Radiations and their interaction with matter

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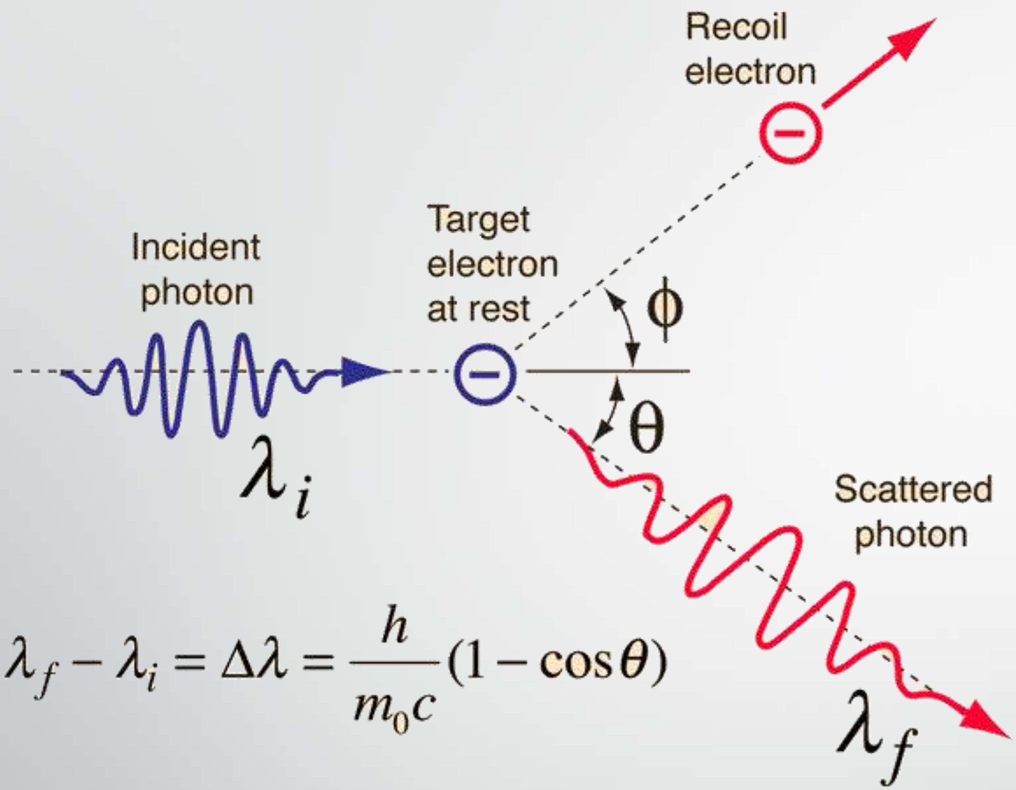
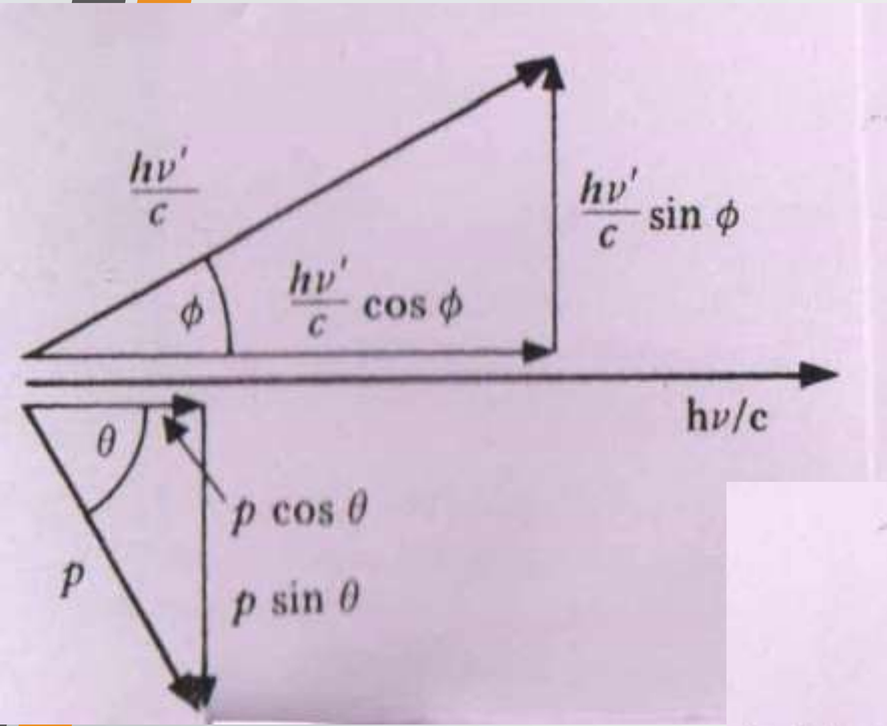
Compton Scattering



