Unit - I
Probability.

Trial: Experiment is known as trial
Event: Outcome of Trial is called event
eg • Tossing of a coin is trial and head/tail appear is called event
• Throw a die is called trial
1 or 2 or 3 or 4 or 5 or 6 appear is called event

Exhaustive Event: All possible events in any trial is called as exhaustive events
Coins : H & T
Dice : 1, 2, 3, 4, 5, 6 are 6 cases

Equally Likely Events: Event are said to be equally likely if there is no reason to give preference to any other.

Mutually Exclusive Event: Events are said to be mutually exclusive/disjoint if no two or more of them can happen simultaneously in same trial
eg Head and Tail are mutually exclusive

Probability: If there are n exhaustive, mutually exclusive, equally likely outcomes of an exp. and m of them are favourable to an event A, then probability of A is defined as

$$P(A) = \frac{m}{n}$$

eg 1, 2, --- 20
Prime no. = $n = 20$ $m = 8$
$P(A) = 8/20 = 2/5$

Sample Space (S) : Set of all possible outcomes of trial is called sample space

Coin $S: \{H, T\}$

Dice $S: \{1, 2, 3, 4, 5, 6\}$

coin $P(S) = P(H) + P(T)$

$= \frac{1}{2} + \frac{1}{2}$

$= 1$

Dice $P(S) = P(1) + \dots + P(6)$

A and B are mutually exclusive.

$A \cap B = \phi$

$P(A \cap B) = P(\phi) = 0$.

Properties
i) $P(A) \geq 0$
ii) $P(S) = 1$
iii) $0 \leq P(A) \leq 1$
iv) $P(\phi) = 0$
v) $A \subseteq B \Rightarrow P(A) \leq P(B)$

Q A dice is thrown once. Find probability of getting a no greater than 3

Sol $P(A) = 3/6 = 1/2$.

Q If fair coin is tossed twice what is probability of getting head in both

Sol $\{HH, TT, HT, TH\}$ $n = 4$ $m = 1$

$P(A) = \frac{1}{4}$

Q An urn contain 7 cream and 5 yellow balls. Two balls are drawn at a time. find probability that both ~~both~~ balls of same clr.

Sol A : Both balls are of same colour

Total Cases $= {}^{12}C_2 = 66$

favourable Cases: ${}^{7}C_2 + {}^{5}C_2$

$= 21 + 10 = 31$

$P(A) = \frac{31}{66}$

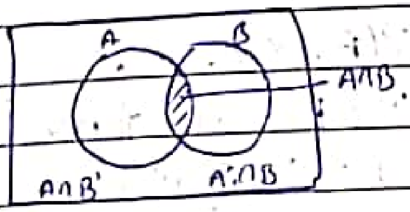
Addition Theorem of Probability

If A and B are two events

then $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Atleast one event occur

$A \cap B'$, $A' \cap B$, $A \cap B$ are exclusive disjoint set



Q Two dice are thrown. Find probability of getting even no of on first die or total of 8

Sol A = Event no. of first die , B = Total of 8

$A = (2, 4, 6) \times (1, 2, 3, 4, 5, 6) = 18$ Cases

$B = (2,6)\ (6,2)\ (4,4) = 3$ case

$P(A) = \frac{18}{36}$ $\quad P(B) = \frac{5}{36}$ $\quad P(A \cap B) = \frac{3}{6}$

$P(A \cup B) = \frac{18}{36} + \frac{5}{36} - \frac{3}{36} = \frac{20}{36} = \frac{5}{9}$

Imp

Proof — Proof of Add^n Theorem

$A \cup B = A \cup (A^c \cap B)$

$\Rightarrow P(A \cup B) = P\{A \cup (A^c \cap B)\}$

$\Rightarrow P(A \cup B) = P(A) + P(A^c \cap B)$

$\Rightarrow P(A \cup B) = P(A) + P(A^c \cap B) + P(A \cap B) - P(A \cap B)$

$\Rightarrow P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Imp — For three Events A, B & C, Add^n thm. given by

A.U.

$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$

Proof — let ~~BAC=X~~. $B \cup C = X$,

$P(A \cup X) = P(A) + P(X) - P(A \cap X)$.

$= P(A) + P(B \cup C) - P\{(B \cup C) \cap A\}$

$= P(A) + P(B) + P(C) - P(B \cap C) + P\{A \cap (B \cup C)\}$

Q. 32

$= P(A) + P(B) + P(C) - P(B \cap C) + P\{(A \cap B) \cup (A \cap C)\}$

$= P(A) + P(B) + P(C) - P(B \cap C) - [P(A \cap B) + P(A \cap C) - P(A \cap B \cap C)]$

$= P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$

Q. If A and B are mutually exclusive events such that $P(A) = 0.3$, $P(A \cup B) = 0.7$ find $P(B)$

Sol: ∵ A and B are mutually exclusive event $P(A \cap B) = 0$, Using addⁿ theorem of probability

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$0.7 = 0.3 + P(B)$$

$$P(B) = 0.4$$

Q. Two dice throw find probability of sum 7 or product 12

Total case $= 6^2 = 36$.

Sol: A = if sum is 7 = (1,6) (6,1) (2,5) (5,2) (3,4) (4,3) = 6

B = Product of no is 12 = (2,6) (6,2) (3,4) (4,3) ~~(1,6)~~ = 4 Case.

$A \cap B$ = (3,4) (4,3) = 2 Case.

