

# Gravity Surveying

# Introduction

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Gravity surveying...

Investigation on the basis of relative variations in the Earth's gravitational field arising from **difference of density** between subsurface rocks

# Application

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- Exploration of fossil fuels (oil, gas, coal)
- Exploration of bulk mineral deposit (mineral, sand, gravel)
- Exploration of underground water supplies
- Engineering/construction site investigation
- Cavity detection
- Glaciology
- Regional and global tectonics
- Geology, volcanology
- Shape of the Earth, isostasy
- Army

# Structure of the lecture

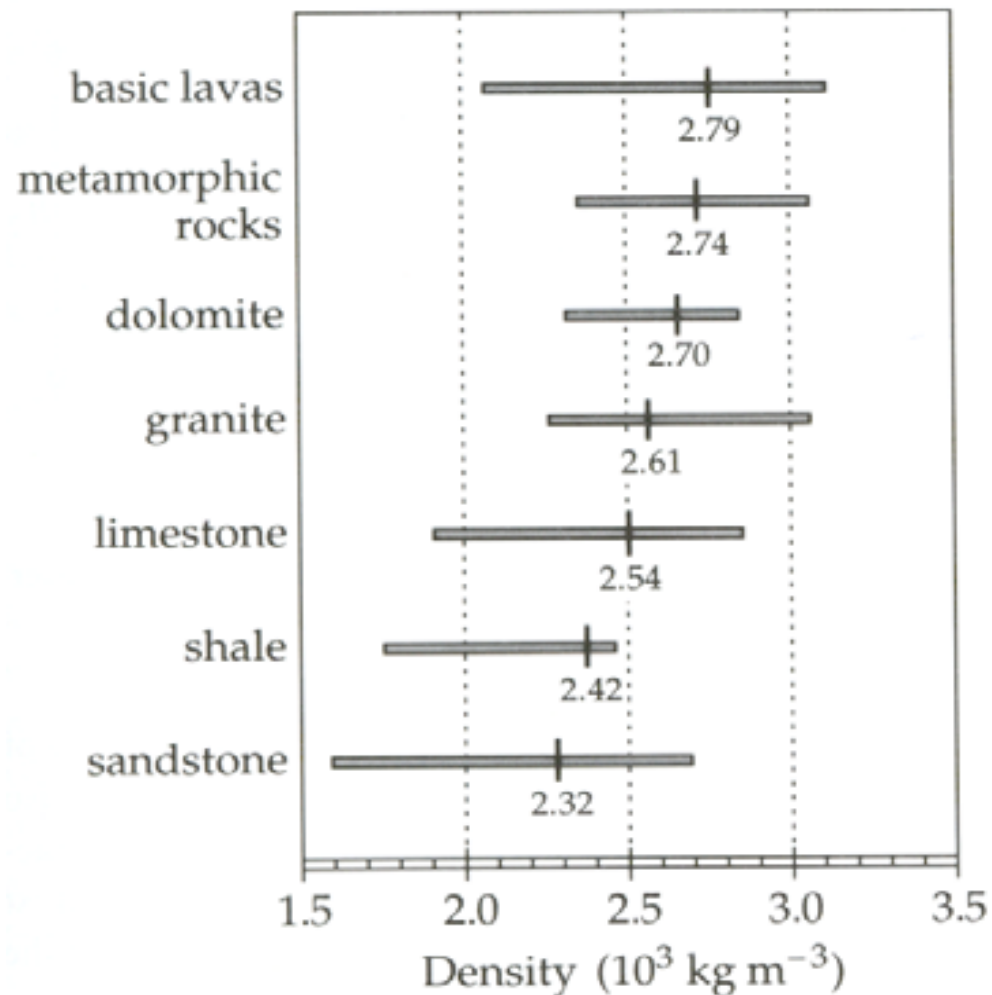
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1. Density of rocks
2. Equations in gravity surveying
3. Gravity of the Earth
4. Measurement of gravity and interpretation
5. Microgravity: a case history
6. Conclusions

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# 1. Density of rocks

# Rock density



Rock density depends mainly on...

- Mineral composition
- Porosity (compaction, cementation)

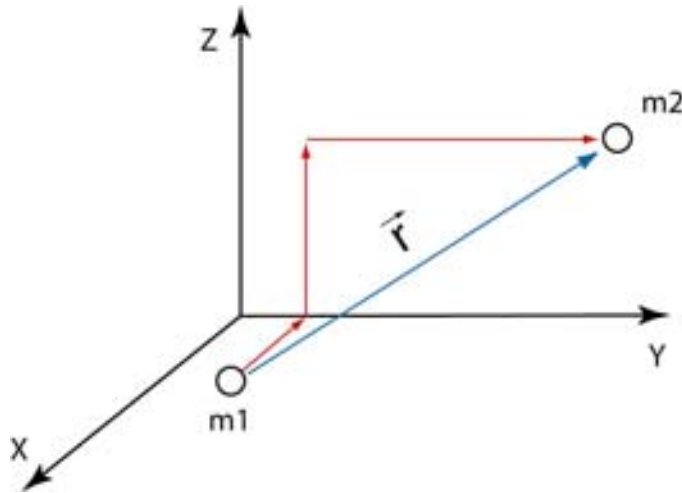
Lab or field determination of density is useful for anomaly interpretation and data reduction

