Unit IV

Forecasting Techniques: Simple Correlation & Regression Analysis, Time Series Analysis-Trend Analysis, Cyclical Analysis, Seasonal Analysis, Irregular Variation.

Correlation:

- Correlation is the relationship that exists between two or more variables.
- If two variables are related to each other in such a way that change in one creates a corresponding change in the other, then the variables are said to be correlated.
- Some of the examples of such relationship are as follows:

1. Relationship between the heights and weights.
2. Relationship between the quantum of rainfall and the yield of wheat.
3. Relationship between the advertising expenditure and sales.

Correlation Analysis:

- It is a statistical technique used to measure the degree and direction of relationship between the variables.

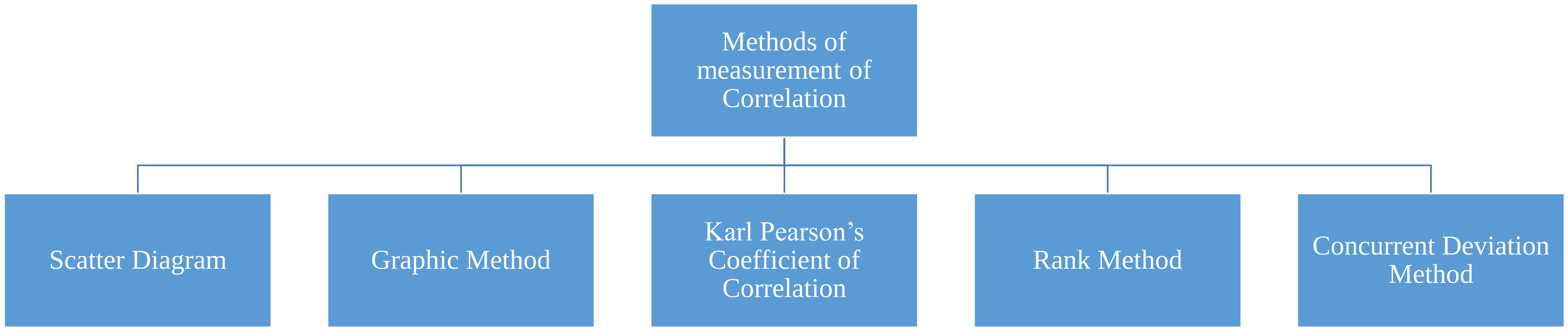
- **TYPES OF CORRELATION ANALYSIS:**

1. **Positive Correlation** (If both the variables vary in the same direction, correlation is said to be positive in other words, One variable increases, the other also increases and vice versa) **and Negative Correlation**(If both the variables vary in opposite direction, the correlation is said to be negative, in other words of one variable increases, the other variable decreases.)

2. **Simple Correlation** (When only two variables are studied) **and Multiple Correlation** (When three or more variables are studied).

3. **Partial Multiple Correlation** (When three or more variables are studied but considers only two variables to be influencing each other and the effect of other influencing variables being held constant) **and Total Multiple Correlation**(When three or more variables are studied without excluding the effect of any variable held constant).

4. **Linear Correlation** (If the amount of change in one variable bears a constant ratio to the amount of change in the other variable) **and Non- Linear Correlation** (It is also known as Curvilinear. If the amount of change in one variable does not bear a constant ratio to the amount of change in the other variable).


Methods of measurement of Correlation
Scatter Diagram
Graphic Method
Karl Pearson’s Coefficient of Correlation
Rank Method
Concurrent Deviation Method

Karl Pearson’s Coefficient of Correlation:

Calculation of Correlation when deviations are taken from Actual Mean

STEP 1: Calculate deviations from the actual mean of X series and denote these deviations by *x*.

STEP 2: Square these deviations and obtain the total i.e. $\sum x^2$.

STEP 3: Calculate the deviations from the actual mean of Y series and denote these deviations by *y*.

STEP 4: Square these deviations and obtain the total i.e. $\sum y^2$.

STEP 5: Multiply the deviation of each variable of X series by the respective deviation of each variable of Y series and obtain the total i.e. $\sum xy$.

STEP 6: Calculate Coefficient of Correlation as follows:

$$\mathbf{r} = \frac{\sum xy}{\sqrt{\sum x^2 \sum y^2}} \qquad \text{OR} \qquad \mathbf{r} = \frac{\sum xy}{N\sigma_x\sigma_y}$$

r = +1	Perfect Positive Correlation	r = -1	Perfect Negative Correlation
$0.75 \leq r < 1$	High Positive Correlation	$-0.75 \geq r > -1$	High Negative Correlation
$0.5 \leq r < 0.75$	Moderate Positive Correlation	$-0.50 \geq r > -0.75$	Moderate Negative Correlation
$0 < r < 0.5$	Low Positive Correlation	$0 > r > -0.50$	Low Negative Correlation
r = 0	No Correlation	r = 0	No Correlation

Calculate Coefficient of Correlation:

X	Y
1	10
2	20
3	30
4	50
5	40

X	x i.e.($X-\bar{X}$)	x^2	Y	y i.e.($Y-\bar{Y}$)	y^2	xy
1	-2	4	10	-20	400	40
2	-1	1	20	-10	100	10
3	0	0	30	0	0	0
4	1	1	50	20	400	20
5	2	4	40	10	100	20
$\sum X = 15$	$\sum x = 0$	$\sum x^2 = 10$	$\sum Y = 150$	$\sum y = 0$	$\sum y^2 = 1000$	$\sum xy = 90$

Mean $(\bar{X})$	**Mean** $(\bar{Y})$
$= \frac{\sum X}{N}$	$= \frac{\sum Y}{N}$
$= \frac{15}{5}$	$= \frac{150}{5}$
= 3	= 30

Calculate Coefficient of Correlation as follows:

$$\mathbf{r} = \frac{\sum xy}{\sqrt{\sum x^2 \sum y^2}}$$

$$\mathbf{r} = \frac{\mathbf{90}}{\sqrt{\mathbf{10*1000}}}$$

= 90/ 100 i.e. +0.9 (There exists high positive correlation)

Calculation of Correlation when deviations are taken from Assumed Mean

STEP 1: Calculate deviations from the assumed mean of X series and denote these deviations by d_x.

STEP 2: Square these deviations and obtain the total i.e. $\sum d_x^2$.

STEP 3: Calculate the deviations from the assumed mean of Y series and denote these deviations by d_y.

STEP 4: Square these deviations and obtain the total i.e.$\sum d_y^2$.

STEP 5: Multiply the deviation of each variable of X series by the respective deviation of each variable of Y series and obtain the total i.e. $\sum d_x d_y$.

STEP 6: Calculate Coefficient of Correlation as follows:

$$\mathbf{r} = \frac{\sum d_x d_y - \frac{\sum d_x \sum d_y}{\boldsymbol{N}}}{\sqrt{\sum d_x^2 - \frac{\left(\sum d_x\right)^2}{\boldsymbol{N}}} * \sqrt{\sum d_y^2 - \frac{\left(\sum d_y\right)^2}{\boldsymbol{N}}}}$$

Calculate Coefficient of Correlation:

X	Y
1	10
2	20
3	30
4	50
5	40

X	d_x i.e.(X- 4)	d_x^2	Y	d_y i.e.(Y-40)	d_y^2	$d_x d_y$.
1	-3	9	10	-30	900	90
2	-2	4	20	-20	400	40
3	-1	1	30	-10	100	10
4	0	0	50	10	100	0
5	1	1	40	0	0	0
$\sum X$= 15	$\sum d_x$ = -5	$\sum d_x^2 = 15$	$\sum Y = 150$	$\sum d_y$= -50	$\sum d_y^2 = 1500$	$\sum d_x d_y = 140$

Calculate Coefficient of Correlation as follows:**r** = $\frac{\sum d_x d_y - \frac{\sum d_x \sum d_y}{N}}{\sqrt{\sum d_x^2 - \frac{(\sum d_x)^2}{N}} * \sqrt{\sum d_y^2 - \frac{(\sum d_y)^2}{N}}}$

$$= \frac{\mathbf{140} - \frac{-5 * -50}{5}}{\sqrt{\mathbf{15} - \frac{(-5)^2}{5}} * \sqrt{\mathbf{1500} - \frac{(-50)^2}{5}}}$$

$$= \frac{\mathbf{140 - (50)}}{\sqrt{\mathbf{15 - 5}} * \sqrt{\mathbf{1500 - 500}}}$$

$$= \frac{\mathbf{90}}{\sqrt{\mathbf{10}} * \sqrt{\mathbf{1000}}}$$

= 90/ 100 i.e. +0.9 (There exists high positive correlation)

Calculation of Correlation when no deviations are taken

STEP 1: Obtain the total of values of variable of X series i.e. $\sum X$.

STEP 2: Square the value of each variable of X series, denote these squared values by X^2 and in the total i.e. $\sum X^2$**.**

STEP 3: Obtain the total of values of variable of Yseries i.e. $\sum Y$.

STEP 4: Square the value of each variable of Y series, denote these squared values by Y^2 and in the total i.e. $\sum Y^2$**.**

STEP 5: Multiply the value of each variable of X series by the respective value of each variable of Y series and obtain the total i.e. $\sum XY$.