$P(A) = \frac{6}{36}$ $P(B) = \frac{4}{36}$ $P(A \cap B) = \frac{2}{36}$.

Using addⁿ theorem of Probability

$$P(A \cup B) = \frac{6}{36} + \frac{4}{36} - \frac{2}{36} = \frac{8}{36} = \frac{2}{9}.$$

Conditional Probability

If there are two events A and B and probability of A is $P(A) \neq 0$, then prob. of occurrence of event B when event A has already occurred is called conditional probability of B and it is denoted by $P\left(\frac{B}{A}\right)$

$$P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)}, \quad P(A) \neq 0$$

Multiplication Theorem:

A and B are two events st $P(A) \neq 0$, $P(B) \neq 0$. then

P(A) - unconditional prob. of A

$$P(A \cap B) = P(A) \cdot P\left(\frac{B}{A}\right)$$

or $$P(A \cap B) = P(B) \cdot P\left(\frac{A}{B}\right)$$

Independent Event :

Two events are said to be independent, if occurrence of one doesnot depend upon the occurrence of the other.

Let two fair coins are tossed

E_1 : event of getting a tail on first coin
E_2 : event of getting a tail on second coin

Cards → With replacement $\frac{^4C_1}{^{52}C_1}$, $\frac{4}{52}$, $\frac{1}{13}$,
→ without replacement $\frac{4}{52}$, $\frac{3}{51}$
I Card, II Card

If A and B are independent events then

$$\boxed{P(A \cap B) = P(A) \cdot P(B)}$$

[∵ $P(A \cap B) = P(A)\, P(B/A)$
$P(A \cap B) = P(A) \cdot P(B)$]

Theorem : If A and B are Independent events then A^c & B, A & B^c, A^c & B^c are also independent event

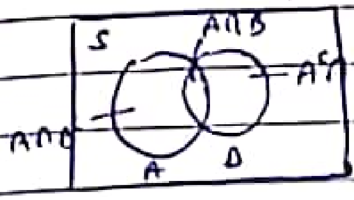
Proof: A and B are independent event then

$P(A \cap B) = P(A)P(B)$

$P(A^c \cap B) = P(B) - P(A \cap B)$

$= P(B) - P(A)(P(B) = [1 - P(A)]P(B)$

$= P(A^c)P(B)$.



ii) $P(A \cap B^c) = P(A) - P(A \cap B)$

$= P(A) - P(A)P(B)$

$= P(A)[1 - P(B)]$

$= P(A)P(B^c)$

$= P[(A \cup B)^c]$ Demorgan

iii) $P(A^c \cap B^c) = 1 - P(A \cup B) = 1 - [P(A) + P(B) - P(A \cap B)]$

$= 1 - P(A) - P(B) + P(A)P(B)$ [Given A and B are ind].

$= [1 - P(A)] - P(B)[1 - P(A)]$

$= [1 - P(B)][1 - P(A)]$

$= P(A^c)P(B^c)$

Q If $P(A) = \frac{1}{4}$, $P(\bar{B}) = \frac{1}{2}$ and $P(A \cup B) = \frac{5}{9}$.

- Find i) $P(A/B)$. ii) Are A and B independent

Soln $P(A) = 1/4$.

$P(\bar{B}) = 1/2 \rightarrow P(B) = 1 - P(\bar{B}) = 1/2$.

$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$

Addn theo. of Prob.

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\frac{3}{4} - \frac{5}{4} = $ $1 - \frac{5}{4}$ $-\frac{3}{4} - \frac{5}{4}$, 27.20

$$P(A \cap B) = \frac{1}{4} + \frac{1}{2} - \frac{5}{9}$$

$$P(A \cap B) = \frac{\cancel{1}}{\cancel{4}} \frac{\cancel{4}}{\cancel{4}} \cdot \frac{7}{36}$$

$$P(A/B) = \frac{4/9}{\cancel{1}/2} = \frac{8}{9} \quad \frac{7/36}{1/2} = \frac{7}{18}$$

$\because P(A) \neq P(A/B)$

ii) unconditional probability of A is not equal to conditional probability of A.

So A and B are not independent

Or $P(A \cap B) = 7/36$

$$P(A)\,P(B) = \frac{\cancel{7}}{4} \times \frac{1}{2} = \frac{1}{8}$$

$P(A \cap B) \neq P(A)\,P(B)$.

A and B are not independent

Q The probability that A hits the target is $1/3$
Probability " B " " " is $1/5$ both fired at target Find the prob. that

i) A doesnot hit the target
ii) Both hit the target
iii) One of them hit the target
iv) Neither hit the target

Sol i) $P(A^c) = 1 - P(A)$

$$= 1 - 1/3 = \frac{2}{3}$$

$A \cup B \rightarrow$ Atleast one event
$A \cap B \rightarrow$ Both event occur
$A^c \cap B \rightarrow$ A doesnot occur but B occur.
$A^c \cap B^c \rightarrow$ None occur.

