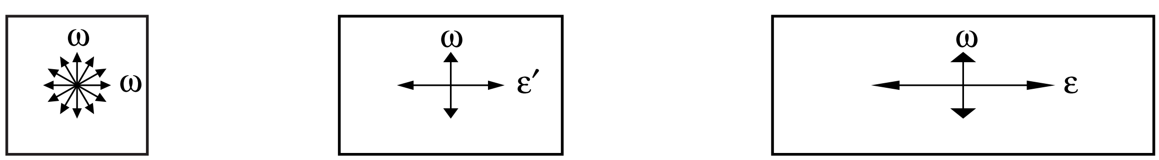
Lecture Notes - Optics 5: Uniaxial Minerals, Interfences Figures, Indicatrix

- Tetragonal and hexagonal crystals all have one unique rotation axis, the **c**-axis, which is a 4-fold, $\bar{4}$, 6-fold, or $\bar{6}$, 3-fold, $\bar{3}$ axis. Measurements of refractive indices for tetragonal and hexagonal crystals demonstrate that the **c**-axis is also a unique direction with respect to the velocity of light. These crystals are said to be **uniaxial** because of the unique optical features of the **c**-axis. For uniaxial crystals, the refractive index is called $\mathbf{n}_\omega$ for light rays with their wave normals parallel to the **c**-axis (vibration direction perpendicular to the **c**-axis). The refractive index is called $\mathbf{n}_\varepsilon$ for light rays with their wave normals along any direction in the plane perpendicular to the **c**-axis (vibration direction parallel to the **c**-axis). Refractive indices for any other vibration direction in a uniaxial crystal are between $\mathbf{n}_\varepsilon$ and $\mathbf{n}_\omega$ in value and are called $\mathbf{n}_{\varepsilon'}$.

- Uniaxial crystals may be either positive or negative, depending on the relative values of $\mathbf{n}_\varepsilon$ and $\mathbf{n}_\omega$. If $\mathbf{n}_\varepsilon > \mathbf{n}_{\varepsilon'} > \mathbf{n}_\omega$, the crystal is **positive**. If $\mathbf{n}_\varepsilon < \mathbf{n}_{\varepsilon'} < \mathbf{n}_\omega$, the crystal is **negative**.

- The vibration directions for a uniaxial crystal in normally incident light may be determined by locating the vertical projection of the **c**-axis of that crystal onto a horizontal plane (which would be parallel to the microscope stage). The vibration directions for the two rays will be parallel to and perpendicular to the projection of the **c**-axis, respectively. Note that based on this recipe, if the **c**-axis is vertical, it will project at a point and the vibration directions will be indeterminate. If a prismatic, uniaxial positive crystal was elongated in the direction of the **c**-axis, slabs cut from the crystal at various angles to the **c**-axis would have one of the following three types of optic orientation:



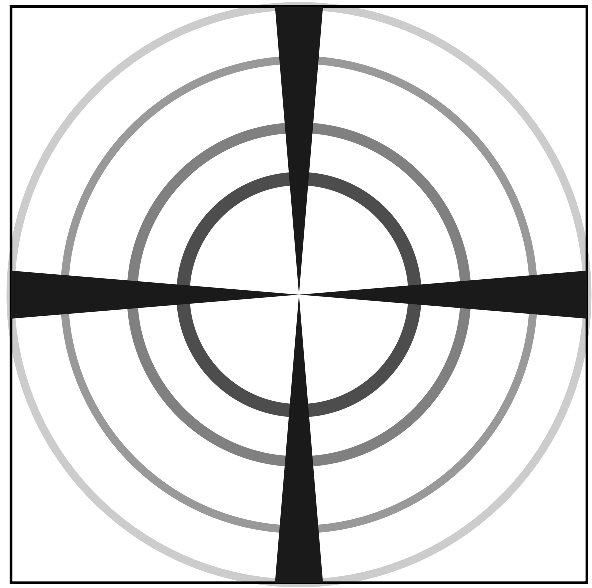
c-axis vertical   c-axis inclined   c-axis horizontal

- Recall that the **retardation** of an anisotropic crystal depends on the thickness of the crystal and on the difference between the refractive indices of the two refracted rays. For most orientations, then, $\Delta = \mathbf{h} \cdot | \mathbf{n}_{\varepsilon'} - \mathbf{n}_\omega |$. The maximum retardation possible for a given crystal in a slab of constant thickness (**h**) is $\Delta(\max) = \mathbf{h} \cdot | \mathbf{n}_\varepsilon - \mathbf{n}_\omega | = \mathbf{h} \cdot \delta$, where $\delta \equiv | \mathbf{n}_\varepsilon - \mathbf{n}_\omega |$ is the **maximum birefringence** of the crystal. The maximum retardation for a uniaxial mineral of will be exhibited by a crystal with its **c**-axis horizontal. The minimum retardation ($\Delta(\min) = 0$) will be exhibited by a crystal with its **c**-axis vertical.