$$\lambda_f - \lambda_i = \Delta\lambda = \frac{h}{m_0 c}(1 - \cos\theta)$$

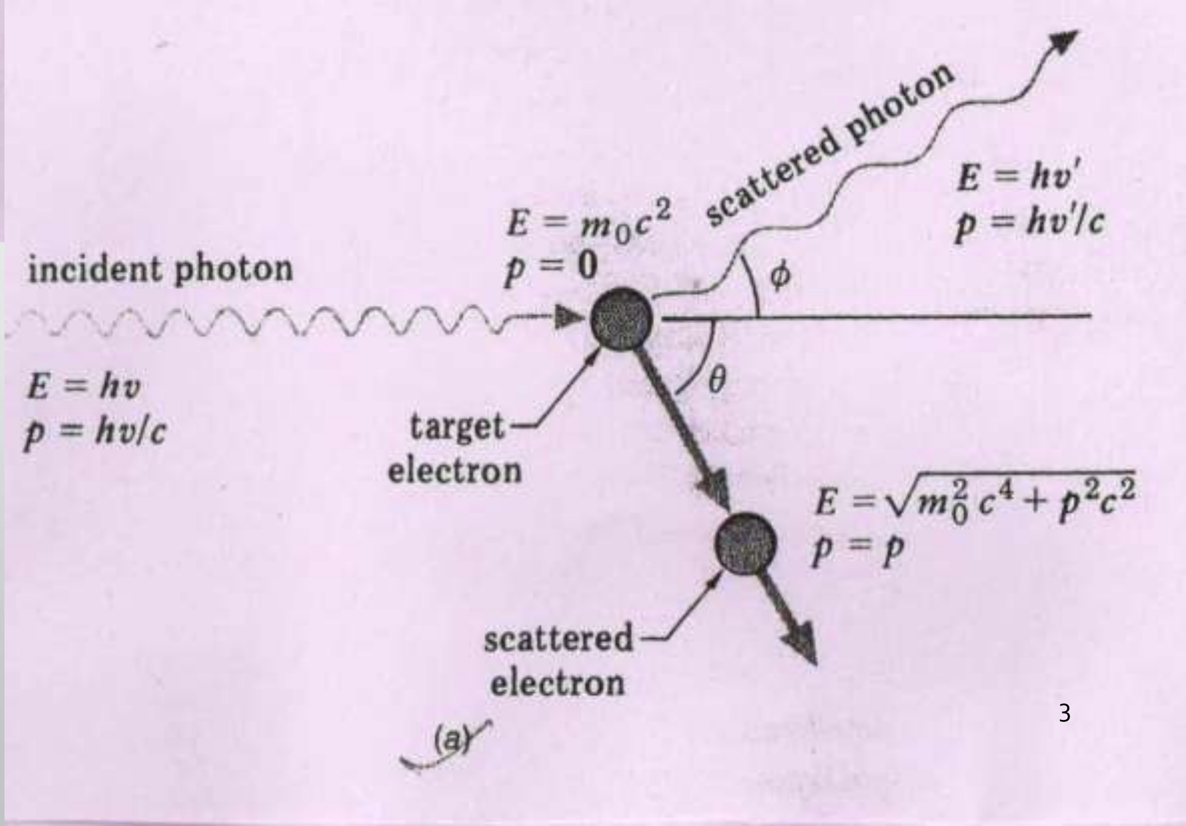
Compton scattering, discovered by Arthur Holly Compton, is the inelastic scattering of a photon by a charged particle, usually an electron. It results in a decrease in energy (increase in wavelength) of the photon (which may be an X-ray or gamma ray photon), called the Compton effect. Part of the energy of the photon is transferred to the recoiling electron. Inverse Compton scattering exists, in which a charged particle transfers part of its energy to a photon.

Compton Scattering



Loss in photon energy = gain in electron energy

$$h\nu - h\nu' = K$$



Compton Scattering

Momentum of a massless particle is related to its energy by the formula

$$E = pc$$

Since the energy of a photon is hν, its momentum is

$$p = \frac{E}{c} = \frac{hv}{c}$$

Initial momentum = final momentum

$$\frac{hv}{c} + 0 = \frac{hv'}{c}\cos\phi + p\cos\theta \ldots\ldots\ldots\ldots\ldots(1)$$

Compton Scattering

and perpendicular to this direction

Initial momentum = final momentum

$$0 = \frac{hv'}{c}\sin\phi - p\sin\theta \ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