ii) $P(A \cap B) = P(A)\,P(B)$ [Both are independent events]

$$= \frac{1}{3} \times \frac{1}{5} = \frac{1}{15}$$

iv) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ [Add^n theorem of prob]

$$= \frac{1}{3} + \frac{1}{5} - \frac{1}{15}$$

$$= \frac{7}{15}$$

~~1 - P(A~~ $P(A^c \cap B^c) = 1 - P(A \cup B)$ (Demorgan)

$$= 1 - \frac{7}{15} = \frac{8}{15}$$

Required probability

iii) $= P(A \cap B^c) + P(A^c \cap B)$

$$= P(A)\,P(\bar{B}) + P(\bar{A})\,P(B)$$

$$= \frac{1}{3}\left[1 - \frac{1}{5}\right] + \frac{1}{5}\left[1 - \frac{1}{3}\right]$$

$$= \frac{4}{15} + \frac{2}{15} = \frac{6}{15} = \frac{2}{5}$$

iv) $P(\bar{A} \cap \bar{B}) = P(\bar{A})\,P(\bar{B})$ [A and B are independent event]

$$= \frac{8}{15}$$

* Q Given n independents events $A_i \{i = 1, 2 \ldots n\}$ with probability p_i then probability of occurrence of atleast one of them is

$$p = 1 - (1-p_1)(1-p_2)\cdots(1-p_n)$$

If problem are given to student then it is supposed they will do independently

Proof $P(A_1 \cup A_2 \cdots \cup A_n) + P(\overline{A_1 \cup A_2 \cdots \cup A_n}) = 1$

$\Rightarrow P(A_1 \cup A_2 \cdots \cup A_n) = 1 - P(\overline{A_1 \cup A_2 \cdots \cup A_n})$

$\Rightarrow P = 1 - [P(\bar{A_1}) \cap P(\bar{A_2}) \cap P(\bar{A_3}) \cdots \cap P(\bar{A_n})]$

Demorgan's Law

and

$= 1 - P(\bar{A_1}) P(\bar{A_2}) \cdots P(\bar{A_n})$ [Because $A_1, A_2 \cdots A_n$ are independent and complements are also indep]

$= 1 - (1 - P(A_1))(1 - P(A_2)) \cdots (1 - P(A_n))$

$= 1 - (1-p_1)(1-p_2) \cdots (1-p_n)$

Q The problem in statistics is given to three students A, B, C whose chance of solving it are $\frac{1}{2}$, $\frac{3}{4}$ and $\frac{1}{4}$. What is probability the problem will be solved if all of them try independently

Sol^n Probability

$P(A) = 1/2 \quad P(B) = 3/4 \quad P(C) = 1/4$

Require Probability

$P(A \cup B \cup C) = 1 - \left(1 - \frac{1}{2}\right)\left(1 - \frac{3}{4}\right)\left(1 - \frac{1}{4}\right)$

$P(A \cup B \cup C) = \frac{29}{32}$

Probability that atleast one of event A and B occur is 0.6 If A and B occur simultaneously with prob 0.2. find $P(\bar{A}) + P(\bar{B})$

Sol: $P(A \cup B) = 0.6$
$P(A \cap B) = 0.2$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$
$0.6 + 0.2 = P(A) + P(B)$
$0.8 = P(A) + P(B)$

$P(\bar{A}) + P(\bar{B}) = 1 - P(A) + 1 - P(B)$
$= 2 - 0.8$
$= 1.2$

[Sum can be greater than 1]
But individual should be less than 1.

Q $P(A) = 0.4$ $P(A \cup B) = 0.7$. A and B are independent then find value of $P(B)$

Ans $P(A \cup B) = P(A) + P(B) - P(A)P(B)$ {Independent events}
$0.7 = 0.4 + x - 0.4x$
$0.7 - 0.4 = 0.6x$
$0.3 = 0.6x$
$x = 0.5$

Atleast → None.

Q. The probability of selection of A is 1/7 and that of B is 1/5. Then find probability that both of them would not be selected

~~Ans:~~ $\cancel{P(A^c} P(A\cap B)^c = \cancel{P(A')} 1 - P(A\cap B)$

$= 1 - P(A)P(B)$

$1 - \frac{1}{5}\cdot\frac{1}{7}$

$= \frac{34}{35}$

Required probability

Ans. $P(\overline{A\cup B}) = \cancel{P(\bar A)\,AP}\, P(\bar A \cap \bar B)$

$= P(\bar A)\,P(\bar B)$ [∵ A and B are independent so $\bar A$ and $\bar B$ are]

$= \cancel{\frac{4}{7}} [1-P(A)]\,[1-P(B)]$

$= \left(1-\frac{1}{5}\right)\left(1-\frac{1}{7}\right)$

$= \frac{24}{35}$.

Q. A bag B_1 has 4 white and 3 black balls. Similarly another bag B_2 has 3 white and 5 black balls. A ball is drawn first bag and without noting its colour is put into 2nd bag then ball is is drawn from 2nd bag. Find prob that it is white.

Solⁿ

B_1	B_2	Case I	Case II
4W + 3B	3W + 5B	4W + 5B	3W + 6B.

Req. Prob $= \cancel{P(W/W)}\; P(WW) + P(BW)$

$$P = \frac{^4C_1}{^7C_1} \cdot \frac{^4C_1}{^9C_1} + \frac{^3C_1}{^7C_1} \cdot \frac{^5C_1}{^9C_1}$$

$$P = \frac{25}{63}$$

P & C $\quad {}^nC_1 = n \quad {}^nC_n = 1 \quad {}^nC_0 = 1 \quad {}^nC_r = {}^nC_{n-r}$.

Q From city population probability of selecting

i) a male ~~of~~ or a smoker is 7/10

ii) a male smoker is $\frac{2}{5}$.

iii) a male, if a smoker is already selected is 2/3.

