STANDARD DEVIATION

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FLOW OF PRESENTATION

- Introduction To Significance Of Standard Deviation
- Computation Of Standard Deviation – Ungrouped Mean
  - ❑ By Taking Deviations From Ungrouped Mean
  - ❑ By Taking Deviation From Grouped Mean
- Computation Of Standard Deviation – Grouped Mean
  - ❑ By Taking Deviations From Ungrouped Mean
  - ❑ By Taking Deviation From Grouped Mean
- Correcting Incorrect Value Of Standard Deviation
- Merits And Limitations Of Standard Deviation.

STANDARD DEVIATION

- **Definition:** Standard deviation is the measure of dispersion of a set of data from its mean. It measures the absolute variability of a distribution; the higher the dispersion or variability, the greater is the standard deviation and greater will be the magnitude of the deviation of the value from their mean.

$$\sigma = \sqrt{\frac{\sum(x-\bar{x})^2}{N}}$$

STANDARD DEVIATION

- Concept was introduced by Karl Pearson in 1893.
- Important and widely used measure of dispersion.
- It is free from those defects which afflicted other methods and satisfies most of the properties of a good measure of dispersion.
- Also known as root-mean square deviation as it is the square root of means of the squared deviations from the arithmetic mean.

SIGNIFICANCE OF STANDARD DEVIATION

- Used to measure risks involved in an investment instrument.
- For investors it is a mathematical basis for decisions to be made regarding their investment in financial market.
- Used in deals involving stocks, mutual funds, ETFs and others.
- Standard Deviation is also known as volatility because it provides a sense of how dispersed the data in a sample is from the mean.

Calculation of Standard Deviation

❖ Ungrouped Data

1. Actual Mean
2. Assumed Mean

❖ Grouped Data

1. Actual Data
2. Assumed Data

Computation of Standard Deviation - Ungrouped Mean

- When deviation is taken from the actual mean, the following is applied:

$$\sigma = \sqrt{\frac{\sum(x-\bar{x})^2}{N}}$$

- When deviation is taken from the assumed mean, the following is applied:

$$\sigma = \sqrt{\frac{\sum d^2}{N}} - \sqrt{\left(\frac{\sum d}{N}\right)^2}$$

$$D = X - A$$

EXAMPLE:

Find the standard deviation from the weekly wages of labour working in the factory:

workers	**A**	**B**	**C**	**D**	**E**	**F**	**G**	**H**	**I**	**J**
Weekly wages	1320	1310	1315	1322	1326	1340	1325	1321	1320	1331

Solution:
Method-I: Actual Mean Method

Workers	Weekly wages	$(x - \bar{x})$	$(x - \bar{x})^2$
A	1320	-3	9
B	1310	-13	169
C	1315	-8	64
D	1322	-1	1
E	1326	+3	9
F	1340	+17	289
G	1325	+2	4
H	1321	-2	4
I	1320	-3	9
J	1331	+8	64
N = 10	**$\sum X = 13230$**	**$\sum(X-) = 0$**	**$\sum(X-)^2 = 622$**

$\bar{X} = \sum x/N = 13230/10 = 1323$

$\sigma = \sqrt{\sum(X-\bar{X})^2}$

$\sigma = \sqrt{622/10} = 7.89$

Method-II: Assumed Mean Method

Workers	Weekly wages	*(X-A)* *A = 1310*	d^2
A	1320	10	100
B	1310	0	00
C	1315	5	25
D	1322	12	144
E	1326	16	256

F	**1340**	**30**	**900**
G	1325	15	225
H	1321	11	121
I	1320	10	100
J	1331	21	441
N = 10	**∑X = 13230**	**130**	**2312**

$$\sigma = \sqrt{((\Sigma d^2)/N)} - \sqrt{(((\Sigma d)/N))^2}$$
$$= \sqrt{(2312/10)} - \sqrt{((130/10)\ )^2}$$
$$= \sqrt{(231.2-169)}$$
$$= \sqrt{62.2} = 7.89$$

Thus the answer is same in both.

NOTE: Assumed mean should be preferred when actual mean is not a whole number, because it simplifies calculations.

Computation of Standard Deviation - Grouped Mean

- When deviation is taken from the actual mean, the following is applied:

$$\sigma = \sqrt{\frac{\Sigma f X^2}{N} - \left(\frac{\Sigma f X}{N}\right)^2}$$

- When deviation is taken from the assumed mean, the following is applied.

$$\sigma = \sqrt{\frac{\Sigma f d^2}{N} - \left(\frac{\Sigma f d}{N}\right)^2}$$

Example

- Calculate the standard deviation from the following distribution of marks by using all the methods.

Marks	No. of Students
1–3	40
3–5	30
5–7	20
7–9	10

SOLUTION:

Method-I: Actual Mean Method

Marks	f	X	fX	fX^2
1–3	40	2	80	160
3–5	30	4	120	480
5–7	20	6	120	720
7–9	10	8	80	640
Total	100		400	2000

$$\sigma = \sqrt{\frac{\sum fX^2}{N} - \left(\frac{\sum fX}{N}\right)^2}$$

= √(2000/(100)-((400)/100)^2)

= √(20-16)

= √4

= 2 marks

Method-II: Taking assumed mean as 2

Marks	f	X	$D=(X-2)$	fD	fD^2
1–3	40	2	0	0	0
3–5	30	4	2	60	120
5–7	20	6	4	80	320
7–9	10	8	6	60	160
Total	100			200	800

$$\sigma = \sqrt{\frac{\sum fd^2}{N} - \left(\frac{\sum fd}{N}\right)^2}$$

$\sigma = \sqrt{(800/(100)-((200)/100)^2)}$

$= \sqrt{(8-4)}$

$= \sqrt{4}$

$= 2$ marks

Correcting Incorrect Value of Standard Deviation

- While calculating the Mean and Standard Deviation , some values may enter wrong during tabulation.

- For example sometimes 15 may be misread as 51 or 159 may be misread as 59, and we may have used these wrong values in the calculation.

- Two ways to correct the error i.e
    - Either to make all calculation afresh, which is very tedious task
    - Correcting the value by adjustments and readjustments of right hand side figures, is easy and less time consuming as compare to the former method.

Merits of standard deviation

- Standard deviation is well defined.
- Its value is always definite
- It is based on all the observation of the data.
- It gives a more accurate idea of how the data is distributed
- It is less affected by the fluctuations of sampling than most other measures of dispersion.
- The squaring of deviations makes them positive and the difficulty about algebraic signs which was experienced in case of mean deviation is not found here.

Uses Of Standard Deviation

- Despite the drawbacks mentioned above the standard deviation is the best measure of dispersion and should be used wherever possible. Just as mean is the best measure of central tendency (leaving exceptional cases) standard deviation is the best measure of dispersion, excepting a few cases where mean deviation or quartile deviation may give better results.
- However since standard deviation gives greater weight to extreme items, it does not find much favor with economists and businessmen who are more interested in the results of the model class.

Demerits of Standard Deviation

- Standard deviation is not easy to calculate, nor is it easily understood. In any case it is more cumbersome in its calculation than either quartile deviation or mean deviation.
- It gives more weight to extreme items and less to those which are near the mean, because the squares of the deviations, which are big in size, would be proportionately greater than the squares of those deviations which are comparatively small. Thus, deviation 2 and 8 are in the ratio of 1:4 but their square i.e, 4 and 64 would be in the ratio of 1 : 16.

THANK YOU