Multiplying Eqs. (1) and (2) by c and rewrite them as

$$hv - hv'\cos\phi = pc\cos\theta$$

$$hv'\sin\phi = pc\sin\theta$$

By squaring each of those equations and adding the new ones together, the angle θ is eliminated, leaving

$$p^2c^2 = (hv)^2 - 2(hv)(hv')\cos\phi + (hv')^2 \ldots\ldots\ldots\ldots(3)$$

Compton Scattering

Next we equate the two expressions for the total energy of a particle

$$E = K + m_0c^2$$

$$E = \sqrt{p^2c^2 + m_0^2c^4}$$

$$or \left(K + m_0c^2\right)^2 = \left(p^2c^2 + m_0^2c^4\right)$$

$$or\ p^2c^2 = K^2 + 2m_0c^2K$$

Since $K = hv - hv'$

We have

$$p^2c^2 = (hv)^2 - 2(hv)(hv') + (hv')^2 + 2m_0c^2(hv - hv')$$

By substituting this value of p^2c^2 in Eq. 3, we have finally obtain

$$2m_0c^2(hv - hv') = 2(hv)(hv')(1-\cos\phi)\text{.............}(4)$$

This relationship is simpler when expressed in terms of wavelength rather than frequency. Dividing above equation by $2h^2c^2$

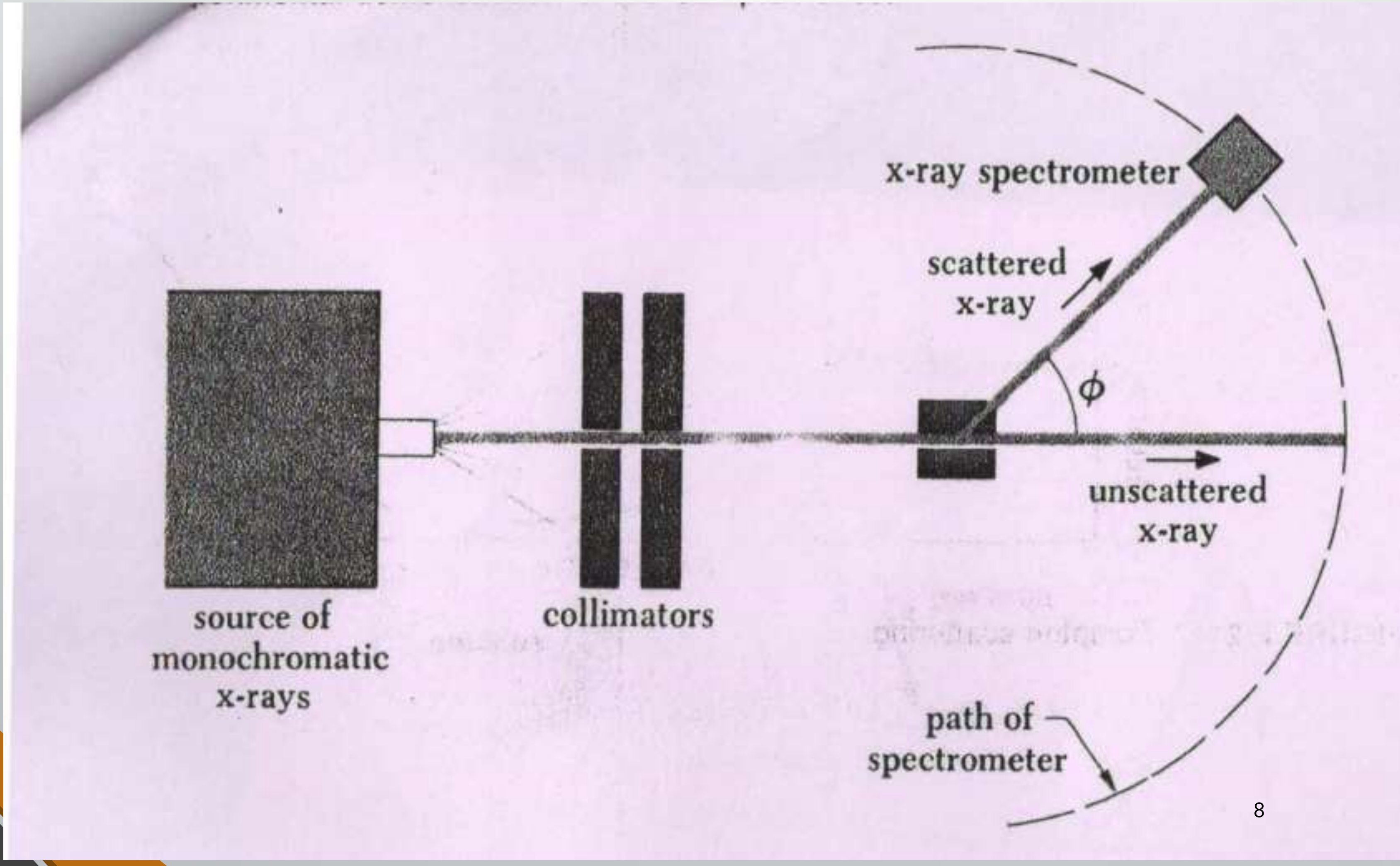
$$\frac{m_0c}{h}\left(\frac{v}{c} - \frac{v'}{c}\right) = \frac{v}{c}\frac{v'}{c}(1-\cos\phi)$$