Find prob. of selecting

a) a non-smoker

b) a male

c) a smoker, if male is first selected

Sol: ~~$P(A \cup B) = \frac{7}{10}$~~

A : a male is selected

B : a smoker is "

$$P(A \cup B) = \frac{7}{10}$$

$$P(A \cap B) = \frac{2}{5}$$

$$P(A/B) = 2/3.$$

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$$P(A/B) = \frac{P(A\cap B)}{P(B)} = \frac{2}{3}.$$

$$= \frac{2/5}{P(B)} = 2/3$$

$$= \boxed{P(B) = \frac{3}{5}}.$$

a) $P(\overline{B}) = 1 - \frac{3}{5}$

$$= \frac{2}{5}.$$

b) ~~P(A) . P(~~ Male P(A).

$$\frac{7}{10} = \frac{3}{5} + P(A) - \frac{2}{5}.$$

$$\frac{\cancel{6}\,5}{10} = P(A)$$

$$P(A) = \frac{1}{2}.$$

c) $P\left(\frac{B}{A}\right) = \frac{2/5}{1/2} = \frac{4}{5}.$

Q) Each coefficient in equation $ax^2 + bx + c = 0$ determined by throwing ~~dice~~ die. Find the prob. that eq^n will have real roots.

Sol: Total cases : $6 \times 6 \times 6 = 216$

roots of eq^n $ax^2 + bx + c = 0$ will have real roots if $b^2 - 4ac \geq 0$ i.e $b^2 \geq 4ac$.

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each of coefficient take 1 to 6 value

Favourable Case

ac	a	c	4ac	b ($b^2 \ge 4ac$)	No. of cases
1	1	1	4	2, 3, 4, 5, 6	$1 \times 5 = 5$
2	{1, 2}	{2, 1}	8	3, 4, 5, 6	$2 \times 4 = 8$
3	{1, 3}	{3, 1}	12	4, 5, 6	$2 \times 3 = 6$
4	{1, 4, 2}	{4, 1, 2}	16	4, 5, 6	$3 \times 3 = 9$
5	{1, 5}	{5, 1}	20	5, 6	$2 \times 2 = 4$
6	{1, 6, 2, 3}	{6, 1, 3, 2}	24	5, 6	$4 \times 2 = 8$
ac = 7 no pairs					
8	{2, 4}	{4, 2}	32	6	$2 \times 1 = 2$
9	3	3	36	6	$1 \times 1 = 1$

~~10~~ Favourable Cases = 43

~~12~~ Req. Prob $= \frac{43}{216}$

Q Out of $(2n+1)$ tickets consecutively numbered three are drawn at random find prob. chance that the no.s on them in AP

Sol^n

Total Cases $= {}^{2n+1}C_3$

$$= \frac{2n+1!}{3!\,(2n-2)!}$$

$$= \frac{(2n+1)(\cancel{2}n)(2n-1)}{3 \cdot \cancel{2} \cdot 1}$$

$$= \frac{n}{2}\,(4n^2-1)$$

Favourable Case

- If $d = 1$.

  1, 2, 3.
  2, 3, 4 $(2n-1)$ Case.
  ⋮
  $2n-1$, $2n$, $2n+1$

- If $d = 2$.

  1, 3, 5.
  2, 4, 6 $(2n-3)$ Cases.
  ⋮
  $2n-3$, $2n-1$, $2n+1$

- If $d = n-1$

  $1, n, 2n-1$.   3 Cases.
  $n+1-2$ $2, n+1, 2n$.
  $3, n+2, 2n+1$.

~~Favorable Cases = (2n-1) + (2n-3) + --- + 3~~

- If $d = n$.

  $1, n+1, 2n+1$ 1 Case.

$+2, 2n+2$

Favourable Case $= 1 + 3 + --- + (2n-3) + (2n-1)$

$= \frac{n}{2}[2n-1+1]$

$= n^2$.

Required prob. $= \dfrac{n^2}{\frac{n}{3}(4n^2-1)} = \dfrac{3n}{4n^2-1}$.

Q. p is prob. that man aged x years will die in a year. Find the prob. that out of n men $A_1, A_2 \ldots A_n$ each aged x, A_1 will die in a year and will be first to die. Prob that none die ⇒ Atleast One

Sol^n Let E_i $(i = 1, 2, \ldots n)$ denote event that A_i dies in year

$P(E_i) = p$

1) Prob. that none of n men $A_1, A_2 \ldots A_n$ die in a year

$$= P(\bar{E}_1 \cap \bar{E}_2 \ldots \cap \bar{E}_n)$$
$$= p(\bar{E}_1)\, p(\bar{E}_2) \ldots p(\bar{E}_n)$$
$$= (1-p)(1-p) \ldots (1-p)$$
$$= (1-p)^n$$

2)

Prob^{that} At least one of $A_1, A_2 \ldots A_n$ die in a year

$$= 1 - P(\bar{E}_1 \cap \bar{E}_2 \ldots \cap \bar{E}_n)$$
$$= 1-(1-p)^n$$

Req. prob. that A_1 will first die $= \left(\frac{1}{n}\right) \cdot \left[1-(1-p)^n\right]$

Boole's inequality for n-events $A_1, A_2, \dots, A_n$

We have.

i) $P(\bigcap_{i=1}^{n} A_i) \geq \sum_{i=1}^{n} P(A_i) - (n-1)$.

ii) $P(\bigcup_{i=1}^{n} A_i) < \sum_{i=1}^{n} P(A_i)$

Proof i) for Two events A_1 and A_2.

