N_a$  $\tau_e = 1/BN_a$ 

Strong injections  $\Delta n_p >> p_{po}$ ٠

$$\frac{\partial \Delta n_p}{\partial t} = B \Delta n_p \Delta p_p = B \left( \Delta n_p \right)^2$$

LEDs modulated under high carrier injection have variable minority carrier concentrations which • lead to distortion of the modulated light output

 $n_p = n_{po} + \Delta n_p$ 

$$\frac{\partial \Delta n_p}{\partial t} = -Bn_p p_p + G_{thermal}$$

 $\frac{\partial \Delta n_p}{\partial \mu} = -B\left(n_p p_p + n_{po} p_{po}\right)$ 

$$\frac{\partial \Delta n_p}{\partial t} = -\frac{\Delta n_p}{\tau_e}$$

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#### **Pn Junction Band Diagram**



Energy band diagrams for a *pn* junction under (a) open circuit, (b) forward bias and (c) reverse bias conditions. (d) Thermal generation of electron hole pairs in the depletion region results in a small reverse current.

#### Quasi-Fermi Energy Levels

<u>Question</u>: Consider an n-type semiconductor at T=300K with carrier concentration of  $n_0=10^{15}$  cm<sup>-3</sup>, ni=10<sup>10</sup> cm<sup>-3</sup>, and  $p_0=10^5$  cm<sup>-3</sup>. In nonequilibrium, assume that the excess carrier concentrations are  $\delta n=\delta p=10^{13}$  cm<sup>-3</sup>

The Fermi level for thermal equilibrium can be determined

$$E_F - E_{Fi} = kT \ln\left(\frac{n_0}{n_i}\right) = 0.2982 \text{ eV}$$

In nonequilibrium, quasi Fermi level for electrons and holes becomes

$$E_{Fn} - E_{Fi} = kT \ln\left(\frac{n_o + \delta n}{n_i}\right) = 0.2984 \text{ eV}$$
$$E_{Fp} - E_{Fi} = kT \ln\left(\frac{p_o + \delta p}{n_i}\right) = 0.179 \text{ eV}$$