And so, since $v/c=1/\lambda$ and $v'/c=1/\lambda'$,

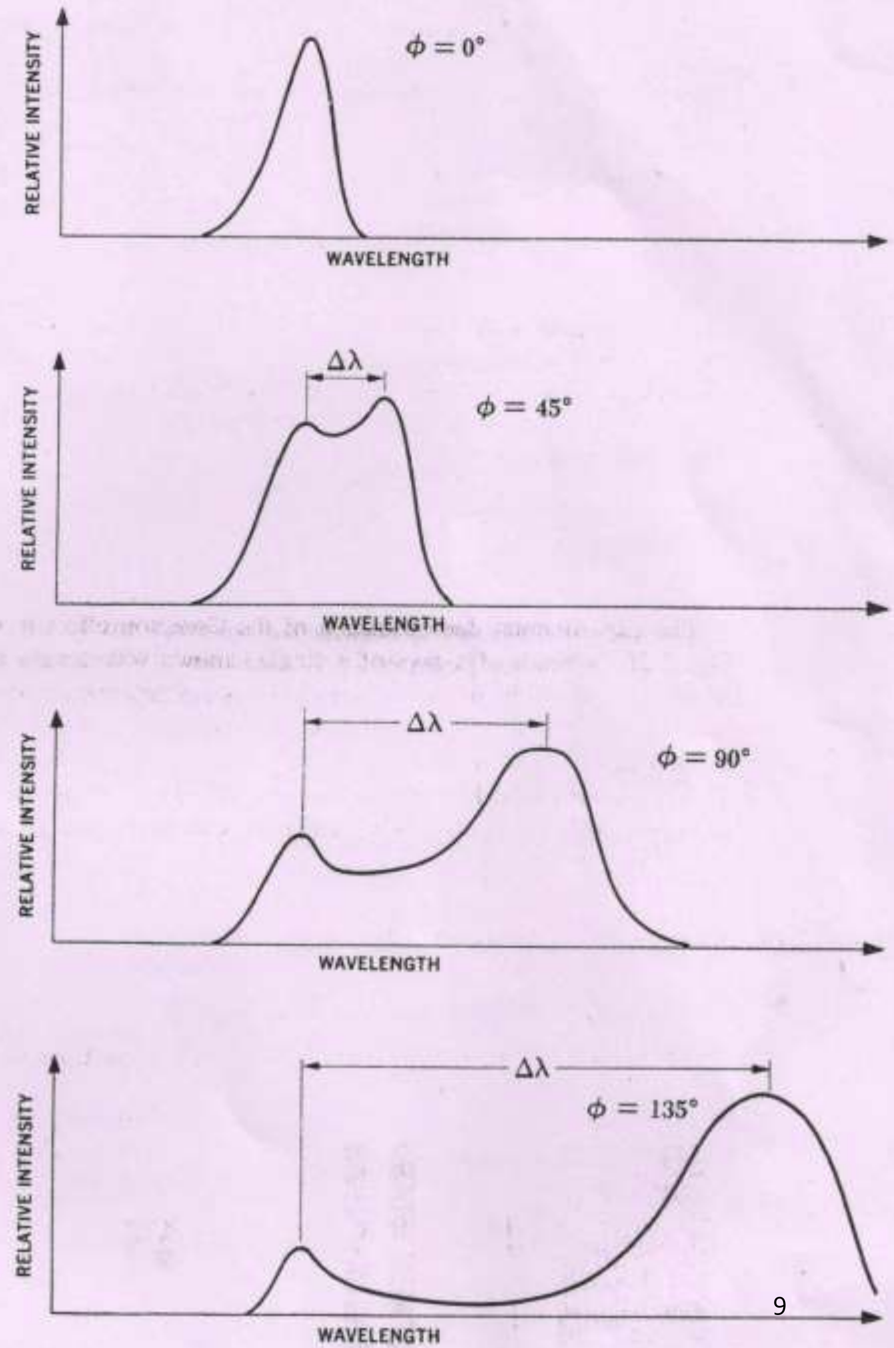
$$\frac{m_0c}{h}\left(\frac{1}{\lambda} - \frac{1}{\lambda'}\right) = \frac{1}{\lambda\lambda'}(1-\cos\phi)$$

$$or \quad \lambda' - \lambda = \frac{h}{m_0c}(1-\cos\phi)$$

Compton Scattering



Compton Scattering



1 Compton scattering

Compton Scattering

The wavelength of the scattered photon is given by well known Compton shift formula