$P(A_1 \cup A_2) = P(A_1) + P(A_2) - P(A_1 \cap A_2) \leq 1$ [By add" then]

~~or $P(A_1) + P(A_2)$~~

Or $P(A_1 \cap A_2) \geq P(A_1) + P(A_2) - 1$. — (1)

result hold for $n=2$.

ii) let the result is true for $n = r$

$$P(\bigcap_{i=1}^{r} A_i) \geq \sum_{i=1}^{r} P(A_i) - (r-1)$$

iii) Now

$$P(\bigcap_{i=1}^{r+1} A_i) = P(\bigcap_{i=1}^{r} A_i \cap A_{r+1})$$

from (1).

$$P(\bigcap_{i=1}^{r+1} A_i) \geq P(\bigcap_{i=1}^{r} A_i) + P(A_{r+1}) - 1.$$

$$\geq \sum_{i=1}^{r} P(A_i) - (r-1) + P(A_{r+1}) - 1$$

$$= \sum_{i=1}^{r} P(A_i) - r + P(A_{r+1})$$

$$= \sum_{i=1}^{r+1} P(A_i) - r.$$

Result is also true for $n = r+1$.

ii) $P\left(\cancel{\bigcap_{i=1}^{n} A_i}\right) \cdot P\left(\bigcup_{i=1}^{n} A_i\right) \leq \sum_{i=1}^{n} P(A_i)$

Using (i)

$$P(\bar{A_1} \cap \bar{A_2} \cap \ldots \cap \bar{A_n}) \geq P(\bar{A_1}) + P(\bar{A_2}) + \ldots + P(\bar{A_n}) - (n-1)$$

$$= [1 - P(A_1)] + [1 - P(A_2)] + \ldots + [1 - P(A_n)] - (n-1)$$

$$= 1 - P(A_1) - P(A_2) - \ldots - P(A_n)$$

$$\Rightarrow P(A_1) + P(A_2) + \ldots + P(A_n) \geq 1 - P[\bar{A_1} \cap \bar{A_2} \cap \bar{A_3} \ldots \cap \bar{A_n}]$$

$$= 1 - P(\overline{A_1 \cup A_2 \ldots \cup A_n})$$

$$= P(A_1 \cup A_2 \ldots \cup A_n)$$

$$P(A_1 \cup A_2 \ldots \cup A_n) \leq P(A_1) + P(A_2) + \ldots + P(A_n)$$

Pairwise Independents :-

For the set of n events $A_1, A_2 ---- A_n$ are said to be pairwise independent if

$$P(A_i \cap A_j) = P(A_i)P(A_j), \quad i \neq j; \quad i,j = 1,2,---n$$

Mutually Independence :

For n-events. If $A_1, A_2, \ldots A_n$ are n events for mutual independence, we have

i) $P(A_i \cap A_j) = P(A_i)P(A_j) \quad i \neq j \quad i,j = 1,2,---n$

ii) $P(A_i \cap A_j \cap A_k) = P(A_i)P(A_j)P(A_k) \quad \left(\begin{matrix} i \neq j \neq k \\ i,j,k = 1,2,---n \end{matrix}\right)$

⋮

$$P(A_1 \cap A_2 --- \cap A_n) = P(A_1)P(A_2)---P(A_n)$$

* Every Mutual Independence is Pairwise Independence But converse not true.

(i) is satisfied by $^nC_2$ cond^n
(ii) — " — by $^nC_3$ — " —

Total cond^n for mutual independence $= {^nC_2} + {^nC_3} + \ldots {^nC_n}$

$\because \; {^nC_0} + {^nC_1} + {^nC_2} + ---- + {^nC_n} = 2^n$
${^nC_2} + {^nC_3} + --- + {^nC_n} = 2^n - n - 1.$

i.e. Total cond^n for mutual independence $= 2^n - n - 1$

Q) Consider 3 events A_1, A_2 & A_3

Total condⁿ for mutual indep. $= 2^3 - 3 - 1$
$= 4.$

$$\begin{cases} P(A_1 \cap A_2) = P(A_1)P(A_2) \\ P(A_1 \cap A_3) = P(A_1)\ P(A_2) \\ P(A_2 \cap A_3) = P(A_2)\ P(A_3) \\ P(A_1 \cap A_2 \cap A_3) = P(A_1)\cdot P(A_2)\cdot P(A_3) \end{cases}$$

Mutual Independence is pairwise independence but converse is not true.

Q) An Urn contains four tickets bearing 112, 121, 211 and 222 and one ticket is drawn.

Let A_i ($i = 1, 2, 3$) be event that i^{th} digit of no of ticket drawn is 1.
Discuss independence of events A_1, A_2, A_3.

Solⁿ A_1 be event that 1^{st} digit of no. is 1
A_2 be event that 2^{nd} digit of no is 1
A_3 —— " —— 3^{rd} —— " —— .

$P(A_1) = \frac{2}{4}$ $P(A_2) = \frac{2}{4}$ $P(A_3) = \frac{2}{4}$.

$P(A_1 \cap A_2) = \frac{1}{4} = P(A_1)P(A_2)$ [112]

$P(A_2 \cap A_3) = \frac{1}{4} = P(A_2)P(A_3)$ [211]

$P(A_1 \cap A_3) = \frac{1}{4} = P(A_1)P(A_3)$ [121]

A_1, A_2, A_3 are pairwise Independence

$P(A_1 \cap A_2 \cap A_3) = P(\phi) = 0 \neq P(A_1)P(A_2)P(A_3)$

as $P(A_1)P(A_2)P(A_3) = \frac{1}{2}\frac{1}{2}\frac{1}{2} = \frac{1}{8}$.

