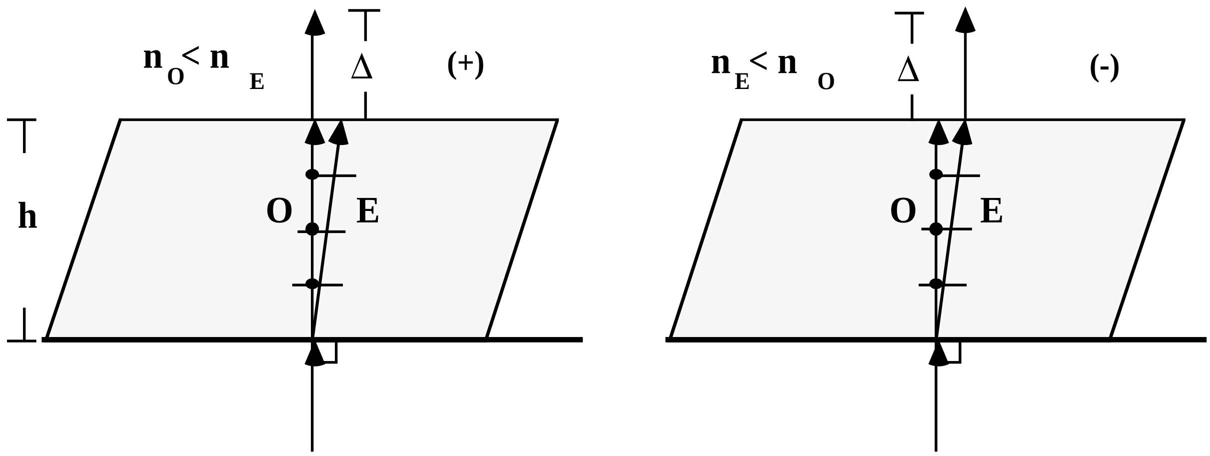
Lecture Notes - Optics 4: Retardation, Interference Colors

- In anisotropic crystals, the two rays of light produced by double refraction travel at different velocities through the crystal. It takes the **slow** ray longer to traverse the crystal than it takes the **fast ray**. The fast ray will have passed through the crystal and traveled some distance Δ beyond the crystal before the slow ray reaches the surface of the crystal. This distance Δ is called the **retardation**.



- The retardation Δ may be calculated as follows. If $\mathbf{t_S}$ is the time in seconds that it takes the slow ray to traverse the crystal and $\mathbf{t_F}$ is the time it takes the fast ray to traverse the crystal, then the distance Δ that the fast ray travels beyond the crystal before the slow ray emerges is

$$\Delta = \mathbf{c}\ (\mathbf{t_S} - \mathbf{t_F}) \qquad \{\text{units: m = (m/s)(s)}\},$$

where **c** is the velocity of light in a vacuum, which is very close to the velocity of light in air. For a crystal of thickness **h** with velocities $\mathbf{v_F}$ and $\mathbf{v_S}$, $\mathbf{t_F}$ and $\mathbf{t_S}$ may be replaced by $\mathbf{h/v_F}$ and $\mathbf{h/v_S}$ {units: (m)/(m/s) = s}, respectively, to give

$$\Delta = \mathbf{c}\left(\frac{\mathbf{h}}{\mathbf{v_S}} - \frac{\mathbf{h}}{\mathbf{v_F}}\right) = \mathbf{h}\left(\frac{\mathbf{c}}{\mathbf{v_S}} - \frac{\mathbf{c}}{\mathbf{v_F}}\right)$$

Recalling the definition of the refractive index **n**, the equation for Δ becomes