STEP 6: Calculate Coefficient of Correlation as follows:

$$\mathbf{r} = \frac{\sum XY - \frac{\sum X \sum Y.}{\boldsymbol{N}}}{\sqrt{\sum X^2 - \frac{(\sum X)2}{\boldsymbol{N}}} * \sqrt{\sum Y^2 - \frac{(\sum Y)2}{\boldsymbol{N}}}}$$

Calculate Coefficient of Correlation:

X	Y
1	10
2	20
3	30
4	50
5	40

X	X^2	Y	Y^2	XY
1	1	10	100	10
2	4	20	400	40
3	9	30	900	90
4	16	50	2500	200
5	25	40	1600	200
$\sum X$= 15	$\sum X^2$ = 55	$\sum Y$ = 150	$\sum Y^2$= 5500	$\sum X$Y = 540

Calculate Coefficient of Correlation as follows:**r** = $\dfrac{\sum XY - \frac{\sum X \sum Y.}{N}}{\sqrt{\sum X^2 - \frac{(\sum X)2}{N}} * \sqrt{\sum Y^2 - \frac{(\sum Y)2}{N}}}$

$$= \frac{540 - \frac{15 * 150}{5}}{\sqrt{55 - \frac{(15)^2}{5}} * \sqrt{5500 - \frac{(150)^2}{5}}}$$

$$= \frac{540 - 450}{\sqrt{55 - 45} * \sqrt{5500 - 4500}}$$

$$= \frac{90}{\sqrt{10} * \sqrt{1000}}$$

= 90/ 100 i.e. +0.9 (There exists high positive correlation)

SPEARKMAN'S RANK CORRELATION:

- Spearman's Rank Correlation uses rank rather than actual observation and makes no assumption about the population from which actual observations are drawn.
- The correlation coefficient between two series of ranks is called 'Rank Correlation Coefficient'
- It is given by the formula:

$$R = 1 - \frac{6\sum D^2}{N^3 - N}$$

Where, R = Rank correlation coefficient

D = Difference of the ranks between paired items in two series

N = Number of pairs of ranks

- In case of Tied Ranks- In case there is more than one item with same value in the series, usually average rank is allotted to each of these items and the factor $\frac{m^3 - m}{12}$ is added for each such tied item to $\sum D^2$. Thus, in case of tied ranks, the modified formula for rank correlation coefficient becomes:

$$R = 1 - \frac{6\left[\sum D^2 + \frac{m^3 - m}{12} \ldots\right]}{N^3 - N}$$

M is Number of items whose ranks are common

CALCULATION OF RANK CORRELATION WHEN ACTUAL RANKS ARE GIVEN BUT EQUAL RANKS HAVE NOT BEEN ASSIGNED TO SOME ENTRIES:

STEP 1: Calculate the differences between two ranks i.e. $(R_1 – R_2)$ and denote these differences by D.
STEP 2: Square these differences and obtain the total i.e. $\sum D^2$.
STEP 3: Calculate Rank Correlation as follows:

$$R = 1 - \frac{6 \sum D^2}{N^3 - N}$$

CALCULATION OF RANK CORRELATION WHEN ACTUAL RANKS ARE GIVEN BUT EQUAL RANKS HAVE BEEN ASSIGNED TO SOME ENTRIES:

STEP 1: Calculate the differences between two ranks i.e.(R_1 – R_2) and denote these differences by D.
STEP 2: Square these differences and obtain the total i.e. $\sum D^2$**.**
STEP 3: Calculate Rank Correlation as follows:

$$R = 1 - \frac{6\left[\sum D^2 + \frac{1}{12}(m^3 - m) + \frac{1}{12}(m^3 - m) + \cdots\right]}{N^3 - N}$$

m = Number of items whose ranks are common.
$\frac{1}{12}(m^3 - m)$ is added to the value of $\sum D^2$ for each group of items with common ranks.

Two judges in a beauty contest ranked the entries as follows:

X	Y
1	5
2	4
3	3
4	2
5	1

X (R_1)	Y (R_2)	D = ($R_1 - R_2$)	$D^2 = (R_1 - R_2)^2$
1	5	-4	16
2	4	-2	4
3	3	0	0
4	2	2	4
5	1	4	16
N = 5			$\sum D^2 = 40$

$$R = 1 - \frac{6\sum D^2}{N^3 - N}$$

$$R = 1 - \frac{6 * 40}{5^3 - 5}$$

$$R = 1 - \frac{240}{120}$$

$$= -1$$

i.e. There exists high negative Coefficient of Correlation.

Three judges in a beauty contest ranked the entries as follows:

X	Y	Z
1	5	3
2	4	5
3	3	2
4	2	1
5	1	4

X (R_1)	Y (R_2)	Z (R_3)	$D^2 = (R_1 – R_2)^2$	$D^2 = (R_2 – R_3)^2$	$D^2 = (R_1 – R_3)^2$
1	5	3	16	4	4
2	4	5	4	1	9
3	3	2	0	1	1
4	2	1	4	1	9
5	1	4	16	9	1
N = 5			$\sum D^2 = 40$	$\sum D^2 =16$	$\sum D^2 = 24$

Rank Correlation between the judgements of 1st and 2nd judges:

$$R = 1 - \frac{6\sum D^2}{N^3 - N} = 1 - \frac{6 * 40}{5^3 - 5} = 1 - \frac{240}{120} = 1 – 2 \qquad = -1$$

Rank Correlation between the judgements of 2nd and 3rd judges:

$$R = 1 - \frac{6\sum D^2}{N^3 - N} \qquad = 1 - \frac{6 * 16}{5^3 - 5} \qquad = 1 - \frac{96}{120} \qquad = 1 - 0.8 \qquad = 0.2$$

Rank Correlation between the judgements of 1st and 3rd judges:

$$R = 1 - \frac{6\sum D^2}{N^3 - N} \qquad = 1 - \frac{6 * 24}{5^3 - 5} \qquad = 1 - \frac{144}{120} \qquad = 1 – 1.2 \qquad = -0.2$$

2nd and 3rd judges have the nearest approach to common tastes in beauty since Coefficient of Correlation is maximum in their judgements.

CALCULATION OF RANK CORRELATION WHEN ACTUAL RANKS ARE NOT GIVEN:

STEP 1: Assign the ranks by taking either highest value as 1 or lowest value as 1 in case of both variables. (In case of equal value of two or more variables, assign the average of the ranks e.g. if two are ranked equal at six places, they are given the rank 6.5 i.e. (6+7)/2)

STEP 2: Calculate the difference between two ranks i.e. (R_1 – R_2) and denote these differences by D.

STEP 3: Square these differences and obtain the total i.e. $\sum D^2$.

STEP 4: Calculate Rank Correlation as follows:

$$R = 1 - \frac{6\left[\sum D^2 + \frac{1}{12}(m^3 - m) + \frac{1}{12}(m^3 - m) + \cdots\right]}{N^3 - N}$$

m = Number of items whose ranks are common.

$\frac{1}{12}(m^3 - m)$ is added to the value of $\sum D^2$ for each group of items with common ranks.

Calculate Rank Correlation Coefficient for the following data:

X	Y
59	79
69	69
39	59
49	49
29	39

X	R_1	Y	R_2	$D = (R_1 – R_2)$	$D^2 = (R_1 – R_2)^2$
59	4	79	5	-1	1
69	5	69	4	1	1
39	2	59	3	-1	1
49	3	49	2	1	1
29	1	39	1	0	0
N = 5					$\sum D^2 = 4$

$$R = 1 - \frac{6\sum D^2}{N^3 - N}$$

$$R = 1 - \frac{6 * 4}{5^3 - 5}$$

$$R = 1 - \frac{24}{120}$$

$$= 0.8$$

Calculate Rank Correlation Coefficient for the following data:

X	Y
49	59
69	59
39	59
49	49
29	39

X	R_1	Y	R_2	$D = (R_1 - R_2)$	$D^2 = (R_1 - R_2)^2$
49	3.5	59	4	-0.5	0.25
69	5	59	4	1	1
39	2	59	4	-2	4
49	3.5	49	2	1.5	2.25
29	1	39	1	0	0
N = 5					$\sum D^2 = 7.50$

$$R = 1 - \frac{6\,[\sum D^2 + \frac{1}{12}(m^3 - m) + \frac{1}{12}(m^3 - m) + \cdots]}{N^3 - N}$$

$$R = 1 - \frac{6\,[7.50 + \frac{1}{12}(2^3 - 2) + \frac{1}{12}(3^3 - 3)]}{5^3 - 5}$$

$$R = 1 - \frac{6\,[7.50 + \frac{1}{12}(6) + \frac{1}{12}(24)]}{5^3 - 5}$$

$$R = 1 - \frac{6\,[7.50 + 0.5 + 2]}{120}$$

R = 1 – 0.5

R = 0.5

- Since item 49 is repeated 2 times in Series X, m = 2
- Since item 59 is repeated 3 times in Series Y, m = 3

CONCURRENT DEVIATION METHOD:

- It is based on the direction of change in the two paired variables.
- The correlation coefficient between two series of direction of change is called Coefficient of Concurrent Deviation.
- It is given by the formula:

$$r_c = \pm\sqrt{\pm\frac{2c - n}{n}}$$

r_c = Coefficient of Concurrent Deviation

c = Number of Positive signs after multiplying the direction of change of X series and Y series

n = Number of pairs of observation compared

Steps involved in Concurrent Deviation Method:

STEP 1: Find out the direction of change of X variable by comparing the current value with the previous value and put the sign + (in case increasing) or sign – (in case decreasing) in the column denoted by Dx.

STEP 2: Assign the ranks by taking either highest value as 1 or lowest value as 1 in case of both the variables.

STEP 3: Multiply the signs of column Dx by signs of column Dy and determine the number of positive signs and denote these numbers by ‘c’.