Theorem: If A, B, C are mutually independent events then $A \cup B$ and C are also mutually independent.

To prove $(A \cup B)$ and C are independent, it is sufficient to prove

$P[(A \cup B) \cap C] = P(A \cup B) \cdot P(C)$.

Pf

$$\begin{aligned} P[(A \cup B) \cap C] &= P[(A \cap C) \cup (B \cap C)] \\ &= P(A \cap C) + P(B \cap C) - P(A \cap C \cap B \cap C) \\ &= P(A)P(C) + P(B)P(C) - P(A)P(B)P(C) \\ &= P(C)[P(A) + P(B) - P(A)P(B)] \\ &= P(A \cup B)P(C). \end{aligned}$$

A, B, C mutually indepe.

Q. If A, B and C are events in sample space if A, B and C are pairwise ind. and also A is independent of $B \cup C$, then A, B, C are mutually indepen.

Soln

$P(A \cap B) = P(A)P(B)$
$P(B \cap C) = P(B)P(C)$
$P(A \cap C) = P(A)P(C)$

$P[A \cap (B \cup C)] = P(A)P(B \cup C)$

$P[A \cap B \cup A \cap C] = P(A \cap B) + P(A \cap C) - P(A \cap B \cap C)$

$P(A)P(B \cup C) = P(A)P(B) + P(A)P(C) - P(A \cap B \cap C)$

$$P(A\cap B\cap C) = P(A)P(B) + P(A)P(C) - P(A)P(B\cap C)$$
$$= P(A)\left[P(B) + P(C) - P(B\cap C)\right]$$
$$\cancel{= P(A)\, P(B\cup C)}$$
$$= P(A)\,P(B\cap C)$$
$$= P(A)P(B)P(C)$$

then

A, B, C are mutually indepe.

Q. A and B throw alternately with pair of ordinary dice. A wins if he throw 6 before 7 and B win if he throw 7 before A throw 6. If A begins; show that his chance of winning is 30/61.

~~Sol. If A~~

1,5 5,1 4,2 2,4 3,3

2,5 5,2 6,1 3,4 4,3 1,6

Sol^n E_1 : event of A showing 6.
E_2 : event of B showing 7.

$P(E_1) = \frac{5}{36}$ $\qquad$ $P(E_2) = \frac{6}{36}$

$P(\bar{E}_1) = \frac{31}{36}$ $\qquad$ $P(\bar{E}_2) = \frac{30}{36}$.

$P(E_1) + P(\bar{E}_1\cap \bar{E}_2\cap E_1) + P(\bar{E}_1\cap \bar{E}_2\cap \bar{E}_1\cap \bar{E}_2\cap E_1) + \cdots$

If A start, then he will have the following mutually exclusive

i) E_1 happen. ii) $\bar{E}_1\cap \bar{E}_2\cap E_1$ iii) $\bar{E}_1\cap \bar{E}_2\cap \bar{E}_1\cap \bar{E}_2\cap E_1$

Required prob. of A to win is -

$$= P(E_1) + P(\bar{E}_1\cap \bar{E}_2\cap E_1) + \cdots$$
$$= \frac{5}{36} + \frac{31}{36}\cdot\frac{30}{36}\cdot\frac{5}{36} + \frac{31}{36}\cdot\frac{30}{36}\cdot\frac{31}{36}\cdot\frac{30}{36}\cdot\frac{5}{36}$$
$$= \frac{5}{36}\left[1 + \frac{31\cdot 30}{36\cdot 36} + \frac{31}{36}\cdot\frac{30}{36}\cdot\frac{31}{36}\cdot\frac{30}{36} \text{———}\right].$$

$$P = \frac{5}{36}\left[1 + \frac{31\cdot 30}{36\cdot 36} + \left(\frac{31\cdot 30}{36\cdot 36}\right)^2 + \cdots\right]$$

$$P = \frac{5}{36} \cdot \frac{1}{1 - \frac{31\cdot 30}{36\cdot 36}} = \frac{5\cdot 6}{61} = \frac{30}{61}$$

$$= \frac{\cancel{5}}{\cancel{36}} \quad \cancel{1}$$

Q A player tosses a coin and is to score one point for every head and 2 coin for every tail turned up. He is to play on until his score reaches or passes n. If p_n is chance of attaining exactly n scores, show that

$$p_n = \frac{1}{2}[p_{n-1} + p_{n-2}]$$

and hence find value of p_n

Soln The score n can be reached the following two mutually exclusive cases

i) By throwing tail when score is n-2
ii) By throwing heat when score is n-1.