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## 2. Equations in gravity surveying

# First Newton's Law

Newton's Law of Gravitation



$$\vec{F} = -\frac{G m_1 m_2}{r^2} \vec{r}$$

$$|r| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

Gravitational constant  $G = 6.67 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$



# Second Newton's Law

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$$\vec{F} = m \vec{a}$$

$$\vec{a} = -\frac{G M}{R^2} \vec{r} = \vec{g}_N$$

$$g_N \cong 9.81 \text{ m/s}^2$$

$g_N$ : gravitational acceleration or „gravity“

for a spherical, non-rotating, homogeneous Earth,  $g_N$  is everywhere the same

$$M = 5.977 \times 10^{24} \text{ kg}$$

mass of a homogeneous Earth

$$R = 6371 \text{ km}$$

mean radius of Earth

# Units of gravity

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- $1 \text{ gal} = 10^{-2} \text{ m/s}^2$
- $1 \text{ mgal} = 10^{-3} \text{ gal} = 10^{-5} \text{ m/s}^2$
- $1 \text{ } \mu\text{gal} = 10^{-6} \text{ gal} = 10^{-8} \text{ m/s}^2$  (precision of a gravimeter for geotechnical surveys)
- Gravity Unit:  $10 \text{ gu} = 1 \text{ mgal}$
- Mean gravity around the Earth:  $9.81 \text{ m/s}^2$  or  $981000 \text{ mgal}$

# Keep in mind...

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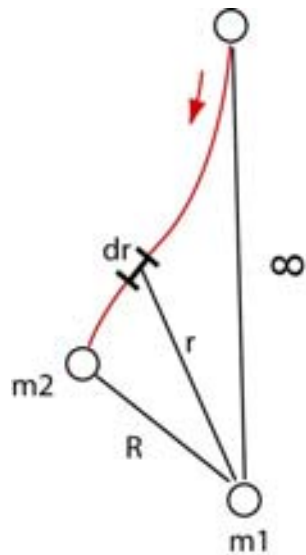
...that in environmental geophysics,  
we are working with values about...

$$0.01\text{-}0.001 \text{ mgal} \approx 10^{-8} - 10^{-9} g_N !!!$$

# Gravitational potential field

The gravitational potential field is conservative (i.e. the work to move a mass in this field is independent of the way)

The first derivative of  $U$  in a direction gives the component of gravity in that direction



$$\nabla U = \vec{g} = \frac{\vec{F}}{m_2} \quad \text{with} \quad \nabla U = \frac{\partial U}{\partial x} \vec{i} + \frac{\partial U}{\partial y} \vec{j} + \frac{\partial U}{\partial z} \vec{k}$$

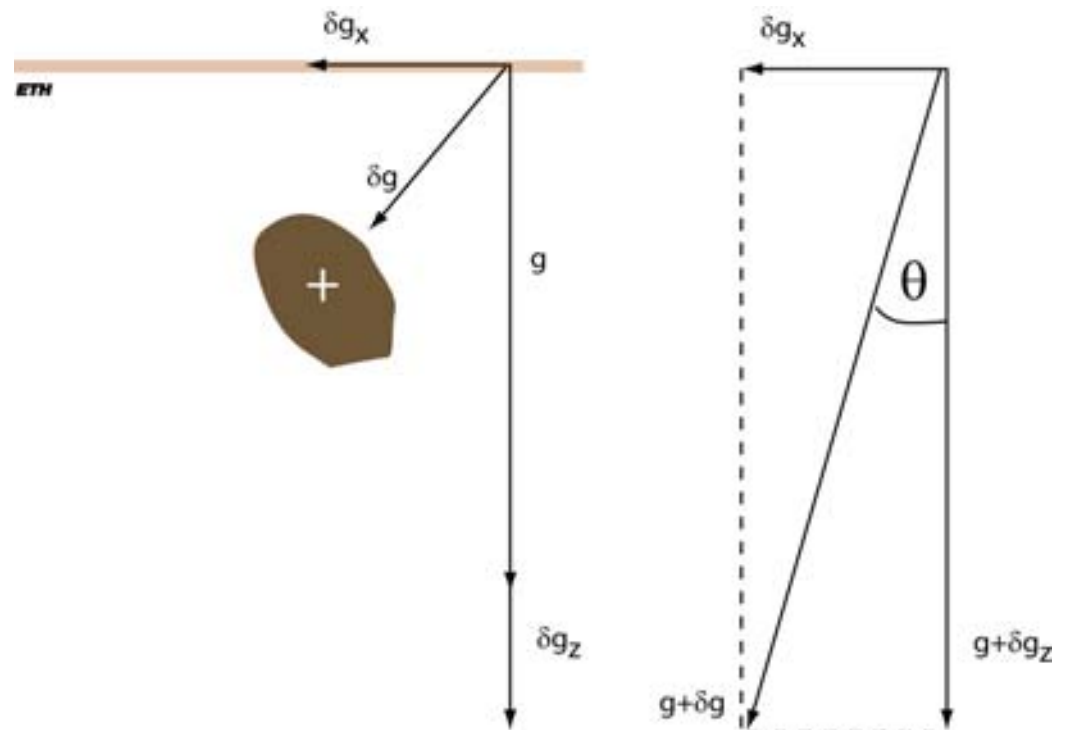
$$U = \int_{\infty}^R \vec{g} \cdot \vec{r} \, dr = -G m_1 \int_{\infty}^R \frac{dr}{r^2} = \frac{G m_1}{R}$$