$$\lambda' - \lambda = \frac{2h}{m_e c}\sin^2\frac{\theta}{2}$$

It is based on two assumptions

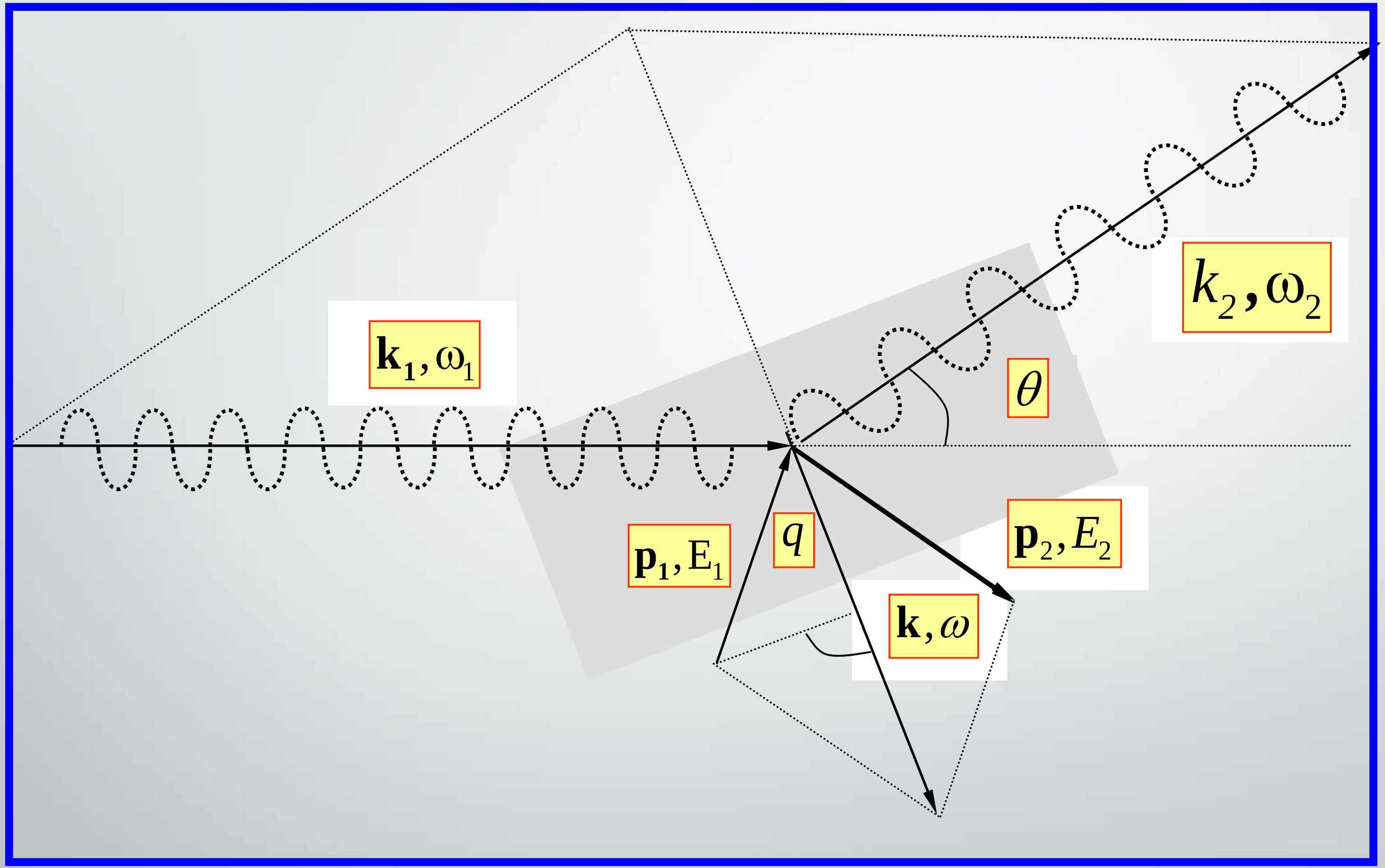
(i) electron is free and

(ii) electron is at rest.

First assumption: Valid if $\hbar\omega_1 >> E_B$

Second assumption: Too simplified since electrons in a solid have a continuous distribution of momenta.