- It is possible to determine whether a uniaxial mineral is positive or negative by a simple optical test using a petrographic microscope. The procedure is as follows:

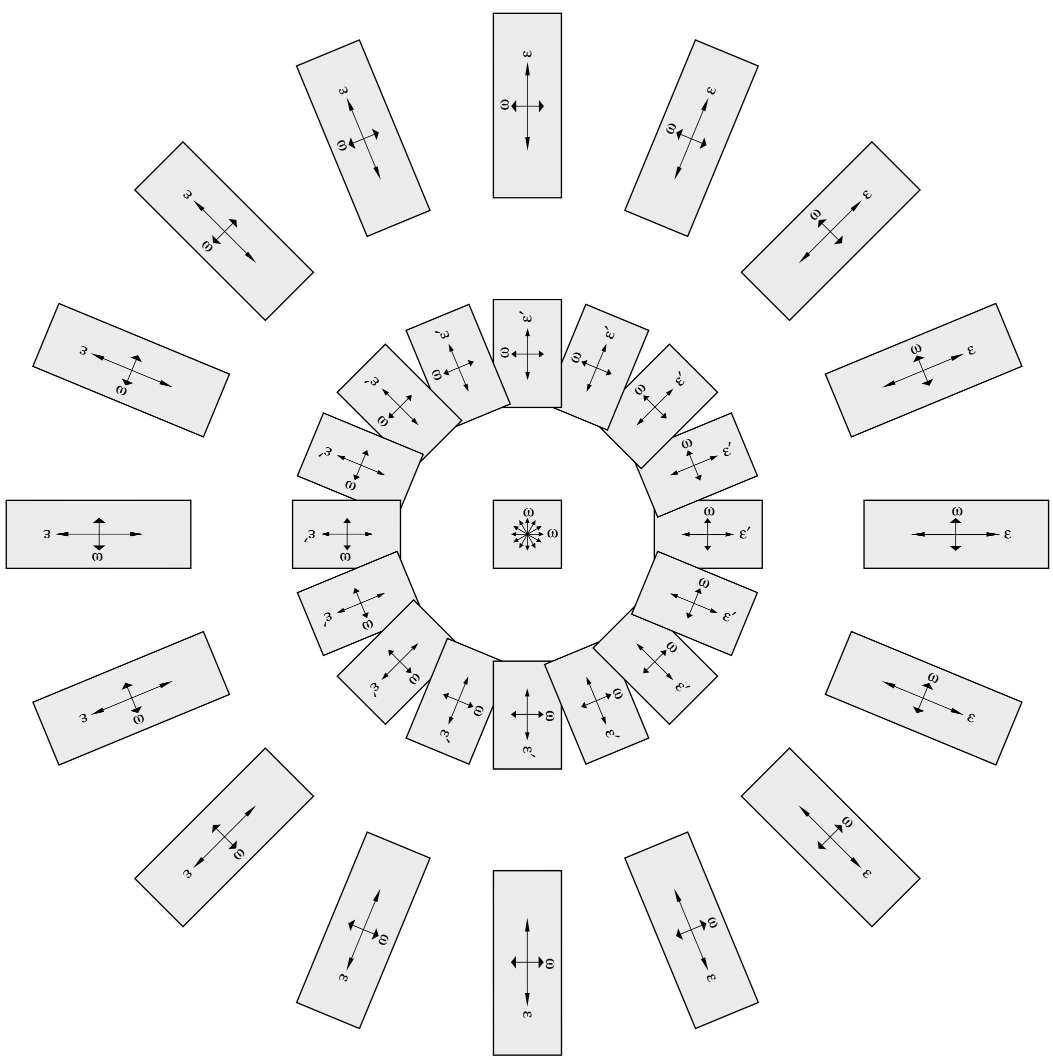
  - Locate a crystal of the mineral that exhibits the **minimum** retardation
  - Carefully focus on the crystal at using a **low power** objective lens
  - Change to a **high power** objective lens and focus again

- Insert the **condenser lens**
- Insert the **Bertrand lens**



At this point you should see a uniaxial **interference figure** that looks something like this: The dark cross is made of lines called **isogyres**. The surrounding circles are colored **isochromes**. The colors of the isochromes occur in the same sequence as the colors in the chart of interference colors. To determine the **optic sign** from the interference figure,

- Insert the λ plate
- If there is **addition** in the **northeast quadrant**, the mineral is **uniaxial positive**
- If there is **subtraction** in the **northeast quadrant**, the mineral is **uniaxial negative**



- The uniaxial interference figure is produced in part by the condenser lens, which illuminates the crystal with a cone of light such that rays of light enter the crystal at many angles in addition to the usual normal incidence. Similarly, the Bertand lens allows us to see rays refracted at many angles. The result is like seeing the interference colors (retardation) of many crystals simultaneously. The orientations of these crystals are shown schematically by the previous figure, as well as in Nesse (Figure 6.15).