# Measurement component

The measured perturbations in gravity effectively correspond to the **vertical component** of the attraction of the causative body

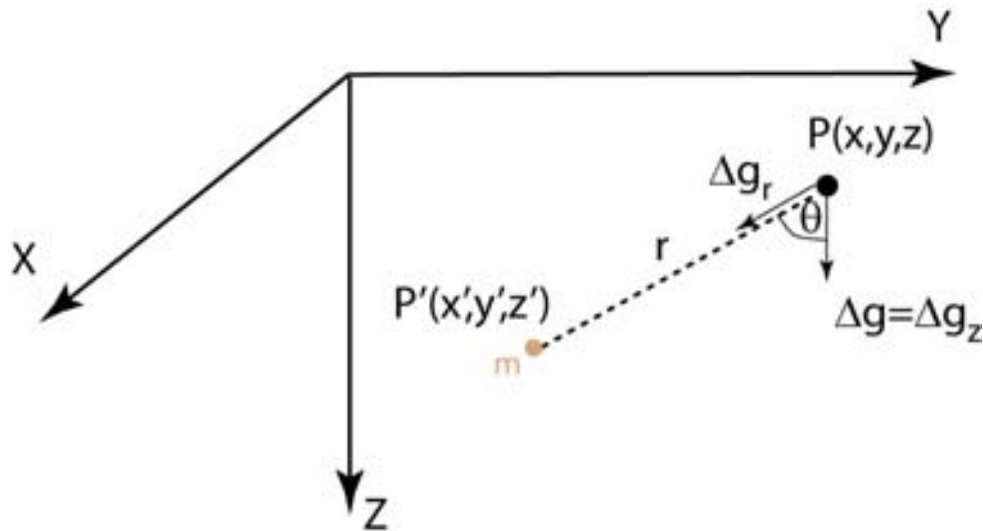
we can show that  $\theta$  is usually insignificant since  $\delta g_z \ll g$   
Therefore...

$$\delta g \approx \delta g_z$$



# Grav. anomaly: point mass

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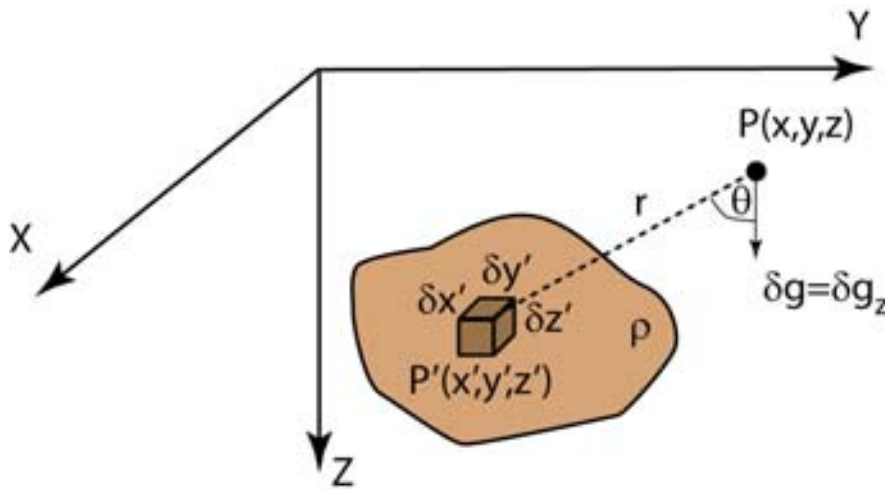


$$\Delta g_r = \frac{Gm}{r^2} \quad \text{from Newton's Law}$$

$$\Delta g = \Delta g_z = \frac{Gm}{r^2} \cos \theta = \frac{Gm(z' - z)}{r^3}$$

# Grav. anomaly: irregular shape

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$$\Delta g = \frac{Gm(z' - z)}{r^3}$$

for  $\delta m = \rho \delta x' \delta y' \delta z'$  we derive:

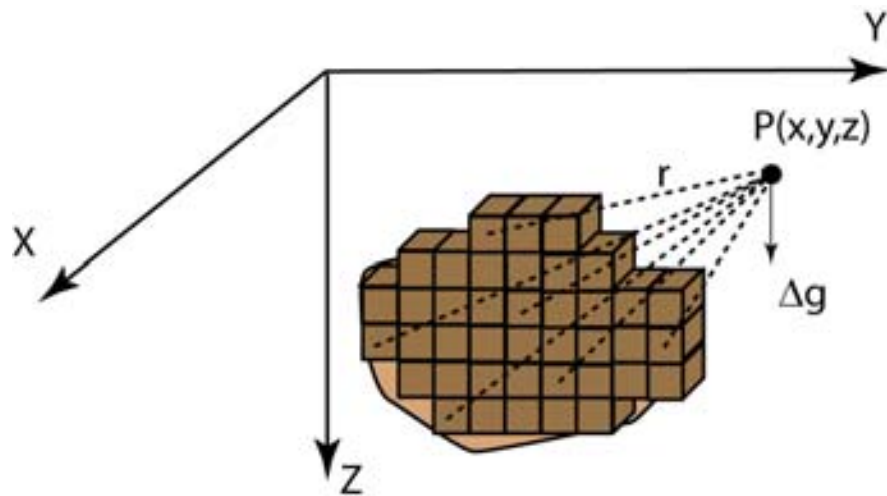
$$\delta g = \frac{G\rho(z' - z)}{r^3} \delta x' \delta y' \delta z'$$

with  $\rho$  the density (g/cm<sup>3</sup>)

$$r = \sqrt{(x' - x)^2 + (y' - y)^2 + (z' - z)^2}$$

# Grav. anomaly: irregular shape

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for the whole body:

$$\Delta g = \sum \sum \sum \frac{G\rho(z' - z)}{r^3} \delta x' \delta y' \delta z'$$

if  $\delta x'$ ,  $\delta y'$  and  $\delta z'$  approach zero:

$$\Delta g = \iiint \frac{G\rho(z' - z)}{r^3} dx' dy' dz'$$

Conclusion: the gravitational anomaly can be efficiently computed! The direct problem in gravity is straightforward:  $\Delta g$  is found by summing the effects of all elements which make up the body



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### 3. Gravity of the Earth

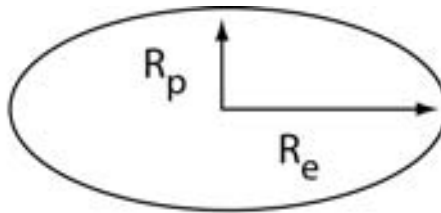
# Shape of the Earth: spheroid

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- Spherical Earth with  $R=6371$  km is an approximation!
- Rotation creates an ellipsoid or a spheroid



