Unit III

Measures of Dispersion: Range, Average Deviation, Standard Deviation, Variance and Coefficient of Variation

DISPERSION:

- Dispersion (also known as Scatter, Spread or Variation) measures the extent to which the items vary from some central value.
- It indicates the scattering of data.

RANGE:

- Range is defined as the difference between the value of largest item and the value of the smallest item included in the distribution.
- Measures of Range may be absolute or relative.

FOR INDIVIDUAL OBSERVATIONS OR DISCRETE SERIES:

1. **Absolute Measure of Range:**

$$R = L - S$$

where L = Largest Value
S = Smallest Value

2. **Relative Measure of Range:**

$$\text{Coefficient of Range} = \frac{L - S}{L + S}$$

Calculate Range and Coefficient of Range:

Roll No.	Marks
1	5
2	15
3	25
4	35
5	45
6	55

Range = L – S

= 55 – 5

= 50

Coefficient of Range = $\frac{L-S}{L+S}$

$= \frac{55-5}{55+5}$

= 0.833

Calculate Range and Coefficient of Range:

Marks	No. of students
5	10
15	20
25	30
35	50
45	40
55	30

Range = L – S

= 55 – 5

= 50

Coefficient of Range = $\frac{\boldsymbol{L-S}}{\boldsymbol{L+S}}$

$= \frac{\mathbf{55-5}}{\mathbf{55+5}}$

= 0.833

For Continuous Series:

1. **Absolute Measure of Range:**

(a) Method I: **R =** $\boldsymbol{U_L - L_S}$

Where $\boldsymbol{U_L =}$ Upper limit of Largest value class

$\boldsymbol{L_S}$ = Lowest limit of Smallest value class

(a) Method II: **R =** $\boldsymbol{M_L - M_S}$

$\boldsymbol{M_L =}$ Mid- point of Largest value class

$\boldsymbol{M_S}$ = Mid-point of Smallest value class

2. **Relative Measure of Range:**

(a) Method I: Coefficient of Range= $\frac{U_L - L_S}{U_L + L_S}$

(b) Method II: Coefficient of Range= $\frac{M_L - M_S}{M + M_S}$

Marks	No. of students	Mid-Point
0-10	10	5
10-20	20	15
20-30	30	25
30-40	50	35
40-50	40	45
50-60	30	55

1. **Absolute Measure of Range:**

(a) Method I: **R** = $60 - 0$

$= 60$

(a) Method II: **R** = $55 - 5$

$= 50$

2. **Relative Measure of Range:**

(a) Method I: Coefficient of Range= $\frac{U_L - L_S}{U_L + L_S}$

$$= \frac{60 - 0}{60 + 0}$$

$$= 1$$

(b) Method II: Coefficient of Range = $\frac{M_L - M_S}{M + M_S}$

$$= \frac{55 - 5}{55 + 5}$$

$$= 0.833$$

Mean Deviation:

- Mean or Average Deviation is the arithmetic Mean of the absolute deviations of all items of the distribution from a measure of central tendency which could be mean or median or sometimes even mode.
- Measures of Mean Deviation may be absolute or relative.

1. Absolute Measure of Mean Deviation

FOR INDIVIDUAL OBSERVATIONS:

$$M.D. = \frac{\sum|D|}{N}$$

Where, M.D. = Mean Deviation

$\sum|D|$ = Deviations of Items from Mean or Median ignoring $\pm$ signs

N = Number of Observations

FOR DISCRETE OBSERVATIONS:

$$\text{M.D.} = \frac{\sum f|D|}{N}$$

Where, M.D. = Mean Deviation

$|D|$= Deviations of Items from Mean or Median ignoring $\pm$ signs

N = Number of total frequencies

$\sum f|D|$ = Total of Product of Deviations and respective frequencies

FOR CONTINUOUS SERIES:

M.D. = $\frac{\sum f|D|}{N}$

Where, M.D. = Mean Deviation

$|D|$= Deviations of Mid-Points of classes from Mean or Median ignoring $\pm$ signs

N = Number of total frequencies

$\sum f|D|$ = Total of Product of Deviations and respective frequencies

II. Relative Measure of Mean Deviation:

(a) Coefficient of M.D. about Mean = $\frac{M.D. about\ Mean}{Mean}$

(b) Coefficient of M.D. about Median = $\frac{M.D. about\ Median}{Median}$

(c) Coefficient of M.D. about Mode = $\frac{M.D. about\ Mode}{Mode}$

Calculate Mean Deviation and Coefficient of Mean Deviation from Median.