STEP 4: Calculate the concurrent deviation method:

$$r_c = \pm\sqrt{\pm\frac{2c - n}{n}}$$

Calculate Coefficient of Concurrent Deviation from the following data:

X	Y
59	79
69	69
39	59
49	49
29	39

X	Direction of Change Dx	Y	Direction of Change Dy	Dx.Dy
59		79		
69	+	69	-	-
39	-	59	-	+
49	+	49	-	-
29	-	39	-	+
	N = 4			C = 2

$$r_c = \pm\sqrt{\pm\frac{2c - n}{n}}$$

$$r_c = \pm\sqrt{\pm\frac{2 * 4 - 4}{4}}$$

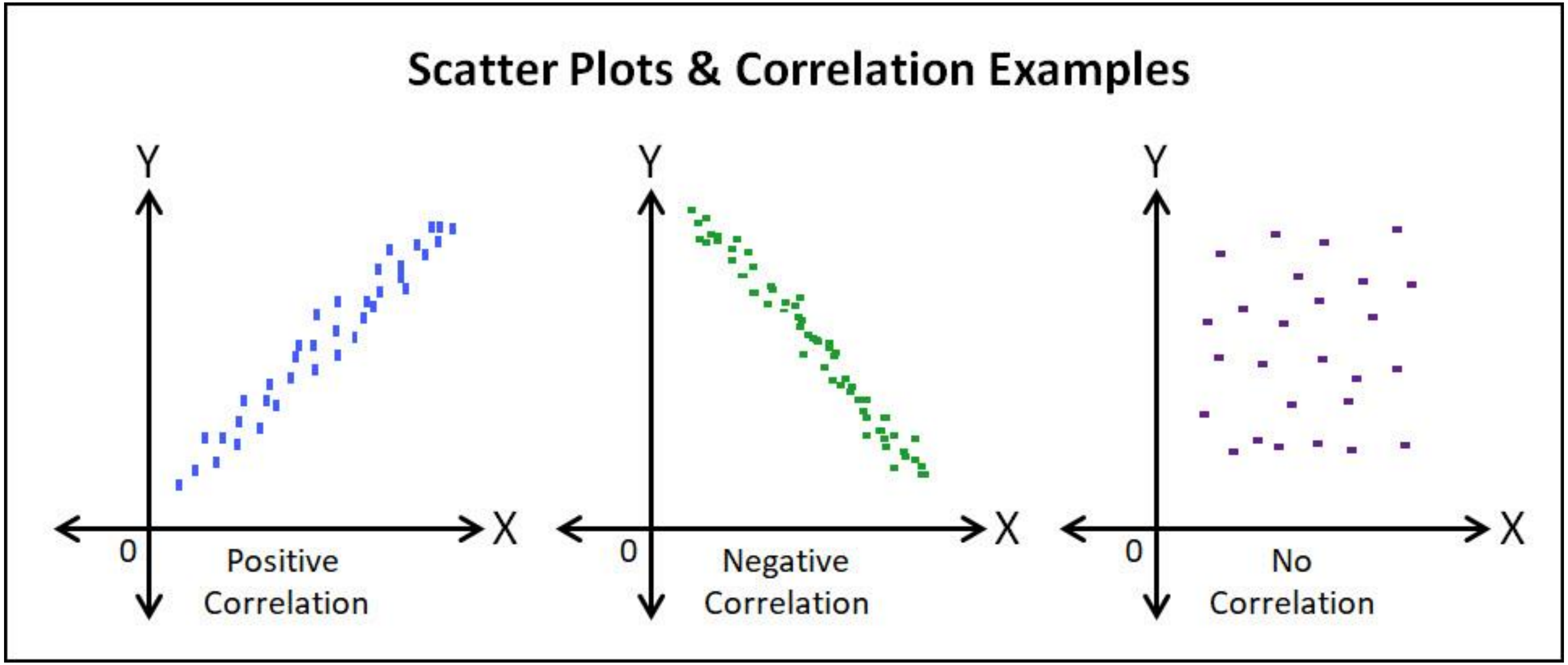
$$r_c = \pm\sqrt{\pm 0}$$

$$r_c = 0$$

SCATTER DIAGRAM :

- It is a diagrammatic representation of bivariate data to ascertain the correlation between two variables.

- STEPS INVOLVED IN THE PREPARATION OF A SCATTER DIAGRAM:
- **STEP 1:** Show one of the variables say X along the horizontal axis OX and the other variable y along the vertical axis OY.
- **STEP 2:** Plot a dot for each pair of X and Y values on a graph paper.
- **STEP 3:** Observe the scatter of the plotted points and form an idea about the degree and direction of correlation.


Scatter Plots & Correlation Examples
Y
X
0
Positive Correlation
Y
X
0
Negative Correlation
Y
X
0
No Correlation

GRAPHIC METHOD:

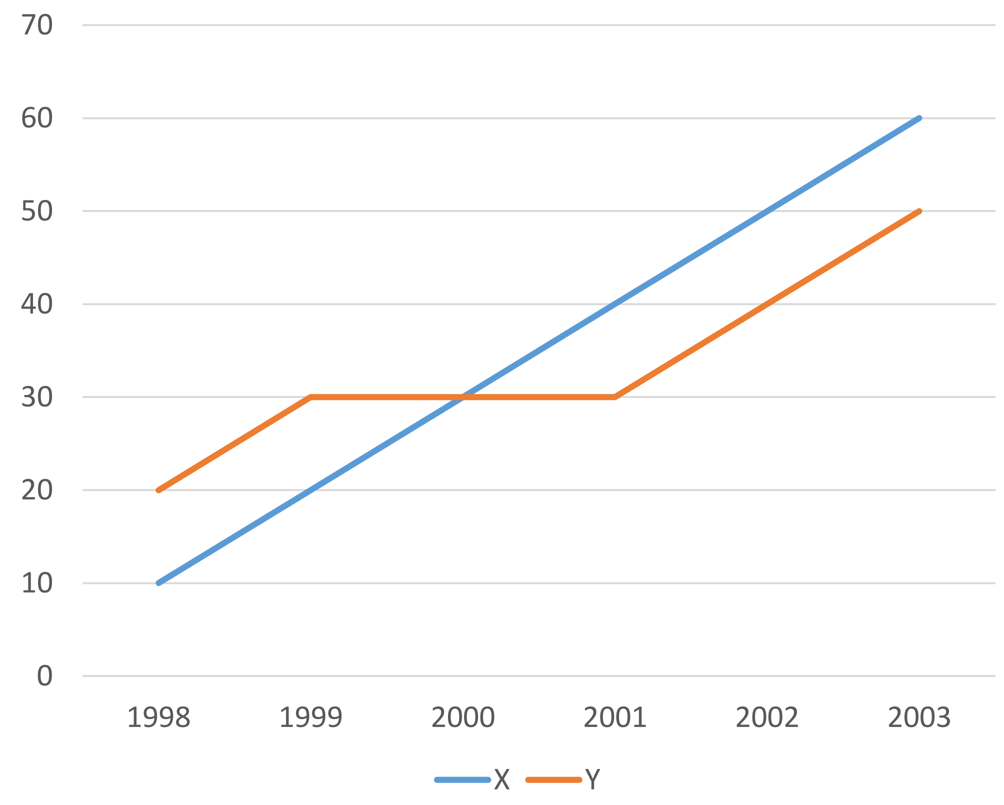
- It is a diagrammatic representation of bivariate data to ascertain the correlation between two variables.

- STEPS INVOLVED IN THE PREPARATION OF A SCATTER DIAGRAM:
- **STEP 1:** Show time horizon along the horizontal axis OX and the variable X and Y along the vertical axis OY.
- **STEP 2:** Plot the dot for each of the individual values of X and join these plotted dots to obtain a curve.
- **STEP 3:** Plot the dot for each of the individual values of Y and join these plotted dots to obtain a curve.
- **STEP 4:** Observe both the curves and form an idea about the direction of correlation.

- Interpretation:

If both the curves move in the same direction (either upward or downward), the correlation is said to be positive and if both the curves move in the opposite direction, correlation is said to be negative.

Year	1998	1999	2000	2001	2002	2003
X (Rs in lakh)	10	20	30	40	50	60
Y (Rs in lakh)	20	30	30	30	40	50



There exists a High Positive Correlation.

REGRESSION ANALYSIS:

- Regression is the measure of average relationship between two or more variables in terms of the original units of the data.
- For example: After having established that two variables (say sales and advertising expenditure) are correlated, one may find out the average relationship between the two to estimate the unknown values of dependent variable (say sales) from the known values of independent variable (say advertising expenditure).

- **REGRESSION ANALYSIS:**
- It is a statistical tool to study the nature and extent of functional relationship between two or more variables and to estimate (or predict) the unknown values of dependent variable from the known values of independent variable.

DEPENDENT VARIABLE : The variable which is predicted on the basis of another variable is called dependent variable or explained variable.

INDEPENDENT VARIABLE : The variable which is used to predict another variable is called independent variable or explanatory variable.

Regression Line of X on Y:

X = a + bY

Where, X = Dependent Variable

Y = Independent Variable

a = X intercept (i.e. value of dependent variable when value of independent variable is zero)

b = Slope of the said line (i.e. the amount of change in the value of the dependent variable per unit change in independent variable.)