Required probability $= \cancel{\tfrac{1}{2}}(\cancel{P_{n-1}}\ p(i) + p(ii)$

$$= p_{n-2} \times \frac{1}{2} + p_{n-1}\frac{1}{2}$$

$$p_n = \frac{1}{2}(p_{n-1} + p_{n-2})$$

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Add^n $\frac{1}{2}P_{n-1}$.

$$P_n + \frac{1}{2}P_{n-1} = \frac{1}{2}P_{n-1} + \frac{1}{2}[P_{n-1} + P_{n-2}]$$

$$P_n + \frac{1}{2}P_{n-1} = \frac{1}{2}P_{n-2} + P_{n-1} = P_{n-1} + \frac{1}{2}P_{n-2}$$

$$\therefore P_n + \frac{1}{2}P_{n-1} = P_{n-1} + \frac{1}{2}P_{n-2}$$

$$P_{n-1} + \frac{1}{2}P_{n-2} = P_{n-2} + \frac{1}{2}P_{n-3}$$

$\vdots$

$$P_3 + \frac{1}{2}P_2 = P_2 + \frac{1}{2}P_1$$

$\longrightarrow$ $P_n + \frac{1}{2}P_{n-1} = P_2 + \frac{1}{2}P_1$

$P_k + \frac{1}{2}P_{k-1}$ score 2 can be obtained by

i) Head in first throw and head in 2nd throw

ii) Tail in 1st throw.

$P_2 = q(i) + p(ii)$

$\Rightarrow P_2 = \frac{1}{2}\times\frac{1}{2} + \frac{1}{2} = \frac{3}{4}$.

$P_1 = \frac{1}{2}$.

Add

$$P_n + \frac{1}{2}P_{n-1} = \frac{3}{4} + \frac{1}{2}\cdot\frac{1}{2} = 1.$$

$$\Rightarrow P_n + \frac{1}{2}P_{n-1} = \frac{2}{3} + \frac{1}{2}\cdot\frac{2}{3}$$

$$\Rightarrow P_n - \frac{2}{3} = \frac{1}{2}\left(\frac{2}{3} - P_{n-1}\right)$$

$$p_n - \frac{2}{3} = \frac{-1}{2}\left(p_{n-1} - \frac{2}{3}\right)$$

also $p_{n-1} - \frac{2}{3} = \frac{-1}{2}\left(p_{n-2} - \frac{2}{3}\right)$

$$p_{n-2} - \frac{2}{3} = \frac{-1}{2}\left(p_{n-3} - \frac{2}{3}\right)$$

$$\vdots$$

$$p_2 - \frac{2}{3} = \frac{-1}{2}\left(p_1 - \frac{2}{3}\right)$$

Multiplying

$$p_n - \frac{2}{3} = \left(\frac{-1}{2}\right)^{n-1}\left(p_1 - \frac{2}{3}\right)$$

$$p_n = \frac{2}{3} + \left(\frac{-1}{2}\right)^{n-1}\left(p_1 - \frac{2}{3}\right)$$

$$p_n = \frac{2}{3} + \left(\frac{-1}{2}\right)^{n-1}\left(\frac{-1}{6}\right)$$

$$= \frac{1}{3}\left[2 + \left(\frac{-1}{2}\right)^{n}\right]$$

Baye's Theorem

State: If $E_1, E_2, \ldots, E_n$ are n mutually disjoint events with $P(E_i) \neq 0$ then for any vector A which is subset of $\cup E_i$

$$P(E_i/A) = \frac{P(E_i)\,P(A/E_i)}{\sum_{i=1}^{n} P(E_i)\,P(A/E_i)}$$

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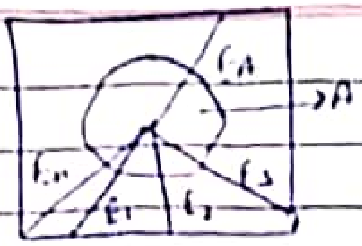
Proof



$A \subset \cup E_i$

$A = A \cap (\cup E_i)$

$A = (A \cap E_1) \cup (A \cap E_2) \ldots$

$A = \bigcup_{i} (A \cap E_i)$

$P(A) = \sum_{i=1}^{n} P(A \cap E_i)$

$$P(A) = P\left[\bigcup_i (A \cap E_i)\right]$$

$$= \sum_{i=1}^{n} P(A \cap E_i) \qquad \left[P(\cup A_i) = \sum P(A_i)\right]$$

$$P(A) = \sum_{i=1}^{n} P(E_i)\, P(A/E_i)$$

$$P(E_i/A) = \frac{P(E_i \cap A)}{P(A)}$$

$$= \frac{P(E_i)\, P(A/E_i)}{\sum_{i=1}^{n} P(E_i)\, P(A/E_i)}$$

i) The probability $P(E_1), P(E_2) \ldots P(E_n)$ are prior prob. b/c they are exist b4 we gain any info. from exp.

ii) Prob. $P(A/E_i)$ are called likelihood b/c they indicate how likely A under consideration is occured

iii) $P(E_i/A) \rightarrow$ Posterior Prob. b/c they are determined after result of exp. are known.

Q In bolt factory machines I, II, III manufacture 25%, 35%, 40% of total production of their output are 5%, 4%, 2% are defective bolts. A bolt is drawn at random from product is found to defective. What are prob. that it was manufactured by machines I, II, III

Sol^n Let E_1, E_2, E_3 event that product belongs to machines I, II & III. A be event that defective bolt

$P(E_1) = \frac{25}{100}$ $P(E_2) = \frac{35}{100}$ $P(E_3) = \frac{40}{100}$

A be event that defective bolt is obtained.