Roll No.	Marks
1	25
2	5
3	55
4	45
5	15
6	35

Marks	Deviations from Median Ignoring signs ($\|D\|$ = X - 30)
5	25
15	15
25	5
35	5
45	15
55	25
N = 6	$\sum\|D\|$= 90

STEP 1: Arrange the size of item in ascending order.

STEP 2: $\frac{N+1}{2}$th item

$= \frac{6+1}{2}$

$= 3.5^{th}$ item

STEP 3: Median = Size of 3.5^{th} item = (3^{rd} + 4^{th} item)/2

= (25+35)/2

= 30

STEP 4: Mean Deviation (M.D.) = $\frac{\sum|D|}{N}$

= 90/6

= 15

Coefficient of M.D. about Median $= \frac{M.D. about\ Median}{Median}$

$= \frac{15}{30}$

$= 0.5$

Calculate Mean Deviation and Coefficient of Mean Deviation from Mean.

Marks
10
20
30
40
50

Marks	**Deviations from Mean Ignoring signs ($\|D\|$ = X - 30)**
10	20
20	10
30	0
40	10
50	20
N = 5	$\sum\|D\|= 60$

Mean

$$\bar{X} = \frac{\sum X}{N}$$

$$= \frac{150}{5}$$

$$= 30$$

$$\text{M.D. from Mean} = \frac{\sum|D|}{N}$$

$$= \frac{60}{5}$$

$$= 12$$

Coefficient of M.D. about Mean $= \frac{M.D. about\ Mean}{Mean}$

$$= \frac{12}{30}$$

$$= 0.4$$

From the following data, calculate Mean Deviation and coefficient of Mean Deviation

Marks	No. of students
5	10
15	20
25	30
35	50
45	40
55	30

Marks	No. of students	Cf
5	10	10
15	20	30
25	30	60
35	50	110
45	40	150
55	30	180

STEP 1: Arrange the size of item in ascending order.

STEP 2: $\frac{N+1}{2}$th item

$= \frac{180+1}{2}$

$= 90.5^{th}$ item

STEP 3: Median = Size of 90.5^{th} item

= 35

Marks	Deviation from Median (X-35)	No. of students	f$\lvert D \rvert$
5	30	10	300
15	20	20	400
25	10	30	300
35	0	50	0
45	10	40	400
55	20	30	600
		180	$\sum f\lvert D \rvert$=2000

STEP 4: Mean Deviation (M.D.) = $\frac{\sum f\lvert D \rvert}{N}$

$=\frac{2000}{180}$

$= 11.11$

Coefficient of M.D. about Median = $\frac{M.D.about\ Median}{Median}$

$= \frac{11.11}{35}$

$= 0.3175$

Marks	No. of Students
0-10	10
10-20	20
20-30	30
30-40	50
40-50	40
50-60	30

Marks	No. of Students	No. of Students (cf)
0-10	10	10
10-20	20	30
20-30	30	60
30-40	50	110
40-50	40	150
50-60	30	180

$\left(\frac{N}{2}\right)$ ***th term*** **=180/2**

= 90th term

Cumulative frequency which includes 90th term = 110

Class corresponding to 110 is 30-40

Median = L+ $\left[\frac{\frac{N}{2} - cf}{f}\right] * i$

= 30+ $\left[\frac{\frac{180}{2} - 60}{50}\right] * 10$

=30+ (30*10) / 50

= 30+6

= 36

Marks	Mid-points	Deviation from Median (X-36)	No. of Students	f$\|D\|$
0-10	5	31	10	310
10-20	15	21	20	420
20-30	25	11	30	330
30-40	35	1	50	50
40-50	45	9	40	360
50-60	55	19	30	570
			N= 180	$\sum f\|D\|$=2040