The values of two constants ‘a’ and ‘b’ can be calculated for the given data of X and Y variable by solving the following two algebraic normal equations:

$$\sum X = Na + b\sum Y$$

$$\sum XY = a\sum Y + b\sum Y^2$$

Regression Line of Y on X:

Y = a + bX

Where, X = Independent Variable

Y = Dependent Variable

a = Y intercept (i.e. value of dependent variable when value of independent variable is zero)

b = Slope of the said line (i.e. the amount of change in the value of the dependent variable per unit change in independent variable.)

The values of two constants ‘a’ and ‘b’ can be calculated for the given data of X and Y variable by solving the following two algebraic normal equations:

$$\sum Y = Na + b\sum X$$

$$\sum XY = a\sum X + b\sum X^2$$

The following data relate to advertising expenditure and sales:

Advertising Expenditure (Rs lakhs)	Sales (Rs lakhs)
1	10
2	20
3	30
4	50
5	40

Calculate:

(a) Find out two Regression Equations.

(b) Estimate the likely sales when advertising expenditure is Rs. 7 lakhs

(c) What should be the advertising expenditure if the firm wants to attain sales target of Rs. 80 lakhs

(d) Calculate Coefficient of correlation:

X	X^2	Y	Y^2	XY
1	1	10	100	10
2	4	20	400	40
3	9	30	900	90
4	16	50	2500	200
5	25	40	1600	200
$\sum X = 15$	$\sum X^2 = 55$	$\sum Y = 150$	$\sum Y^2 = 5500$	$\sum XY = 540$

(a) Regression Line of X on Y: **X = a + bY**

$$\sum X = Na + b\sum Y$$
$$\sum XY = a\sum Y + b\sum Y^2$$

15 = 5a + 150b
540 = 150a + 5500b
By solving the above equations:
a= 0.3
b= 0.09

X = 0.3 + 0.09Y

Regression Line of Y on X: **Y = a + bX**

$$\sum Y = Na + b\sum X$$

$$\sum XY = a\sum X + b\sum X^2$$

150 = 5a + 15b

540 = 15a + 55b

By solving the above equations:

a= 3

b= 9

Y = 3 + 9X

(b) Sales (Y) when Advertising Expenditure (X) is Rs 7 lakhs

Y = 3+ 9*7

= 3 + 63

= 66

(c) Advertising Expenditure (X) to attain sales (Y) target of 80 lakhs

X = 0.3 + 0.09*80

= 0.3 + 7.2

= 7.5

(d) Coefficient of correlation

$$r = \sqrt{b_{xy} \times b_{yx}}$$

$$r = \sqrt{0.09 \times 9}$$

$$r = +0.9$$

Regression Equations by taking deviations from the Actual Mean:

Regression Line of X on Y:

$$(X-\overline{X})=b_{xy}(Y-\overline{Y})$$

$$OR$$

$$(X-\overline{X})=r\frac{\sigma_x}{\sigma_y}(Y-\overline{Y})$$

$$b_{xy}=r\frac{\sigma_x}{\sigma_y}$$

$$where, r\frac{\sigma_x}{\sigma_y}=\frac{\sum xy}{\sum y^2}$$

r= Coefficient of correlation between two variables X and Y

Regression Line of Y on X:

$$(Y-\overline{Y}) = b_{yx}(X-\overline{X})$$

$$OR$$

$$(Y-\overline{Y}) = r\frac{\sigma_y}{\sigma_x}(X-\overline{X})$$

$$b_{yx} = r\frac{\sigma_y}{\sigma_x}$$

$$where, r\frac{\sigma_y}{\sigma_x} = \frac{\sum xy}{\sum x^2}$$

r= Coefficient of correlation between two variables X and Y

The following data relate to advertising expenditure and sales:

Advertising Expenditure (Rs lakhs)	Sales (Rs lakhs)
1	10
2	20
3	30
4	50
5	40

Find out two Regression Equations.

X	x i.e. $(X-\overline{X})$	x^2	Y	y i.e. $(Y-\overline{Y})$	y^2	xy
1	-2	4	10	-20	400	40
2	-1	1	20	-10	100	10
3	0	0	30	0	0	0
4	1	1	50	20	400	20
5	2	4	40	10	100	20
15		10	150		1000	90

$$\overline{X} = \frac{\sum X}{N}$$

$$\overline{X} = \frac{15}{5}$$

$$\overline{X} = 3$$

$$\overline{Y} = \frac{\sum Y}{N}$$

$$\overline{Y} = \frac{150}{5}$$

$$\overline{Y} = 30$$

Regression Line of X on Y:

$$b_{xy} = r\frac{\sigma_x}{\sigma_y} = \frac{90}{1000} = 0.09$$

$$X - \overline{X} = b_{xy}(Y - \overline{Y})$$

$$X - 3 = 0.09(Y - 30)$$

$$X - 3 = 0.09Y - 2.7$$

$$X = 0.3 + 0.09Y$$

Regression Line of Y on X:

$$b_{yx} = r\frac{\sigma_y}{\sigma_x} = \frac{90}{10} = 9$$

$$Y - \overline{Y} = b_{yx}(X - \overline{X})$$

$$Y - 30 = 9(X - 3)$$

$$Y - 30 = 9X - 27$$

$$Y = 3 + 9X$$

Regression Equations by taking deviations from the Assumed Mean:

Regression Equation of X on Y

$$(X - \overline{X}) = b_{xy}(Y - \overline{Y})$$

$$OR$$

$$(X - \overline{X}) = r\frac{\sigma_x}{\sigma_y}(Y - \overline{Y})$$

$$b_{xy} = r\frac{\sigma_x}{\sigma_y}$$

$$where, r\frac{\sigma_x}{\sigma_y} = \frac{\sum d_x d_y - \frac{\sum d_x \times \sum d_y}{N}}{\sum d_y{}^2 - \frac{(\sum d_y)^2}{N}}$$

$$where, d_x = X - A_x$$

$$and, d_y = Y - A_y$$

Regression Line of Y on X:

$$(Y-\overline{Y}) = b_{yx}(X-\overline{X})$$

$$OR$$

$$(Y-\overline{Y}) = r\frac{\sigma_y}{\sigma_x}(X-\overline{X})$$

$$b_{yx} = r\frac{\sigma_y}{\sigma_x}$$

$$where, r\frac{\sigma_y}{\sigma_x} = \frac{\sum d_x d_y - \frac{\sum d_x \times \sum d_y}{N}}{\sum d_x^{\,2} - \frac{(\sum d_x)^2}{N}}$$

$$where, d_x = X - A_x$$

$$and, d_y = Y - A_y$$

The following data relate to advertising expenditure and sales:

Advertising Expenditure (Rs lakhs)	Sales (Rs lakhs)
1	10
2	20
3	30
4	50
5	40

Find out two Regression Equations.

X	d_x i.e. $(X-4)$	d_x^2	Y	d_y i.e. $(Y-40)$	d_y^2	$d_x d_y$
1	-3	9	10	-30	900	90
2	-2	4	20	-20	400	40
3	-1	1	30	-10	100	10
4	0	0	50	10	100	0
5	1	1	40	0	0	0
15		15	150		1500	140

$$\overline{X} = \frac{\sum X}{N}$$

$$\overline{X} = \frac{15}{5}$$

$$\overline{X} = 3$$

$$\overline{Y} = \frac{\sum Y}{N}$$

$$\overline{Y} = \frac{150}{5}$$

$$\overline{Y} = 30$$

$$b_{xy} = r\frac{\sigma_x}{\sigma_y}$$

$$where, r\frac{\sigma_x}{\sigma_y} = \frac{\sum d_x d_y - \frac{\sum d_x \times \sum d_y}{N}}{\sum d_y{}^2 - \frac{(\sum d_y)^2}{N}}$$

$$r\frac{\sigma_x}{\sigma_y} = \frac{140 - \frac{-5 \times -50}{5}}{1500 - \frac{(-50)^2}{5}}$$

$$r\frac{\sigma_x}{\sigma_y} = \frac{140 - 50}{1500 - 500}$$

$$r\frac{\sigma_x}{\sigma_y} = \frac{90}{1000}$$

$$r\frac{\sigma_x}{\sigma_y} = 0.09$$

Regression Line of X on Y:

$$b_{xy} = r\frac{\sigma_x}{\sigma_y} = \frac{90}{1000} = 0.09$$

$$X - \overline{X} = b_{xy}(Y - \overline{Y})$$

$$X - 3 = 0.09(Y - 30)$$

$$X - 3 = 0.09Y - 2.7$$

$$X = 0.3 + 0.09Y$$

$$b_{yx} = r\frac{\sigma_y}{\sigma_x}$$

$$where, r\frac{\sigma_y}{\sigma_x} = \frac{\sum d_x d_y - \frac{\sum d_x \times \sum d_y}{N}}{\sum d_x{}^2 - \frac{(\sum d_x)^2}{N}}$$

$$r\frac{\sigma_y}{\sigma_x} = \frac{140 - \frac{-5 \times -50}{5}}{15 - \frac{(-5)^2}{5}}$$

$$r\frac{\sigma_x}{\sigma_y} = \frac{140 - 50}{15 - 5}$$

$$r\frac{\sigma_x}{\sigma_y} = \frac{90}{10}$$

$$r\frac{\sigma_x}{\sigma_y} = 9$$

Regression Line of Y on X:

$$b_{yx} = r\frac{\sigma_y}{\sigma_x} = \frac{90}{10} = 9$$

$$Y - \overline{Y} = b_{yx}(X - \overline{X})$$

$$Y - 30 = 9(X - 3)$$

$$Y - 30 = 9X - 27$$

$$Y = 3 + 9X$$

Calculate Regression Coefficient of Y on X in each of the following cases:

(a) $b_{xy} = 0.09$; $r = 0.9$
(b) $r = 0.8$; $\sigma_x = 3$

$\sigma_y = 4$

(c) Regression equation of Y on X: 45X-5Y+15=0
(d) $b_{xy} = 0.6$; $r = 0.8$; Variance of X=9 **(Calculate variance of Y)**

(a)

$$r = \sqrt{b_{xy} \times b_{yx}}$$

$$0.9 = \sqrt{0.09 \times b_{yx}}$$

$$(0.9)^2 = 0.09 \times b_{yx}$$

$$\frac{0.81}{0.09} = b_{yx}$$

$$b_{yx} = 9$$

(b)

$$b_{yx} = r\,\frac{\sigma_y}{\sigma_x}$$

$$b_{yx} = 0.8\,\frac{4}{3}$$

$$b_{yx} = 1.067$$

Regression equation of Y on X: 45X-5Y+15=0

$$45X - 5Y + 15 = 0$$

$$5Y = 45X + 15$$

$$Y = 3 + 9X$$

$$\text{Thus, } b_{yx} = 9$$

$b_{xy} = 0.6$; $r = 0.8$; Variance of X=9 **(Calculate variance of Y)**

$$\sigma_x = \sqrt{\sigma_x{}^2} = \sqrt{9} = 3$$

$$b_{xy} = r\frac{\sigma_x}{\sigma_y}$$

$$0.6 = 0.8\frac{3}{\sigma_y}$$

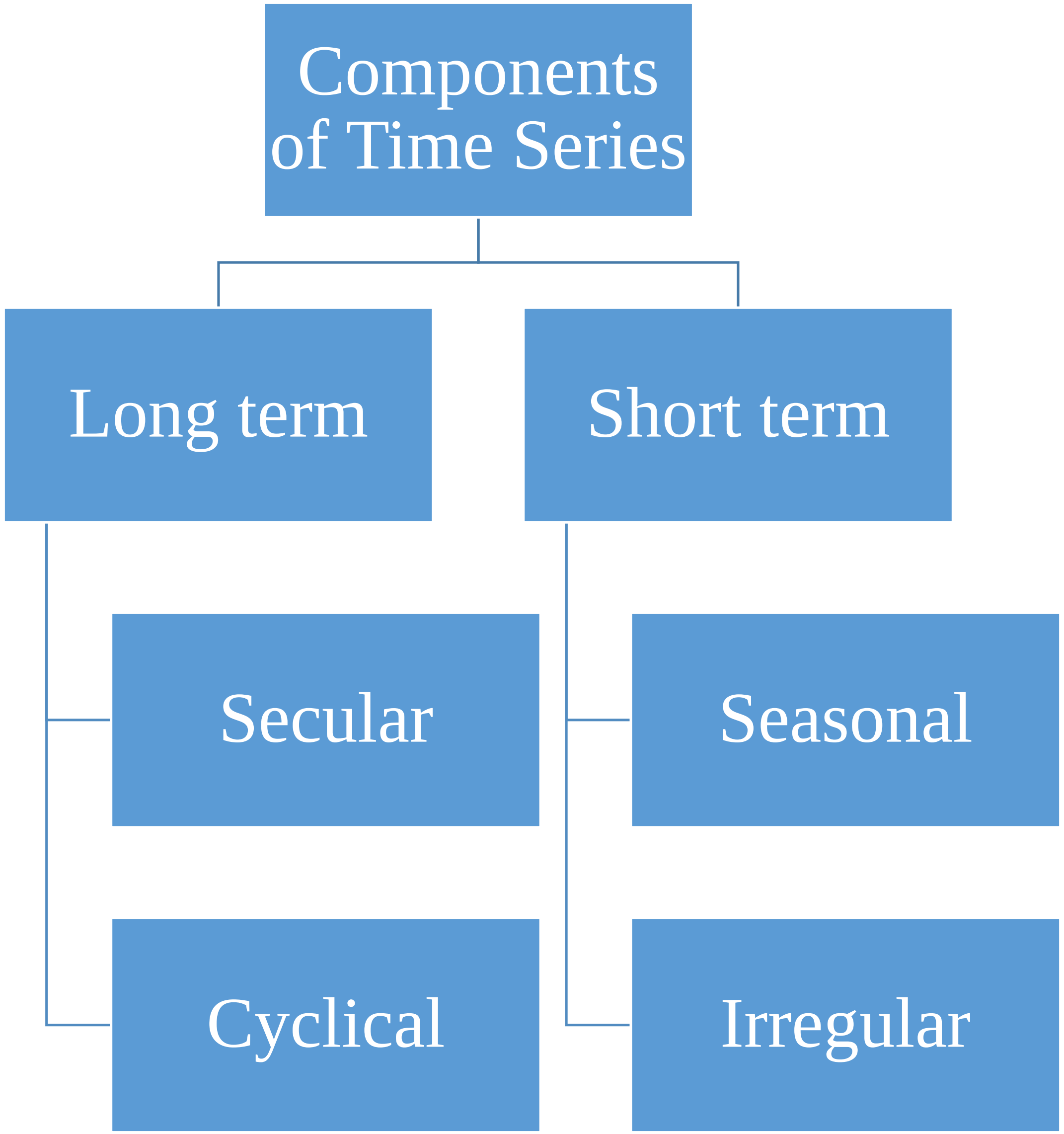
$$\sigma_y = \frac{0.8 \times 3}{0.6}$$

$$\sigma_y = 4$$

ANALYSIS OF TIME SERIES:

- Time series is a set of observations taken at specified times, usually at ‘equal intervals’.
- It is an arrangement of the statistical data in accordance with the time of occurrence.
- Examples of Time Series include the following:

1. Yearly National Income Data for the last 5,6,7 years or some other time period.
2. Yearly production of steel data for the last 5,6,7 years or some other time period.
3. Yearly population data for the last 5,6,7 years or some other time period.
4. Yearly sales data for the last 5,6,7 years or some other time period.
5. Number of deaths/births/accidents taking place during a certain period.


Components of Time Series
Long term
Short term
Secular
Cyclical
Seasonal
Irregular

Components of Time Series:

- By components, we mean the kinds or categories of fluctuations.
- Fluctuations can be seen when the values of a phenomenon are observed at different periods of time.

Secular Trend (T):

- It is the basic tendency of steady movements in a set of observations to move in an upward or downward or constant direction over a fairly long period of time.
- For Example: Long term changes per capita income, technological improvements, growth of population etc.

Seasonal Variations (S):

- These are the regular periodic changes which take place within a period of less than a year and may take place daily, weekly, monthly or quarterly.
- It may be due to climatic changes or due to the changes in the pattern of consumption or production etc.
- For example: sale of raincoats may go up during the rainy season and go down during the dry season, higher sales of woolen garments during winter season and so on.

Cyclical Variations (C):

• Also termed as business cycles, are the periodic movements in the time series.
•The frequency of occurrence of these cycles is not uniform. Such variations happen during intervals of time of more than one year, say, once in 3 to 7 years.
•Each cycle consists of four stages: (a) Boom (b) Recession (d) Depression (c) Recovery
• A businessman with the knowledge of cyclical variations can prepare himself for facing a period of recession by taking appropriate actions in advance. Similarly, he can proactively be ready to take advantage of the recovery and boom in the business.

Irregular Fluctuations (I):

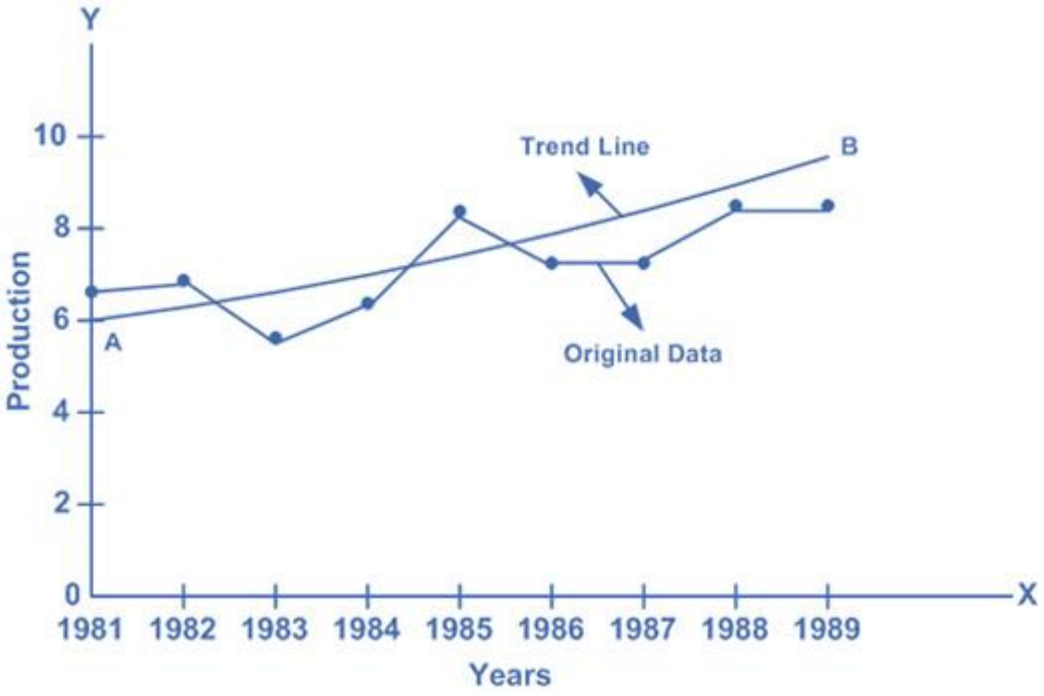
•This is the residual factor that accounts for deviations of actual time series from what we would expect from trend, cyclical and seasonal components.
• Irregular fluctuations occur accidently or by happening of chance.
•There does not exist any possibility of their occurrence.
•For example: fluctuations occurring due to war, earthquakes, flood, strike etc.
•Usually they are short term variations but some time their impact is so powerful that they may create new cycles or movements.
• Because of uncertainty in their character they cannot be properly forecasted.