Compton scattering in real materials



In a.u. e = ħ = m = 1, c = 137.036

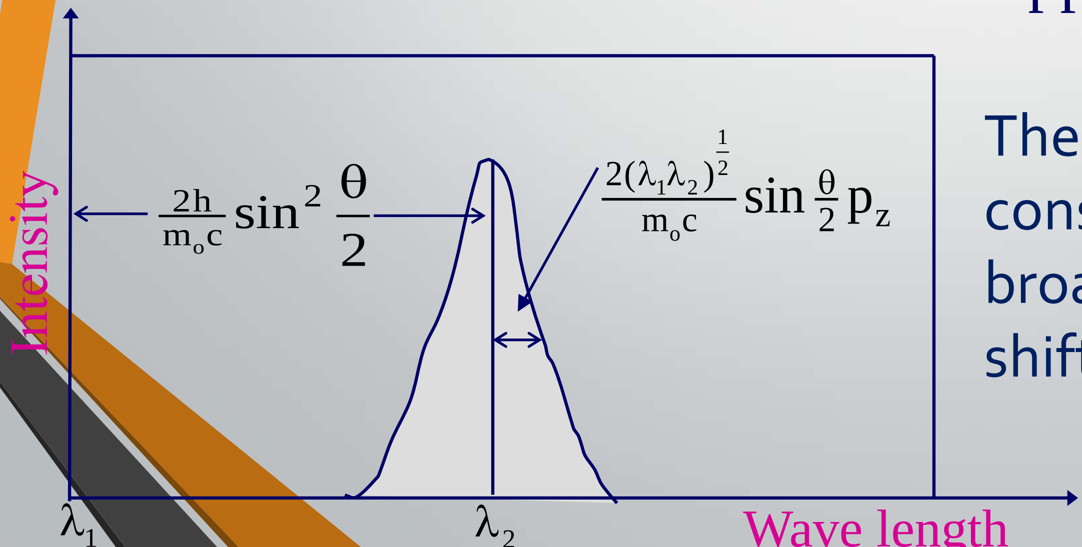
1 a.u. = 1.99x10^{-24} kg m sec^{-1}

Compton scattering in real materials

For real type of material, it is found that the Compton shift Δλ can be given as

$$\boxed{\Delta\lambda = \frac{2h}{m_0 c}\sin^2\frac{\theta}{2} + 2(\lambda_1\lambda_2)^{1/2}\left(\frac{p_z}{m_0 c}\right)\sin\frac{\theta}{2}}$$

Recoil term   Doppler term



The Compton profile can be considered as the Doppler broadening of the Compton shifted line.

Compton profile

If $\psi(\vec{\mathbf{r}})$ is the electron wave function in position space, then electron wave function in momentum space can be derived as

$$\chi(\vec{p}) = \frac{1}{(2\pi\hbar)^{\frac{3}{2}}} \int \psi(\vec{r}) \exp\left(-i \frac{\vec{p}.\vec{r}}{\hbar}\right) d^3\vec{r}$$

Conversation of energy and momentum (in a.u.) dictates as:

$$\vec{k}_1 + \vec{p}_1 = \vec{k}_2 + \vec{p}_2$$

$$\omega_1 + E_1 = \omega_2 + E_2$$

Compton profile

In Compton scattering, only scattered particle is detected, therefore:

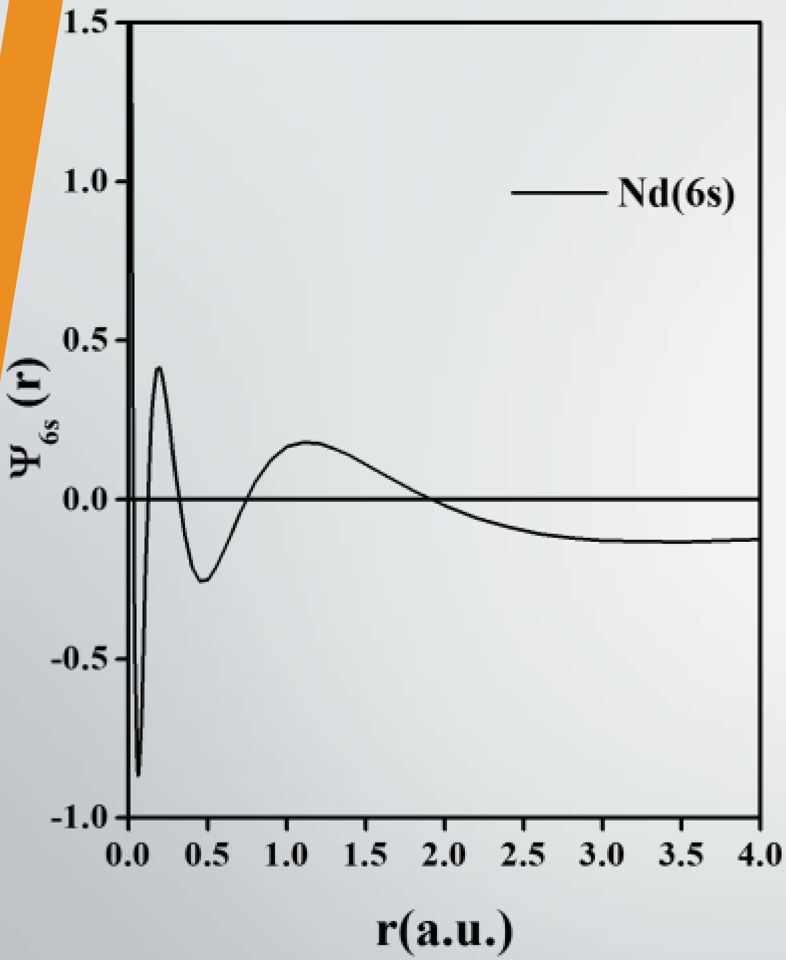
$$\Delta\omega=\omega_2-\omega_1=E_2-E_1=\frac{(\vec{k}+\vec{p}_1)^2}{2m}-\frac{p_1^2}{2m}=\frac{\left|\vec{k}^2\right|}{2m}+\frac{\vec{k}.\vec{p}_1}{m}$$