Mean Deviation (M.D.) = $\frac{\sum f|D|}{N}$

$= \frac{2040}{180}$

$= 11.33$

Coefficient of M.D. about Median = $\frac{M.D. about\ Median}{Median}$

$= \frac{11.33}{36}$

$= 0.3148$

STANDARD DEVIATION:

- Standard Deviation is the square root of the arithmetic mean of the squares of deviations of all items of the distribution from arithmetic mean.
- Measures of Standard Deviation may be absolute or relative.

Absolute Measure of Standard Deviation:

i. For individual observation:

Actual Mean Method	Assumed Mean Method
$\sigma = \sqrt{\frac{\sum x^2}{N}}$ Where, σ = Standard Deviation X = (X-$\bar{X}$) i.e. deviations taken from Actual mean x^2 = Squares of deviation N = Total of observations	$\sigma = \sqrt{\frac{\sum d^2}{N} - \left(\frac{\sum d}{N}\right)^2}$ Where, σ = Standard Deviation d = (X-A) i.e. deviations taken from Assumed mean d^2 = Squares of deviation N = Total of observations

ii. FOR DISCRETE SERIES:

Actual Mean Method	Assumed Mean Method	Step Deviation Method
$\sigma = \sqrt{\frac{\sum fx^2}{N}}$ Where, σ = Standard Deviation X = (X-$\bar{X}$) i.e. deviations taken from Actual mean x^2 = Squares of deviation fx^2 = Product of square of deviation and respective frequency N = Total of observations	$\sigma = \sqrt{\frac{\sum fd^2}{N} - \left(\frac{\sum fd}{N}\right)^2}$ Where, σ = Standard Deviation d = (X-A) i.e. deviations taken from Assumed mean d^2 = Squares of deviation fd^2 = Product of square of deviation and respective frequency N = Total of observations	$\sigma = \sqrt{\frac{\sum fd^2}{N} - \left(\frac{\sum fd}{N}\right)^2} * c$ Where, σ = Standard Deviation d = $\frac{(X-A)}{c}$ i.e. deviations taken from Assumed mean and divided by common factor (c) d^2 = Squares of deviation fd^2 = Product of square of deviation and respective frequency N = Total of observations

CONTINUOUS SERIES:

Actual Mean Method	Assumed Mean Method	Step Deviation Method
$\sigma = \sqrt{\frac{\sum fx^2}{N}}$ Where, σ = Standard Deviation x = (m-$\bar{X}$) i.e. deviations taken from Actual mean x^2 = Squares of deviation fx^2 = Product of square of deviation and respective frequency N = Total of observations	$\sigma = \sqrt{\frac{\sum fd^2}{N} - \left(\frac{\sum fd}{N}\right)^2}$ Where, σ = Standard Deviation d = (m-A) i.e. deviations taken from Assumed mean d^2 = Squares of deviation fd^2 = Product of square of deviation and respective frequency N = Total of observations	$\sigma = \sqrt{\frac{\sum fd^2}{N} - \left(\frac{\sum fd}{N}\right)^2} * c$ Where, σ = Standard Deviation d = $\frac{(m-A)}{c}$ i.e. deviations taken from Assumed mean and divided by common factor (c) d^2 = Squares of deviation fd^2 = Product of square of deviation and respective frequency N = Total of observations

Relative Measure of Standard Deviation :

Coefficient of Variation = $\frac{Standard\ Deviation}{Arithmetic\ Mean} * 100$

VARIANCE:

- Variance is the arithmetic mean of the squares of deviations of all items of the distributions from arithmetic mean.

OR

- Variance is the square of the standard deviation

OR

- Standard Deviation is the square root of the variance

- Variance = σ^2

OR

- $\sigma = \sqrt{Variance}$

- **INTERPRETATION OF VARIANCE:**

Smaller the value of Variance	**Lesser the variability or greater the uniformity in the population**
Larger the value of Variance	Greater the variability or lesser the uniformity in the population

From the following data, calculate Standard Deviation, Coefficient of Variation and Variance.