Measurement of Secular Trend (or Long Term Trend):

1. Free Hand Curve Method
2. Semi Average Method
3. Moving Average Method
4. Least Square Method

Free Hand Curve Method:

- This is the simplest and easy method for measuring long term trend.
- In this method, the values of time series are plotted on a graph paper and a curve emerges.
- Thereafter another smoothened curve is drawn in a manner that it may accurately describe the general long run tendency of the data.



Semi Average Method:

STEP 1: Divide the given data into two equal parts.

(In case of Even Number of Observations)- Two equal parts will be first three years.

(In case of Odd Number of Observations)- Two equal parts are generally made by omitting the middle year.

STEP 2: Calculate the arithmetic mean of each part and thus obtain two points

STEP 3: Plot each point at the mid-point of the class interval covered by the respective part.

STEP 4: Draw a straight line by joining these two plotted points. This line gives us the requires trend line.

Fit a trend line by the method of semi-averages to the data given below.

Year	Sales (Rs Lakhs)
2010	200
2011	120
2012	280
2013	240
2014	160
2015	320
2016	360
2017	400
2018	320
2019	360
2020	360

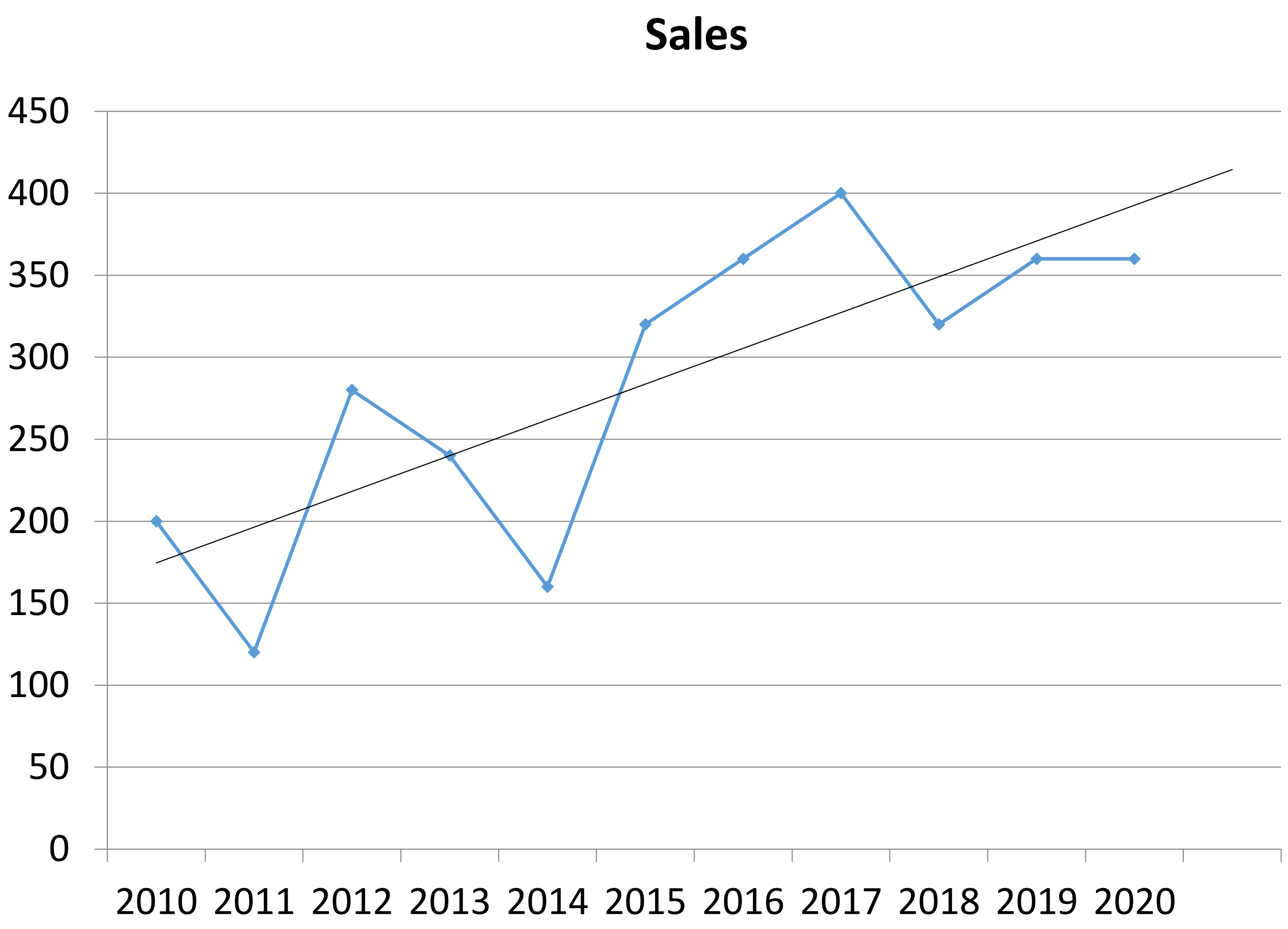
Since eleven years are given, the middle year (2015) shall be left out and an average of the first 5 years and the last 5 years shall be obtained.

Average of first 5 years $= \dfrac{200+120+280+240+160}{5}$

$= 200$

Average of last 5 years $= \dfrac{360+400+320+360+360}{5}$

$= 360$


Sales
450
400
350
300
250
200
150
100
50
0
2010
2011
2012
2013
2014
2015
2016
2017
2018
2019
2020

MOVING AVERAGE METHOD:

STEP 1: Select a period for moving average which is equal to or greater than the average length of the cycle in the data so that the cycle may entirely be removed. For example: 3-yearly moving average, 5-yearly moving average.

STEP 2: Take arithmetic mean of the values for a certain time span. For 3-yearly moving average: (1^{st} year+2^{nd} year+3^{rd} year)/3

STEP 3: Place the average at the centre of that time span.

STEP 4: Repeat the process by dropping the first period's value and adding the value of the next which had not been added previously. For 3-yearly moving average: (2^{nd} year+3^{rd} year+4^{th} year)/3. This process is continued until the series is exhausted.

From the following data estimate the trend values by taking 5-yearly moving averages.

Year	Sales (Rs Lakhs)
2010	200
2011	120
2012	280
2013	240
2014	160
2015	320
2016	360
2017	400
2018	320
2019	360
2020	360

Year	Sales (Rs Lakhs)	3 yearly total	3 yearly moving average
2010	200	-	-
2011	120	600	200
2012	280	640	213.33
2013	240	680	226.67
2014	160	720	240
2015	320	840	280
2016	360	1080	360
2017	400	1080	360
2018	320	1080	360
2019	360	1040	346.67
2020	360	-	-

LEAST SQUARE METHOD:

$$Y_c = a + bX$$

$$a = \frac{\sum Y}{N}$$

$$b = \frac{\sum XY}{\sum X^2}$$

Fit a straight line Trend:

Year	Sales (Rs in Lakhs)
2010	60
2011	72
2012	75
2013	65
2014	80
2015	85
2016	95

Year	Sales (Rs in Lakhs) Y	X (Deviations from Year)	X^2	XY	Y_c
2010	60	-3	9	-180	61.42
2011	72	-2	4	-144	66.28
2012	75	-1	1	-75	71.14
2013	65	0	0	0	76
2014	80	1	1	80	80.86
2015	85	2	4	170	85.72
2016	95	3	9	285	90.58
			28	**136**	