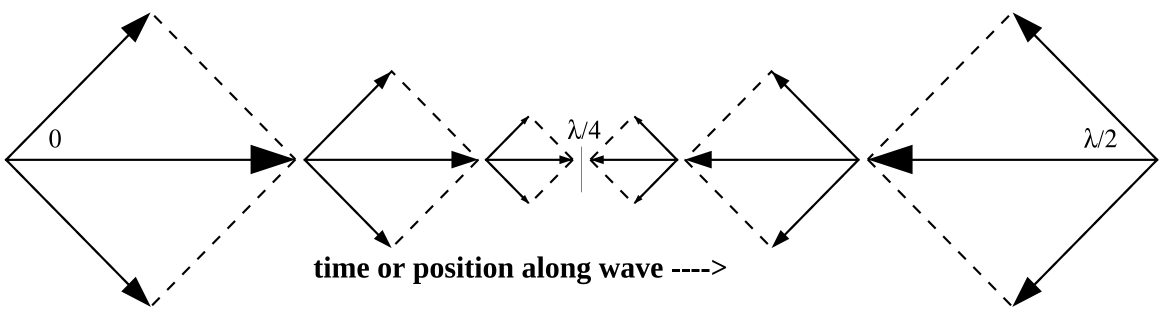
$$\Delta = \mathbf{h}\ (\mathbf{n_S} - \mathbf{n_F}).$$

Because refractive indices are dimensionless, Δ will be in the same units as **h**, normally nanometers (nm). Note that the difference in path length for the **O** and **E** rays has been neglected in this calculation. In fact, for calcite the angle is only about 5°, so the path length difference is only about a factor of 0.005. For most other minerals the angle is much smaller.

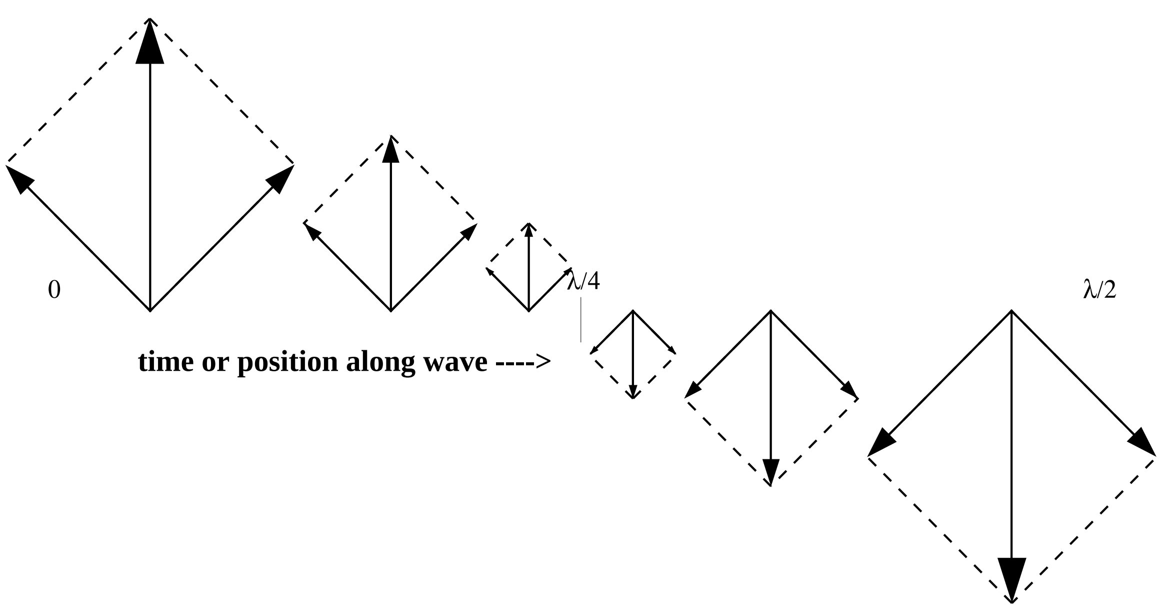
- The **birefringence** of a mineral grain is defined as the absolute value of the difference between the refractive indices of the two rays $|\mathbf{n_S - n_F}|$ for that grain. The **maximum birefringence** of a mineral is defined as the difference δ between the largest and smallest refractive indices for that mineral. Because thin sections are always the same thickness (**h=**3000 nm), the birefringence for a mineral in a particular orientation should be the same in all thin sections. Retardation for a particular mineral will be greatest when the mineral is oriented so that the two rays have the maximum and minimum refractive indices for the mineral.