Roll No.	**Marks**
1	5
2	15
3	25
4	35
5	45
6	55

Roll No.	Marks
1	5
2	15
3	25
4	35
5	45
6	55
N = 6	$\sum X$ = 180

$$\text{Mean} = \frac{\sum X}{N}$$

$$= \frac{180}{6}$$

$$= 30$$

Roll No.	Marks	x i.e. (X-30)	x^2
1	5	-25	625
2	15	-15	225
3	25	-5	25
4	35	5	25
5	45	15	225
6	55	25	625
N = 6	$\sum X = 180$	0	$\sum x^2 = 1750$

Standard Deviation	Coefficient of Variance	Variance
$= \sqrt{\frac{\sum x^2}{N}}$	$= \frac{\sigma}{Mean} * 100$	$= \sigma^2$
$= \sqrt{\frac{1750}{6}}$	$= \frac{17.078}{30} * 100$	$= (17.078)^2$
= 17.078	= 56.928%	= 291.667

From the following data, calculate Standard Deviation, Coefficient of Variation and Variance.

Marks	No. of students
5	10
15	20
25	30
35	50
45	40
55	30

Marks	No. of students	Fx
5	10	50
15	20	300
25	30	750
35	50	1750
45	40	1800
55	30	1650
	N = 180	$\sum fX = 6300$

$$\text{Mean} = \frac{\sum fX}{N}$$

$$= \frac{6300}{180}$$

$$= 35$$

Marks	No. of students	x i.e. (X-35)	x^2	fx^2
5	10	-30	900	9000
15	20	-20	400	8000
25	30	-10	100	3000
35	50	0	0	0
45	40	10	100	4000
55	30	20	400	12000
	N = 180			$\sum fx^2 = 36000$

Standard Deviation	Coefficient of Variance	Variance
$\sigma = \sqrt{\frac{\sum fx^2}{N}}$	$= \frac{\sigma}{Mean} * 100$	$= \sigma^2$
$= \sqrt{\frac{36000}{180}}$	$= \frac{14.142}{35} * 100$	$= (14.142)^2$
$= \sqrt{200}$	= 40.406%	= 200
= 14.142		

From the following data, calculate Standard Deviation, Coefficient of Variation and Variance.

Marks	No. of students
0-10	10
10-20	20
20-30	30
30-40	50
40-50	40
50-60	30

Marks	M	No. of students	D i.e, (X-40) (A=40)	d^2	fd	fd^2
0-10	5	10	-35	1225	-350	12250
10-20	15	20	-25	625	-500	12500
20-30	25	30	-15	225	-450	6750
30-40	35	50	-5	25	-250	1250
40-50	45	40	5	25	200	1000
50-60	55	30	15	225	450	6750
		N = 180	-60		-900	40500

Mean	Standard Deviation	Coefficient of Variance	Variance
$A+\frac{\sum fd}{N}$	$\sigma = \sqrt{\frac{\sum fd^2}{N} - \left(\frac{\sum fd}{N}\right)^2}$	$= \frac{\sigma}{Mean} * 100$	$= \sigma^2$
$40+\frac{-900}{180}$	$= \sqrt{\frac{40500}{180} - \left(\frac{-900}{180}\right)^2}$	$= \frac{14.142}{35} * 100$	$= (14.142)^2$
35	$= \sqrt{225 - 25}$	= 40.406%	= 200
	= 14.142		

Calculate Coefficient of Variation if Median is 23, Mode is 29, Variance is 100

Coefficient of Variation = $\frac{\sigma}{Mean}$*100

Standard Deviation = $\sqrt{Variance}$

= $\sqrt{100}$

= 10

Mode = 3 Median – 2 Mean

29 = 3*23 – 2*Mean

2* Mean = 69 – 29

Mean = 40/2

Mean = 20

Coefficient of Variation = $\frac{10}{20}$*100

= 50%