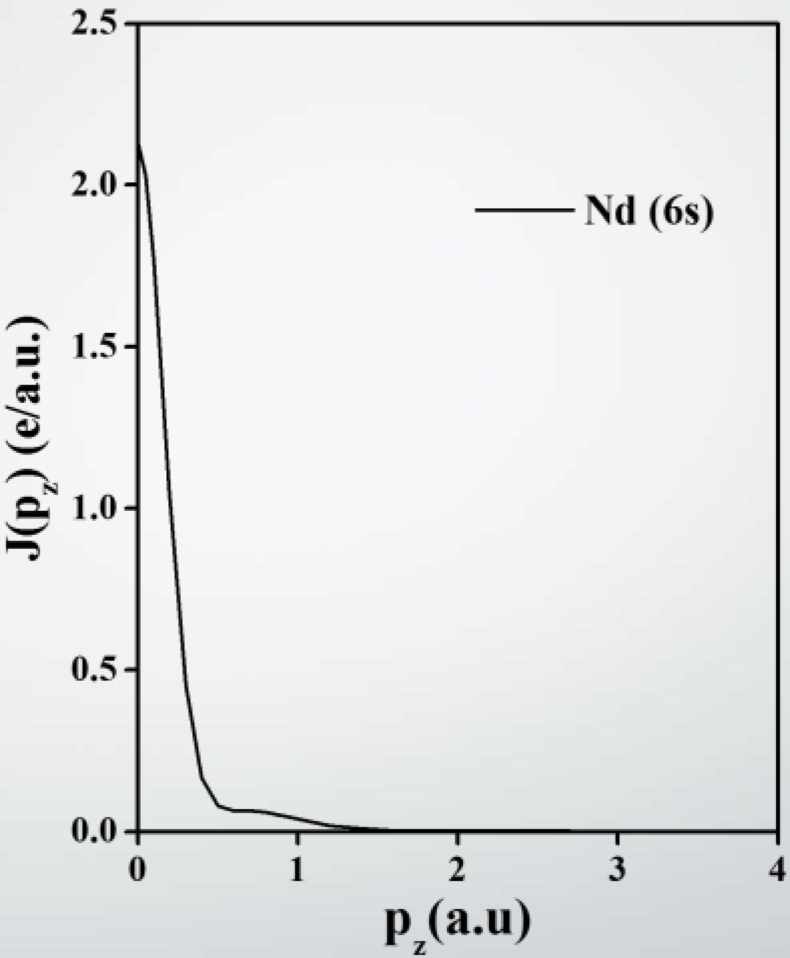
For solids, the electron momentum density $\rho(\vec{p})$ is

$$\rho(\vec{p})=\sum\left|\chi_i(\vec{p})\right|^2$$

Then Compton profile is calculated by

$$J(p_z)=\iint\rho(\vec{p})\,dp_x dp_y$$





Wave function Ψ(r) of 6s electrons of Nd

Corresponding Compton profile $J(p_z)$ of 6s electrons of Nd

Use of Compton scattering

Physics and Chemistry:

✓ Electronic structure of materials; i.e.
Metals: Electronic states, Fermi surface topology
Alloys : Charge transfer, Fermi surface topology, bonding, etc.
Compounds : Bonding, charge transfer, structural parameters, etc.

Biology:

✓ Bone mineral density (BMC)
✓ Protein bonding in HIV positive and Plague, etc. affected patients.
✓ Cancer tissues

Others: Civil Engineering, Chemical Engineering, Electrical/ Electronics Engineering, Mining……….

TECHNIQUES

- Incident photon:

1. X-rays (Cu K_a, Mo K_a, Ag K_a)
2. γ-rays (^{241}Am, ^{137}Cs, ^{123}Te, ^{51}Cr, ^{198}Au)
3. Synchrotron radiation

- Detection system:

1. Scintillation detectors
2. Solid state germanium detectors

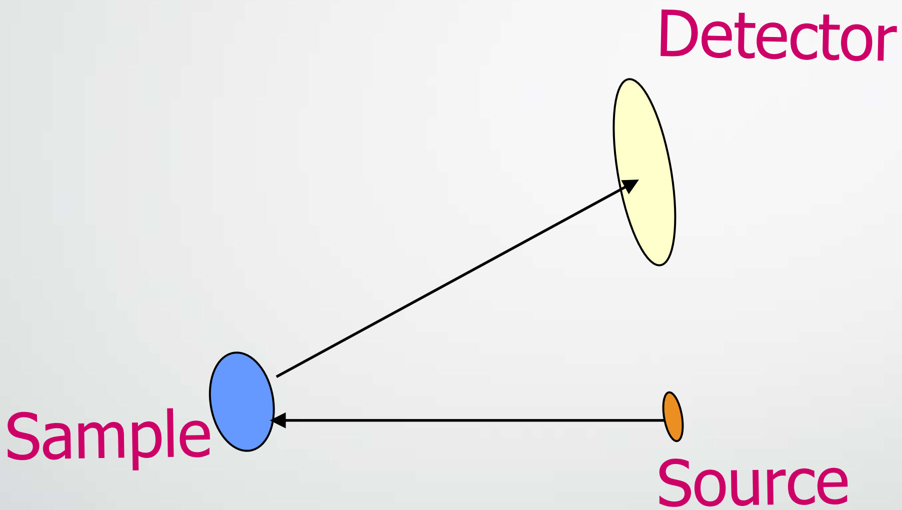
LIMITATION OF X-RAY COMPTON SPECTROMETER