$$Y_c = a + bX$$

$$a = \frac{532}{7}$$

$$a = 76$$

$$b = \frac{\sum XY}{\sum X^2}$$

$$b = \frac{136}{28}$$

$$b = 4.86$$

$$Y_c = a + bX$$

$$76 + (4.86 * -3) = 61.42$$

$$76 + (4.86 * -2) = 66.28$$

$$76 + (4.86 * -1) = 71.14$$

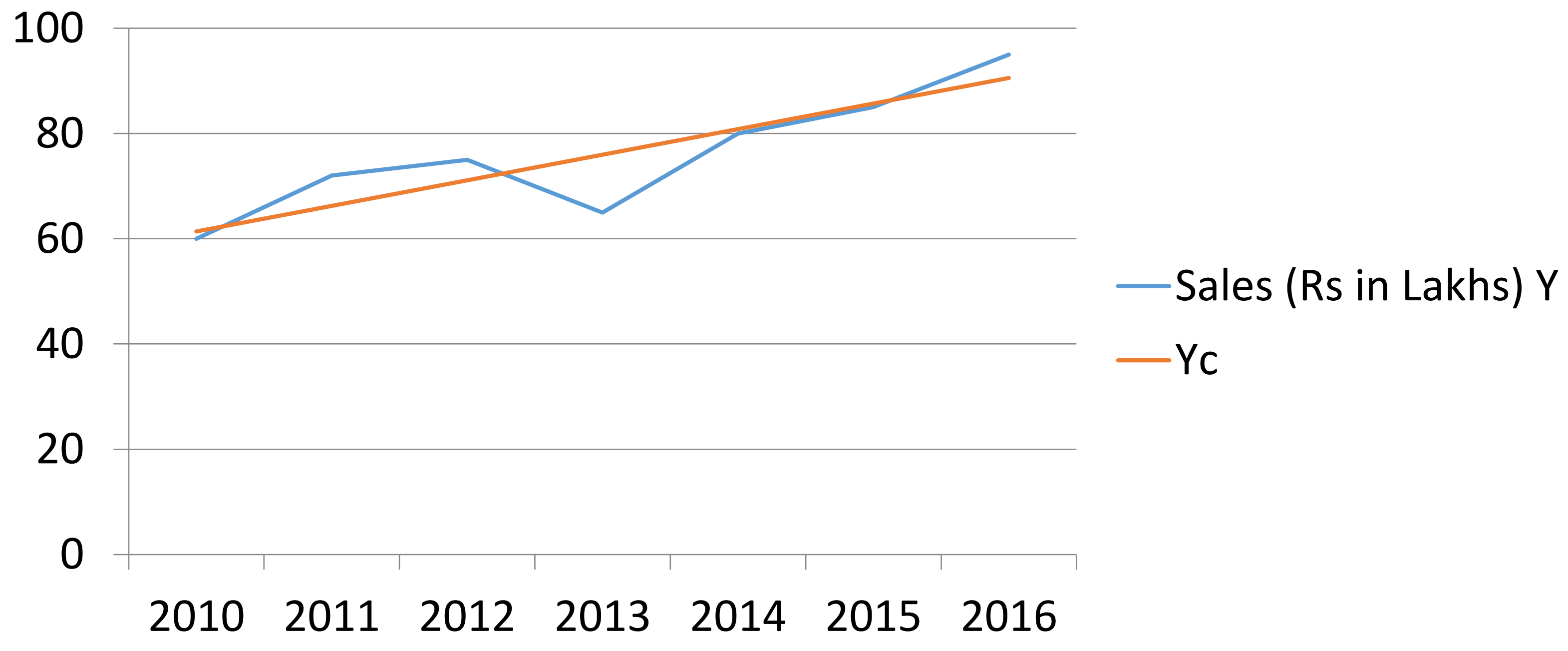
$$76 + (4.86 * 0) = 76$$

$$76 + (4.86 * 1) = 80.86$$

$$76 + (4.86 * 2) = 85.72$$

$$76 + (4.86 * 3) = 90.58$$

Year	Sales (Rs in Lakhs) Y	Y_c
2010	60	61.42
2011	72	66.28
2012	75	71.14
2013	65	76
2014	80	80.86
2015	85	85.72
2016	95	90.58



Fit a straight line Trend:

Year	Sales (Rs in Lakhs)
2010	56
2011	55
2012	51
2013	47
2014	42
2015	38
2016	35
2017	32

Year	Sales (Rs in Lakhs) Y	D (Deviations)	X (D *2)	X^2	XY	Y_c
2010	56	-3.5	-7	49	-392	57.52
2011	55	-2.5	-5	25	-275	53.80
2012	51	-1.5	-3	9	-153	50.08
2013	47	-0.5	-1	1	-47	46.36
2014	42	0.5	1	1	42	42.64
2015	38	1.5	3	9	114	38.92
2016	35	2.5	5	25	175	35.20
2017	32	3.5	7	49	224	31.48
8	**356**			**168**	**-312**	**356**

$$Y_c = a + bX$$

$$a = \frac{356}{8}$$

$$a = 44.5$$

$$b = \frac{\sum XY}{\sum X^2}$$

$$b = \frac{-312}{168}$$

$$b = -1.86$$

$$Y_c = a + bX$$

$$44.5 + (-1.86 * -7) = 57.52$$

$$44.5 + (-1.86 * -5) = 53.80$$

$$44.5 + (-1.86 * -3) = 50.08$$

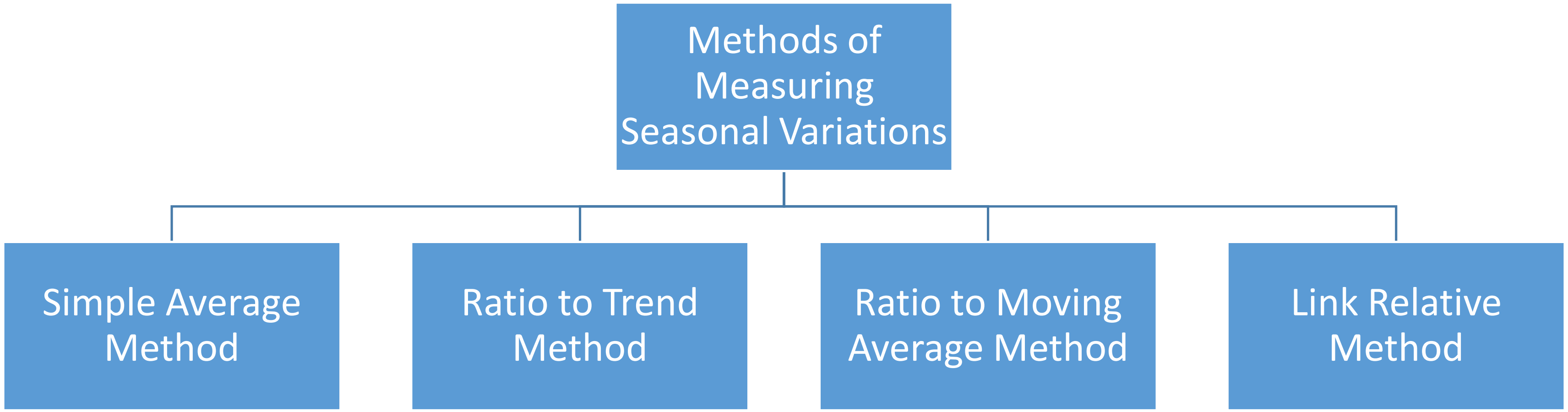
$$44.5 + (-1.86 * -1) = 46.36$$

$$44.5 + (-1.86 * 1) = 42.64$$

$$44.5 + (-1.86 * 3) = 38.92$$

$$44.5 + (-1.86 * 5) = 35.20$$

$$44.5 + (-1.86 * 7) = 31.48$$


Methods of Measuring Seasonal Variations
Simple Average Method
Ratio to Trend Method
Ratio to Moving Average Method
Link Relative Method

•**SIMPLE AVERAGE METHOD:**

STEP 1: Arrange the given data by years and weeks/months/quarters.

STEP 2: Calculate the total for each week/month/quarter for the given number of years.

STEP 3: Calculate weekly/monthly/quarterly average by dividing each total by the number of years for which data are given.

STEP 4: Calculate an average of averages by dividing the total of weekly averages by 52 or monthly averages by 12 or quarterly averages by 4.

STEP 5: Calculate the seasonal indices (SI) as follows:

SI (under Multiplicative Model) =Average/ Grand Average * 100

SI (under Additive Model) = Average – Average of Averages

Calculate Seasonal Indices by Simple Average Method under Multiplicative Model and Additive Model from the following data:

Year	Quarterly Sales (Rs. Lakhs)			
	I	II	III	IV
2012	16	32	8	24
2013	24	48	12	36
2014	32	64	16	48
2015	72	144	36	108
2016	56	112	28	84

		Year	Quarterly Sales (Rs. Lakhs)				
			I	II	III	IV	
		2012	16	32	8	24	
		2013	24	48	12	36	
		2014	32	64	16	48	
		2015	72	144	36	108	
		2016	56	112	28	84	
A	Total		200	400	100	300	
B	Quarterly Average (A/No. of years)		40	80	20	60	200
C	Average of Averages[(40+80+20+60)/4]		50	50	50	50	
D	Seasonal Index(under Multiplicative Model)[B*100/C]		80	160	40	120	
E	Seasonal Index (under Additive Model) [B-C]		-10	30	-30	10	

RATIO TO TREND METHOD (OR PERCENTAGE TO TREND METHOD)

STEP 1: Calculate Yearly totals for each year.

STEP 2: Calculate quarterly average by dividing yearly total by 4.

STEP 3: Calculate trend values by applying the method of least square.

STEP 4: Calculate quarterly increment as follows:

Quarterly Increment = (Value of b) / 4

STEP 5: Calculate quarterly trend values as follows:

Quarterly Trend Values for Q_1 = Trend Value –(Quarterly Increment *1.5)

Quarterly Trend Values for Q_2 = Trend Value –(Quarterly Increment *.5)

Quarterly Trend Values for Q_3 = Trend Value +(Quarterly Increment *.5)

Quarterly Trend Values for Q_4 = Trend Value +(Quarterly Increment *1.5)

STEP 6: Under Multiplicative Model:

(a) Calculate given quarterly values as % of quarterly trend values.

(b) Calculate total of above percentage for each quarter.

(c) Calculate average of total of above percentages for each quarter.

(d) Calculate grand average of averages of each quarter.

(e) Calculate adjusted seasonal indices if total of average indices is not equal to 400.

S.I. =Average/Grand Average *100

Under Additive Model:

(a) Calculate the difference between the given quarterly values and quarterly trend values.
(b) Calculate total of above differences.
(c) Calculate average of total above differences
(d) Calculate an average of averages
(e) Calculate the adjusted Seasonal Indices as follows if total of average indices is not equal to zero.