- When the two rays of light emerge from an anisotropic crystal, they will recombine (following the rules of vector addition) to produce a resultant ray. If there were no retardation, the resultant ray would be identical to the incident ray. No light would pass the analyzer and the crystal would appear dark (extinct). However, retardation leads to a new resultant that does have an electric vector component that will pass the analyzer. If the light source is monochromatic, the crystal will appear lighter or darker, depending on the retardation. If the light source is polychromatic (white light), the crystal will exhibit **interference colors**.

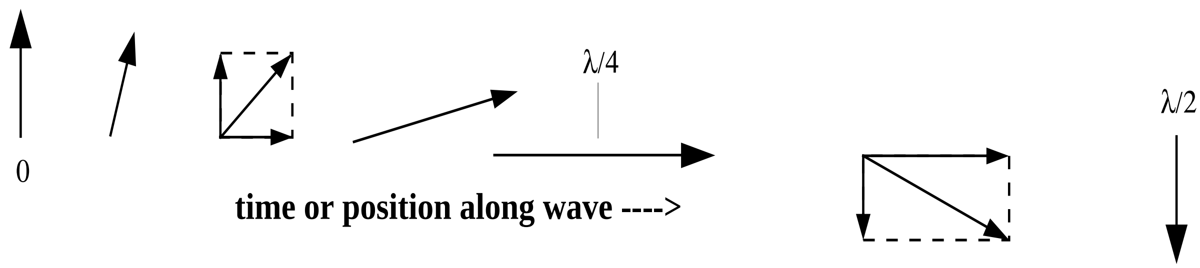
- To understand the origin of interference colors, we must examine the electric vectors at various points along a pair of light waves (emerging from an anisotropic crystal) and the resultant light wave. If the two rays of monochromatic light are **in phase**, the resultant wave will have the



same plane of polarization as the incident wave:
If the two rays of monochromatic light are out of phase due to retardation, then the resultant

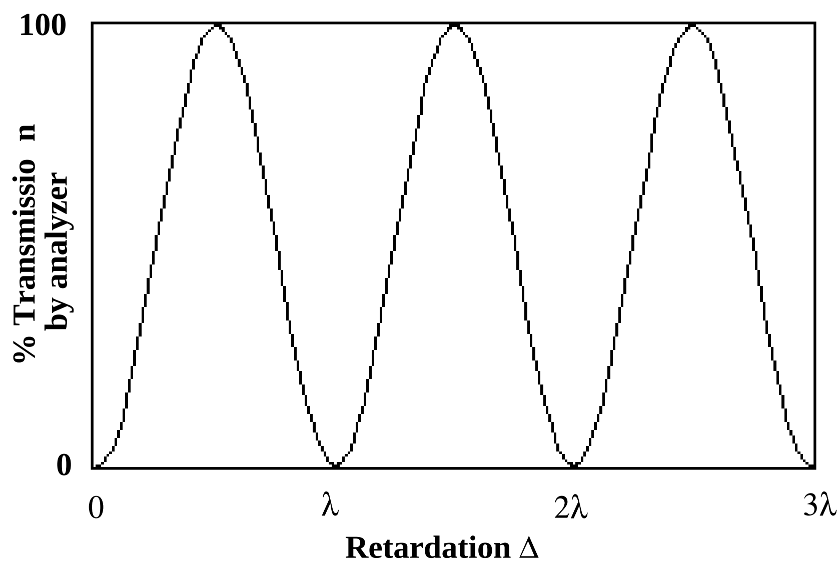


wave will have a new orientation. If the two rays are **λ/2** out of phase, the resultant will be:



If the two rays are **λ/4** out of phase, the resultant will be circularly polarized:

- Transmission of the resultant wave when the analyzer (the upper polarizing filter) is in place will depend on the orientation(s) of the resultant vibration directions with respect to the orientation of the analyzer. In most cases, some of the resultant wave is transmitted and interference colors are observed. However, if the one of the vibration directions of the crystal is parallel to that of the polarizer, then all of the light will pass through the crystal maintaining the analyzer's plane of polarization. Because there is in effect only one ray in this case, there is no interference when the light emerges from the crystal and, therefore, no interference color. **Extinction** is the dark appearance of a crystal between crossed polarizers when a vibration direction in the crystal is parallel to the vibration direction of the polarizer. Anisotropic crystals will become extinct four times as the stage of a polarizing microscope is rotated 360°. The **maximum** amount of light will be transmitted by the analyzer when stage is rotated **45°** from an extinction position.