sphere



spheroid

$$\frac{R_e - R_p}{R_e} = \frac{1}{298.247}$$

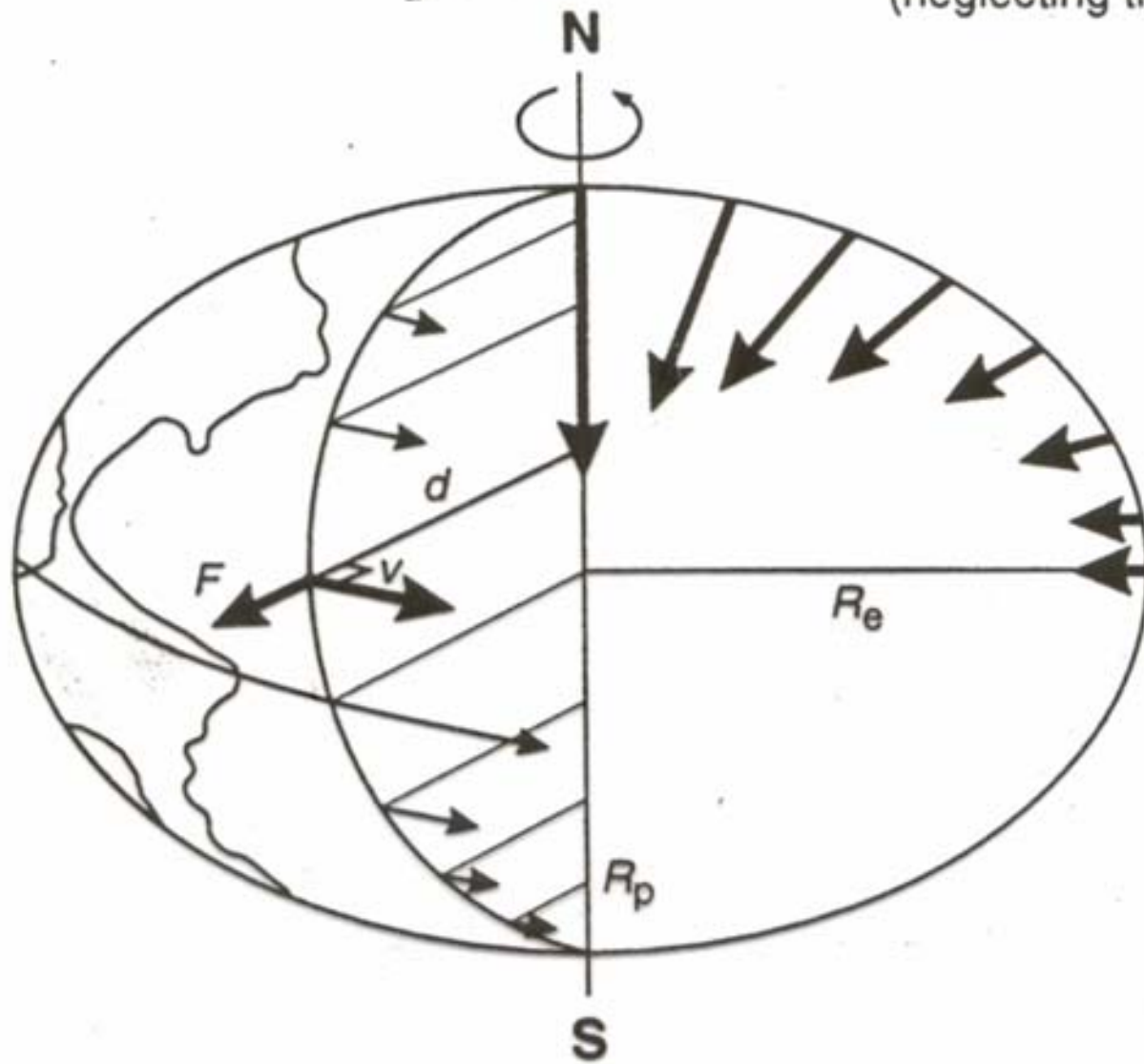
Deviation from a spherical model:

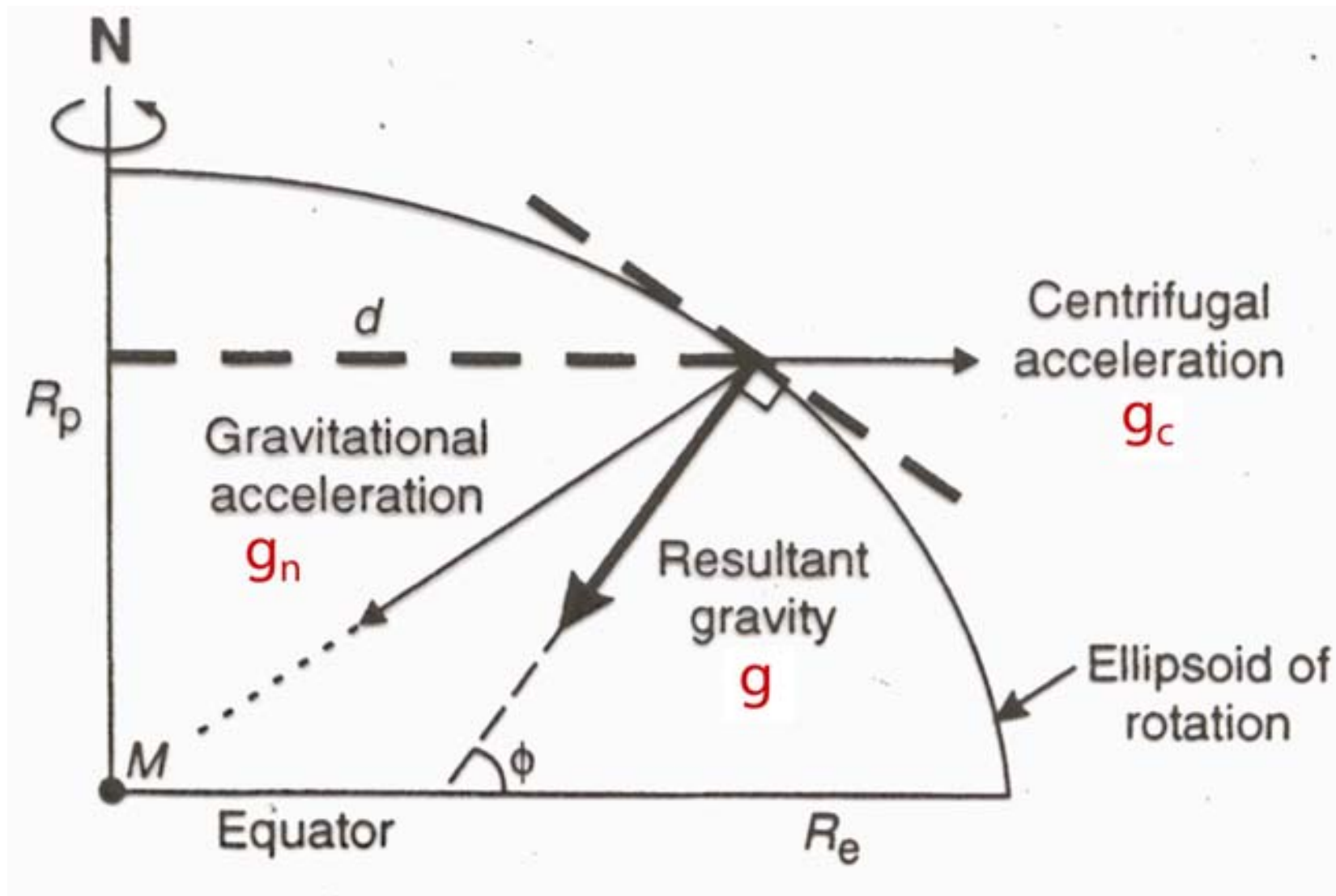
$$R_e - R = 7.2 \text{ km}$$

$$R - R_p = 14.3 \text{ km}$$

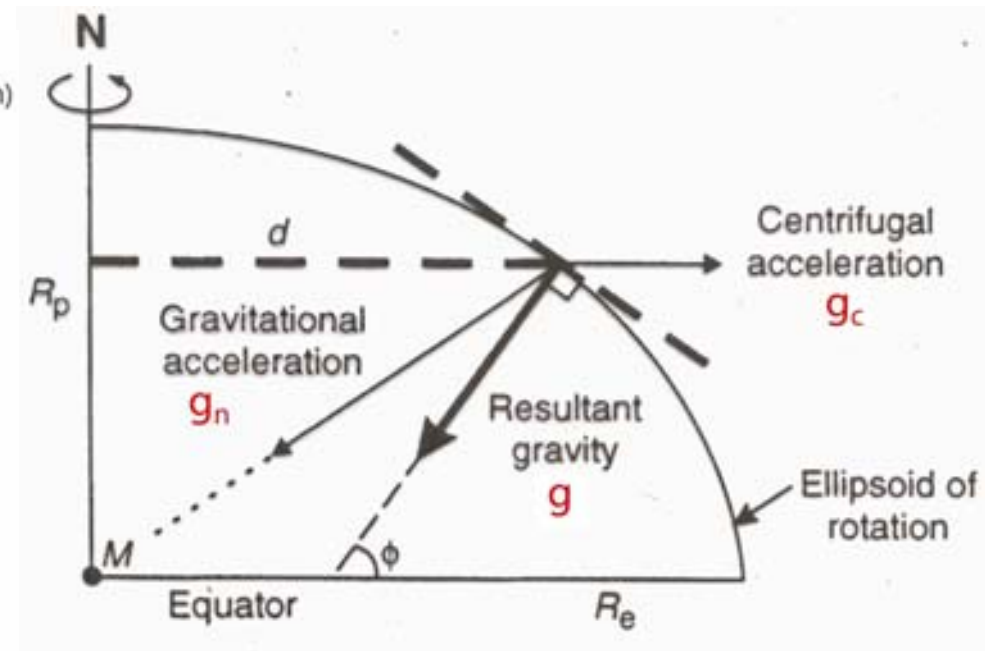
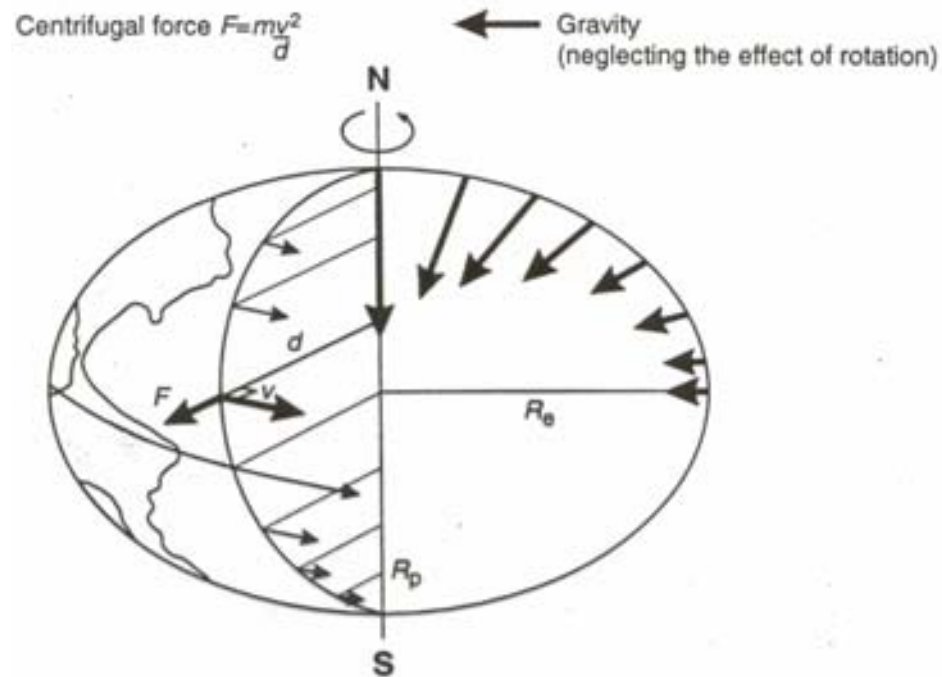
Centrifugal force  $F = m\frac{v^2}{d}$