Adjusted Seasonal Indices = Average Index- Grand Average

Calculate Seasonal Indices by Ratio to Trend Method:

Year	Quarterly Sales (Rs. Lakhs)			
	I	II	III	IV
2012	8	16	24	32
2013	48	36	24	12
2014	48	16	32	64
2015	72	108	144	36
2016	56	28	84	112

STEP 1: Conversion of quarterly data into yearly data and then calculation of quarterly average.

Year	2012	2013	2014	2015	2016
Yearly Sales	80	120	160	360	280
Quarterly Average (Yearly Sales/4)	20	30	40	90	70

STEP 2: Fitting a Straight Line Trend by the method of least squares.

Year	Deviation form 2014	X^2	Sales (Y)	XY	Y_c
2012	-2	4	20	-40	18
2013	-1	1	30	-30	34
2014	0	0	40	0	50
2015	1	1	90	90	66
2016	2	4	70	140	82
N=5	0	10	250	160	

$$a = \frac{\sum Y}{N} = \frac{250}{5} = 50$$

$$b = \frac{\sum XY}{\sum X^2} = \frac{160}{10} = 16$$

STEP 3: Quarterly Increment = 16/4 =4

STEP 4: Calculation of Quarterly Trend Values

Year	Quarterly Sales (Rs. Lakhs)			
	I	II	III	IV
2012	12 i.e.(18-4*1.5)	16 i.e.(18-4*.5)	20 i.e.(18+4*.5)	24 i.e.(18+4*1.5)
2013	28 i.e. (34-4*1.5)	32 i.e. (34-4*.5)	36 i.e. (34+4*.5)	40 i.e. (34+4*1.5)
2014	44 i.e. (50-4*1.5)	48 i.e. (50-4*.5)	52 i.e. (50+4*.5)	56 i.e. (50+4*1.5)
2015	60 i.e. (66-4*1.5)	64 i.e. (66-4*.5)	68 i.e. (66+4*.5)	72 i.e. (66+4*1.5)
2016	76 i.e.(82-4*1.5)	80 i.e.(82-4*.5)	84 i.e.(82+4*.5)	88 i.e.(82+4*1.5)

STEP 5: Calculation of Seasonal Indices by Ratio to Trend Method under Multiplicative Model.

		Year	Quarterly Sales (Rs. Lakhs)				
			I	II	III	IV	
		2012	66.67 (8/12*100)	100(16/16*100)	120(24/20*100)	133.33((32/24*100)	
		2013	171.43	112.5	66.67	30	
		2014	109.09	33.33	61.54	114.29	
		2015	120	168.75	211.76	50	
		2016	73.69	35	100	127.27	
A	Total		540.88	449.58	559.97	454.89	
B	Quarterly Average (A/5)		108.17	89.91	111.99	90.97	401.04
C	Grand Average[401.04/4]		100.26	100.26	100.26	100.26	
D	Seasonal Index(under Multiplicative Model)[B*100/C]		107.89	89.68	111.69	90.74	400

STEP 6: Calculation of Seasonal Indices by Ratio to Trend Method under Additive Model.

		Year	Quarterly Sales (Rs. Lakhs)				
			I	II	III	IV	
		2012	-4 (8-12)	0(16-16)	4(24-20)	8(32-24)	
		2013	20	4	-12	-28	
		2014	4	-32	-20	8	
		2015	12	44	76	-36	
		2016	-20	-52	0	24	
A	Total		12	-36	48	-24	
B	Quarterly Average or Seasonal Index (A/5)		2.4	-7.2	9.6	-4.8	0

LINK RELATIVE METHOD:

STEP 1: Calculate Link Relatives for each season as follows:

Link Relative = (Current Season's Figure)/ (Previous Season's Figure) *100

STEP 2: Calculate the average of the link relatives for each season.

STEP 3: Calculate chain relatives on the basis of first season as follows:

Chain Relative = (Current Season's Average of Link Relative * Previous Season's Chain Relative)/100

Note: Chain Relative for 1st season is taken as 100.

STEP 4: Calculate Chain relative of the first season on the basis of last season as follows:

First Season's Chain Relative = (First season's average of Link Relative* Last Season's Chain Relative)/100

STEP 5: Calculate Adjusted factor as follows:

For 2nd Quarter	**1*(First Season's Chain Relative -100)/4**
For 3rd Quarter	**2*(First Season's Chain Relative -100)/4**
For 4th Quarter	**3*(First Season's Chain Relative -100)/4**

STEP 6: Calculate corrected/Adjusted chain relatives by deducting the adjustment factor from the chain relative.

STEP 7: Calculate average of corrected chain relatives.

STEP 8: Calculate Seasonal Indices as follows:

Seasonal Indices = Corrected Chain Relatives/Average of corrected chain relatives*100

Calculate Seasonal Indices by Link Relative Method from the following data:

Year	Quarterly Sales (Rs. Lakhs)			
	I	II	III	IV
2012	16	32	8	24
2013	24	48	12	36
2014	32	64	16	48
2015	72	144	36	108
2016	56	112	28	84

		Year	Quarterly Sales (Rs. Lakhs)				
			I	II	III	IV	
		2012	-	200	25	300	
		2013	100	200	25	300	
		2014	88.89	200	25	300	
		2015	150	200	25	300	
		2016	51.85	200	25	300	
A	Total of Quarterly Figures		390.741	1000	125	1500	
B	Arithmetic Average [Total/No. of Quarterly Figures]		97.6852	200	25	300	
C	Chain Relatives		100	200 [(200*100)/100]	50 [(25*200)/100]	150 [(300*50)/100]	
D	Adjusted Factor		-	11.6319 {1*[(97.6852*150)/100]-100}/4	23.264 {2*[(97.6852*150)/100]-100}/4	34.896 {3*[(97.6852*150)/100]-100}/4	
E	Corrected Chain Relatives [C-D]		100	188.368	26.736	115.104	430.208
F	Average Corrected Chain Relative[430.208/4]		107.552	107.552	107.552	107.552	
G	Seasonal Index [(E*100)/F]		92.98	175.141	24.86	107.02	400

Ratio to Moving Average Method (or Percentage of Moving Average Method):

STEP 1: Calculate trend values by applying the method of 4 quarterly moving average.

STEP 2: Under Multiplicative Model:

(a) Calculate given quarterly values as % of quarterly moving average.
(b) Calculate total of above percentages for each quarter.
(c) Calculate average of total of above percentages for each quarter.
(d) Calculate grand average of averages of each quarter.
(e) Calculate adjusted Seasonal Indices if total of average indices is not equal to 400.

Seasonal Index = Average / Grand Average *100

STEP 3: Under Additive Model:

(a) Calculate the difference between the given quarterly values and quarterly moving averages.
(b) Calculate total of above differences for each quarter.
(c) Calculate average of total above differences for each quarter.
(d) Calculate grand average of averages of each quarter.
(e) Calculate the adjusted Seasonal Indices as follows if total of average is not equal to zero.

Adjusted Seasonal Indices = Average Index- Grand Average

Calculate Seasonal Indices by Ratio to Moving Average Method:

Year	Quarterly Sales (Rs. Lakhs)			
	I	II	III	IV
2012	8	16	24	32
2013	48	36	24	12
2014	48	16	32	64
2015	72	108	144	36
2016	56	28	84	112

Year	Quarter	Value	4-Quarterly Moving Total	4-Quarterly Moving Average	4-Quarterly Moving Average Centred
2012	I	8	-	-	-
	II	16	-	-	-
	III	24	80	20	25
	IV	32	120	30	32.5
2013	I	48	140	35	35
	II	36	140	35	32.5
	III	24	120	30	30
	IV	12	120	30	27.5
			100	25	

Year	Quarter	Value	4-Quarterly Moving Total	4-Quarterly Moving Average	4-Quarterly Moving Average Centred
2014	I	48	108	27	26
	II	16	160	40	33.5
	III	32	184	46	43
	IV	64	276	69	57.5
2015	I	72	388	97	83
	II	108	360	90	93.5
	III	144	344	86	88
	IV	36	264	66	76
2016	I	56	204	51	58.5
	II	28	280	70	60.5
	III	84	-	-	-
	IV	112	-	-	-

STEP 2: Calculation of Seasonal Indices by Ratio to Moving Average Method under Multiplicative Model.

		Year	Quarterly Sales (Rs. Lakhs)				
			I	II	III	IV	
		2012	-	-	96	98.46	
		2013	137.14	110.77	80	43.67	
		2014	184.62	47.76	74.42	111.30	
		2015	86.75	115.51	163.64	47.37	
		2016	95.73	46.28	-	-	
A	Total		504.24	320.32	414.06	300.8	
B	Quarterly Average (A/4)		126.06	80.08	103.515	75.2	384.855
C	Grand Average[384.855/4]		96.214	96.214	96.214	96.214	
D	Seasonal Index [B*100/C]		131.021	83.2313	107.589	78.159	400

STEP 3: Calculation of Seasonal Indices by Ratio to Moving Average Method under Additive Model.

		Year	Quarterly Sales (Rs. Lakhs)				
			I	II	III	IV	
		2012	-	-	-1	-0.5	
		2013	13	3.5	-6	-15.5	
		2014	22	-17.5	-11	6.5	
		2015	-11	14.5	56	-40	
		2016	-2.5	-32.5	-	-	
A	Total		21.5	-32	38	-49.5	
B	Quarterly Average (A/4)		5.375	-8	9.5	-12.375	-5.5
C	Grand Average[-5.5/4]		-1.375	-1.375	-1.375	-1.375	
D	Seasonal Index(under Additive Model)[B-C]		6.75	-6.625	10.875	-11	0