- The variation of refractive index with direction may be shown graphically in three dimensions by a surface called the **indicatrix**. This surface is constructed from the end points of lines extending in all directions from an origin of the crystallographic coordinate system drawn so that the length of each line proportional to the refractive index for light vibrating in that direction.

- According to **Neumann's Principle**, the symmetry elements of any physical property of a crystal must include the symmetry elements of the point group of that crystal. Because the variation of refractive index with vibration direction must follow Neumann's Principle, the symmetry elements of the indicatrix for any crystal must include all the symmetry elements of its crystal class. The physics of light further constrains the indicatrix to be a second order surface (refractive index is a second rank tensor property), that is a surface that can be described by an equation of the form:

$$\left(\frac{\mathbf{X}}{\mathbf{n}_\alpha}\right)^2 + \left(\frac{\mathbf{Y}}{\mathbf{n}_\beta}\right)^2 + \left(\frac{\mathbf{Z}}{\mathbf{n}_\gamma}\right)^2 = 1$$

  where $\mathbf{n}_\alpha$, $\mathbf{n}_\beta$, and $\mathbf{n}_\gamma$ are refrative indices for light vibrating parallel to the X, Y, and Z directions of a cartesian coordinate system.

- There are only five qualitatively different types of indicatrices that satisfy these constraints:

Isometric System	$\mathbf{n}_\alpha = \mathbf{n}_\beta = \mathbf{n}_\gamma \equiv \mathbf{n}$			Sphere
Hexagonal System Tetragonal System	$\mathbf{n}_\alpha = \mathbf{n}_\beta \equiv \mathbf{n}_\omega$, $\mathbf{n}_\gamma \equiv \mathbf{n}_\varepsilon$ **c**-axis = $\mathbf{n}_\varepsilon$	$\mathbf{n}_\varepsilon > \mathbf{n}_\omega$ $\mathbf{n}_\varepsilon < \mathbf{n}_\omega$	**(+)** **(–)**	Prolate elipsoid of revolution Oblate elipsoid of revolution
Orthorhombic System Monoclinic System Triclinic System	$\mathbf{n}_\alpha \le \mathbf{n}_\beta \le \mathbf{n}_\gamma$	$(\mathbf{n}_\gamma - \mathbf{n}_\beta) \ge (\mathbf{n}_\beta - \mathbf{n}_\alpha)$ $(\mathbf{n}_\gamma - \mathbf{n}_\beta) \le (\mathbf{n}_\beta - \mathbf{n}_\alpha)$	**(+)** **(–)**	$\mathbf{2V}_\gamma > \mathbf{2V}_\alpha$ General elipsoid $\mathbf{2V}_\alpha > \mathbf{2V}_\gamma$ General elipsoid

- The optical indicatrix may be used as an aid in determining the vibration directions for normally incident light in anisotropic crystals. Imagine a plane passing through the indicatrix in the same orientation with respect to the coordinate system of the indicatrix as the bottom surface of the crystal is oriented with respect to the crystallographic coodinate system. (These two coordinate systems have a fixed relative orientation for any crystal.) The intersection of the plane with the indicatrix surface will be an elipse. **The two vibration directions will be the semiaxes of the elipse.**

- Every elipsoid has at least one central cross section that is a circle. Spheres have an infinite number of circular cross sections. Elipsoids of revolution have one. All other elipsoids have two. Crystals cut parallel to the circular cross section exhibit no retardation between crossed polarizing filters: they remain extinct upon rotation of the microscope stage. For this reason, the direction perpendicular to the circular cross section is called an **optic axis. Uniaxial** crystals have one optic axis. **Biaxial** crystals have two optic axes.

- The acute angle between the two optic axes in biaxial crystals is called the indicatrix **2V**. For **biaxial positive** crystals, the acute angle is centered on **Z** and the acute **2V** is **2V**$_\gamma$. For **biaxial negative** crystals, the acute angle is centered on **X** and the acute **2V** is **2V**$_\alpha$.