← Gravity  
(neglecting the effect of rotation)





The Earth's ellipsoidal shape, rotation, irregular surface relief and internal mass distribution cause gravity to vary over its surface



$$g = g_n + g_c = G \left( \frac{M}{R^2} - \omega^2 R \cos \phi \right)$$

- From the equator to the pole:  $g_n$  increases,  $g_c$  decreases
- Total amplitude in the value of  $g$ : 5.2 gal

# Reference spheroid

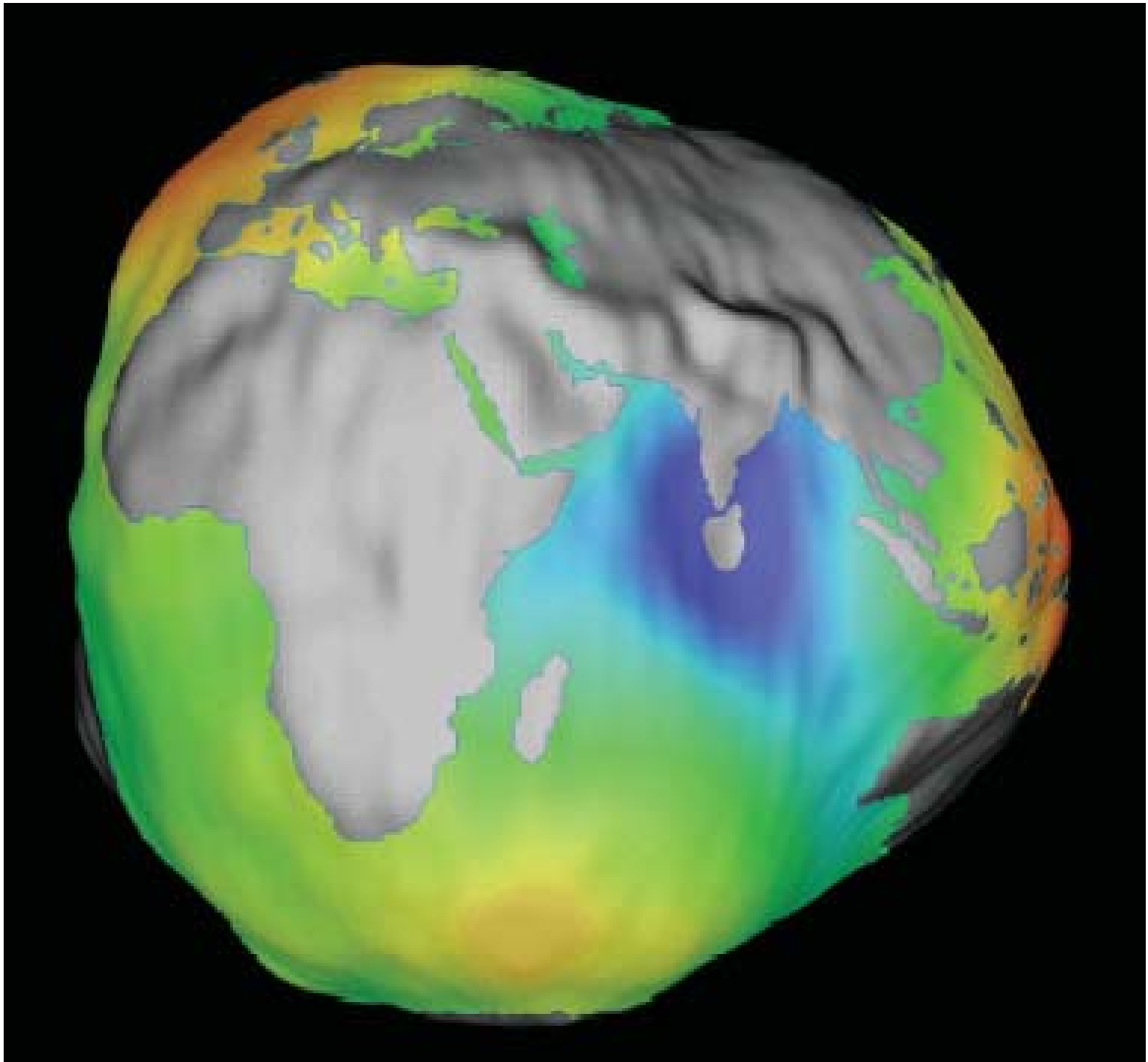
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- The reference spheroid is an oblate ellipsoid that approximates the mean sea-level surface (geoid) with the land above removed
- The reference spheroid is defined in the Gravity Formula 1967 and is **the model used in gravimetry**
- Because of lateral density variations, the geoid and reference spheroid do not coincide