- For monochromatic light illuminating a crystal at 45° from extinction, the intensity of the light transmitted by the analyzer as a function of the retardation is given by this -> graph. Note that no light passes the analyzer when the retardation is an integral number of wavelengths for the wavelength of light used. This effect can be observed by viewing a quartz wedge between crossed polarizers in sodium light. Retardation for the quartz wedge increases with thickness so that a series of parallel dark bands (for Δ = λ, 2λ, etc.) can be observed.

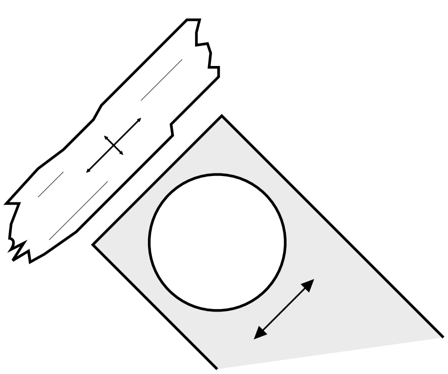
- Because the light source in our microscopes is not monochromatic, the actual interference colors observed result from the summation of dark bands for all visible wavelengths. The characteristic sequence of colors as a function of retardation is shown as the **chart of interference colors** in

Nesse and elsewhere. You will have seen these colors on soap bubbles and oil slicks, where they are produced by the interference of light waves reflected off the front and back surfaces of these films. However, in these cases no polarization or retardation is involved; the colors are due to destructive interference of the two (out of phase) reflected rays. Note that interference colors are **not** the same as the rainbow or spectrum produced from white light by a prism or a diffraction grating.

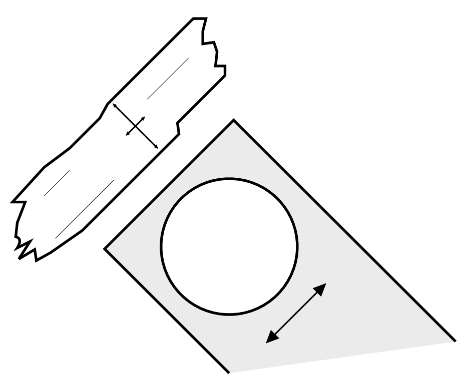
- Retardation is a function of the mineral, its orientation, and its thickness. If the thickness is doubled, so is the retardation. Similarly, if a second crystal of the same mineral with the same orientation is placed on top of the crystal being studied, the retardation will increase. In fact, if a second crystal of any mineral is placed on top with its **slow** vibration direction parallel to the **slow** vibration direction of the crystal being studied, the retardation will increase. This effect is called **addition** of retardation.

- Petrographic microscopes are equipped with a **quartz plate** designed to be placed in the light path above the crystal with the **slow** vibration direction of the quartz crystal oriented at **45°** to the planes of the polarizing filters. The slow vibration direction of the plate is indicated on the plate by a double pointed arrow or similar mark. Use of this plate permits identification of the **slow** and **fast** vibration directions of a crystal by watching for **addition** or **subtraction** of the retardation. The thickness of the quartz plate is selected to add to (or subtract from) the retardation exactly **550 nm**. Older microscopes were equipped with a similar plate made of gypsum (of a different thickness!) that caused the same amount (550 nm) of addition (or subtraction).

- Crystals that occur in a prismatic form may be characterized has having the slow direction either parallel to their long dimension or perpendicular to it. The former are called **length slow** and the latter **length fast**. These two cases may be easily distinguished by insertion of a quartz plate when the long direction of the mineral is 45° from the planes of the polarizer. If the crystal has no long direction, this test is not possible.



Length Slow
Retardation Increases
"Addition"



Length Fast
Retardation Decreases
"Subtraction"