given $P(E_1) = \frac{25}{100}$, $P(E_2) = \frac{35}{100}$

$P(E_3) = \frac{40}{100}$

$P(A/E_1) = \frac{5}{100}$, $P(A/E_2) = \frac{4}{100}$

$P(A/E_3) = \frac{2}{100}$

From using Baye's theorem

$$P(E_1/A) = \frac{P(E_1)\,P(A/E_1)}{P(E_1)\,P(A/E_1) + P(E_2)\,P(A/E_2) + P(E_3)\,P(A/E_3)}$$

$$= P(E_1/A) = \frac{\frac{25}{100} \times \frac{5}{100}}{\frac{25}{100} \times \frac{5}{100} + \frac{35}{100} \times \frac{4}{100} + \frac{40}{100} \times \frac{2}{100}}$$

$$= \frac{25 \times 5}{125 + 140 + 80} = \frac{125}{345} \quad \frac{25}{69}$$

$$= \frac{5}{1} \quad \frac{25}{69}$$

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You
11/12/2019, 4:24 pm

Q. Urns I, II & III are as follows
1 W, 2 B & 3 R balls
2 W, 1 B & 1 R balls
4 W, 5 B & 3 R balls
One urn is chosen at random & two balls drawn from it. They happen to be white & red. What is prob. that they came from urns I, II or III ?

Ans. Let E_1, E_2 & E_3 be the events that urn I, II & III is choosen

$P(E_1) = 1/3$

$P(E_2) = 1/3$

$P(E_3) = 1/3$

A : balls are white & red

$$P(A/E_1) = \frac{{}^1C_1 \times {}^3C_1}{{}^6C_2} = \frac{1 \times \frac{3!}{2!}}{\frac{6!}{4!\,2!}}$$

$$P(A/E_1) = \frac{3! \times 4!}{6!} = \frac{3 \times 2}{6 \times 5} = \frac{1}{5}$$

$$P(A/E_2) = \frac{{}^2C_1 \times {}^1C_1}{{}^4C_2} = \frac{2}{4!} \times 2!\,2!$$

$$P(A/E_2) = \frac{2}{4 \times 3 \times 2} \times 2 \times 2 = \frac{2}{6} = \frac{1}{3}$$

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You
11/12/2019, 4:24 pm

$$P\left(\frac{A}{E_3}\right) = \frac{{}^4C_1 \times {}^3C_1}{{}^{12}C_2} = \frac{4 \times 3}{\frac{12!}{10!\,2!}}$$

$$P\left(A/E_3\right) = \frac{4 \times 3 \times 2}{12 \times 11} = \frac{2}{11}$$

Using Baye's theorm

$$P\left(E_1/A\right) = \frac{P(E_1)\,P(A/E_1)}{P(A)}$$

where $P(A) = P(E_1)\,P(A/E_1) + P(E_2)\,P(A/E_2) + P(E_3)\,P(A/E_3)$

$$P(A) = \frac{1}{3}\cdot\frac{1}{5} + \frac{1}{3}\cdot\frac{1}{3} + \frac{1}{3}\cdot\frac{2}{11}$$

$$= \frac{11 + 55 + 10}{3 \times 5 \times 11} = \frac{76}{165}$$

$$P\left(E_1/A\right) = \frac{1/3 \cdot 1/5}{\frac{76}{3 \times 5 \times 11}} =$$

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Q In answering a question on Multiple choice test a student either know the ans. or he guesses. Let p be prob. that he know the ans. and 1-p prob. that he guesses. Assume that a student who guess at ans. will be correct with prob. $1/5$. Where 5 is no. multiple choice alternative. What is prob. that student know the answer to question given he answered it correctly.

Ans E₁ : Student knows the right ans.
E_2 : Student guess the ans.
A : Student get correct ans.

$P(E_1) = p$ $\quad$ $P(E_2) = 1-p$ $\quad$ $P\left(\frac{A}{E_2}\right) = \frac{1}{5}$ $\quad$ $P\left(\frac{A}{E_1}\right) = 1$

sure event

$P(E_1/A) = ?$

Using Baye's Theorem:

$$P(E_1/A) = \frac{P(E_1)\,P(A/E_1)}{P(E_1)\,P(A/E_1) + P(E_2)\,P(A/E_2)}$$

$$= \frac{p}{p + (1-p)\frac{1}{5}} = \frac{5p}{4p+1}.$$

Q. In 2002, there will three candidates of position of principale Mr. Chatterji, Mr. Ayangar, Dr. Singh whose chance of getting the ~~events~~ appointment 4 : 2 : 3. The prob that Mr. Chatterji if selected would introduce co-education in clg is 0.3. The prob of Mr C and Mr. A doing same are 0.5 and 0.8

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What is prob. that there will be coeducation in clg in 2003.

b) If there is coeducation in clg in 2003. what is prob. that Dr. Singh is principle.

Ans

E_1 : Chatterji is selected.
E_2 : Mr Ayangar "
E_3 : Dr. Singh "

A : Introduction of coed in clg is 2003

$P(E_1) = \frac{4}{9}$, $P(E_2) = \frac{2}{9}$ $P(E_3) = \frac{3}{9}$.

$$P(A) = P(E_1)P(A/E_1) + P(E_2)P(A/E_2) + P(E_3)P(A/E_3)$$
$$= \frac{4}{9} \times 0.3 + \frac{2}{9} \times 0.5 + \frac{3}{9} \times 0.8$$
$$= \frac{1.2 + 1 + 2.4}{9} = 0.51 = \frac{23}{45}$$

$$P(E_3/A) = \frac{P(E_3)\,P(A/E_3)}{\cancel{0.51}\ \frac{4.6}{9}}$$
$$= \frac{\frac{3}{9} \times 0.8}{\frac{4.6}{9}} = \frac{2.4}{4.6} = 0.52 = \frac{12}{23}.$$