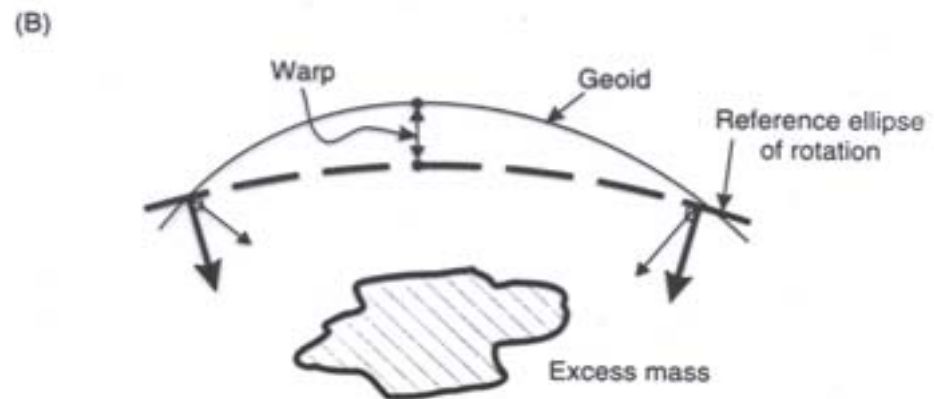
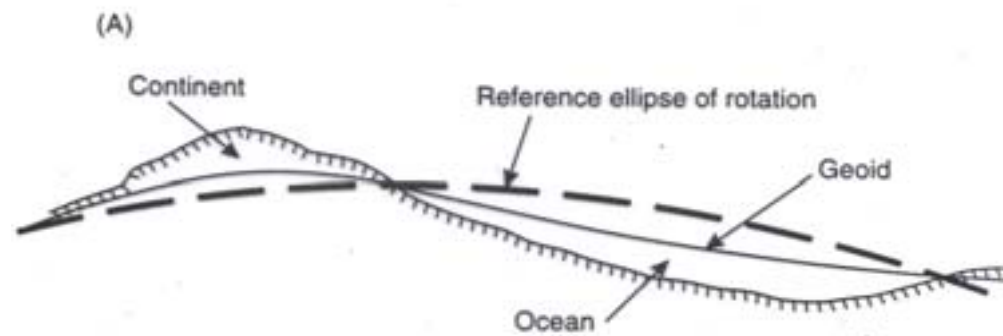
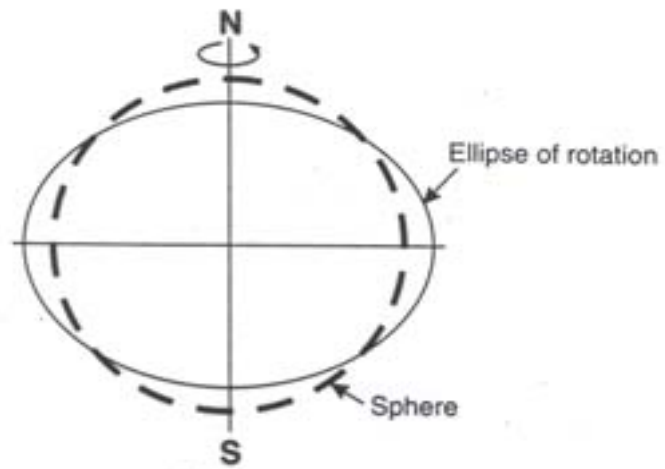
# Shape of the Earth: geoid

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- It is the sea level surface (equipotential surface  $U=\text{constant}$ )
- The geoid is everywhere perpendicular to the plumb line







# Spheroid versus geoid

Geoid and spheroid usually do not coincide (India -105m, New Guinea +73 m)