- Lack of mono-energetic X-rays source
- Low energy of incident radiation
- Small ratio of Compton to photoelectric cross-section
- Time consuming
- Low resolution, controlled by the size of slit

ADVANTAGES OF γ -RAY COMPTON SPECTROMETER RELATIVE TO X-RAY METHOD

- Highly monochromatic
- Wide range of incident energy radiation based on choice of radioisotopes.
- Better resolution

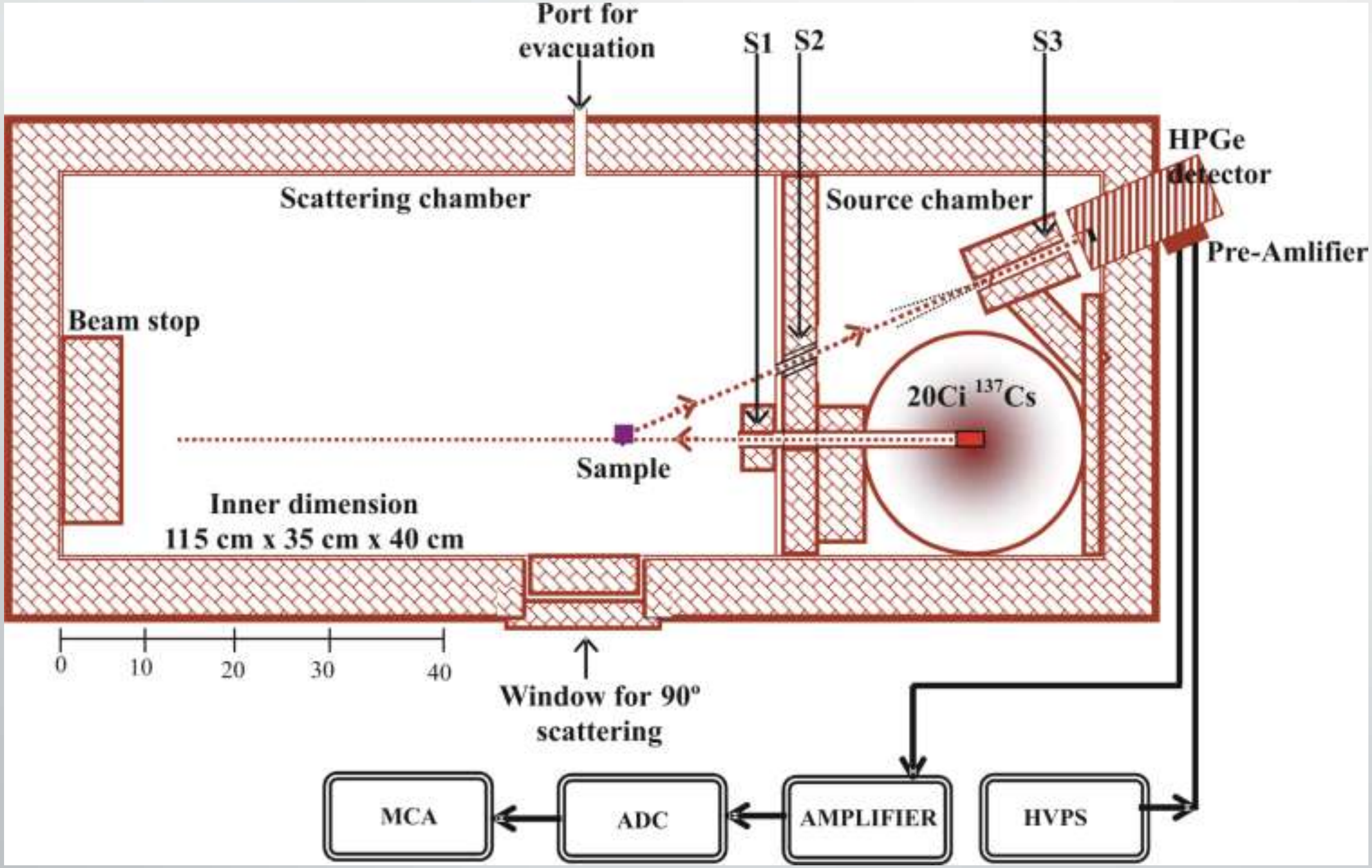
HOW TO DESIGN COMPTON SPECTROMETER



Design Criteria

1. Selection of source
2. Biological shielding of source
3. Maximum possible Compton count rate
4. Best possible resolution
5. Maximum possible scattering angle
6. Maintenance conditions
7. Data collection
8. Data processing

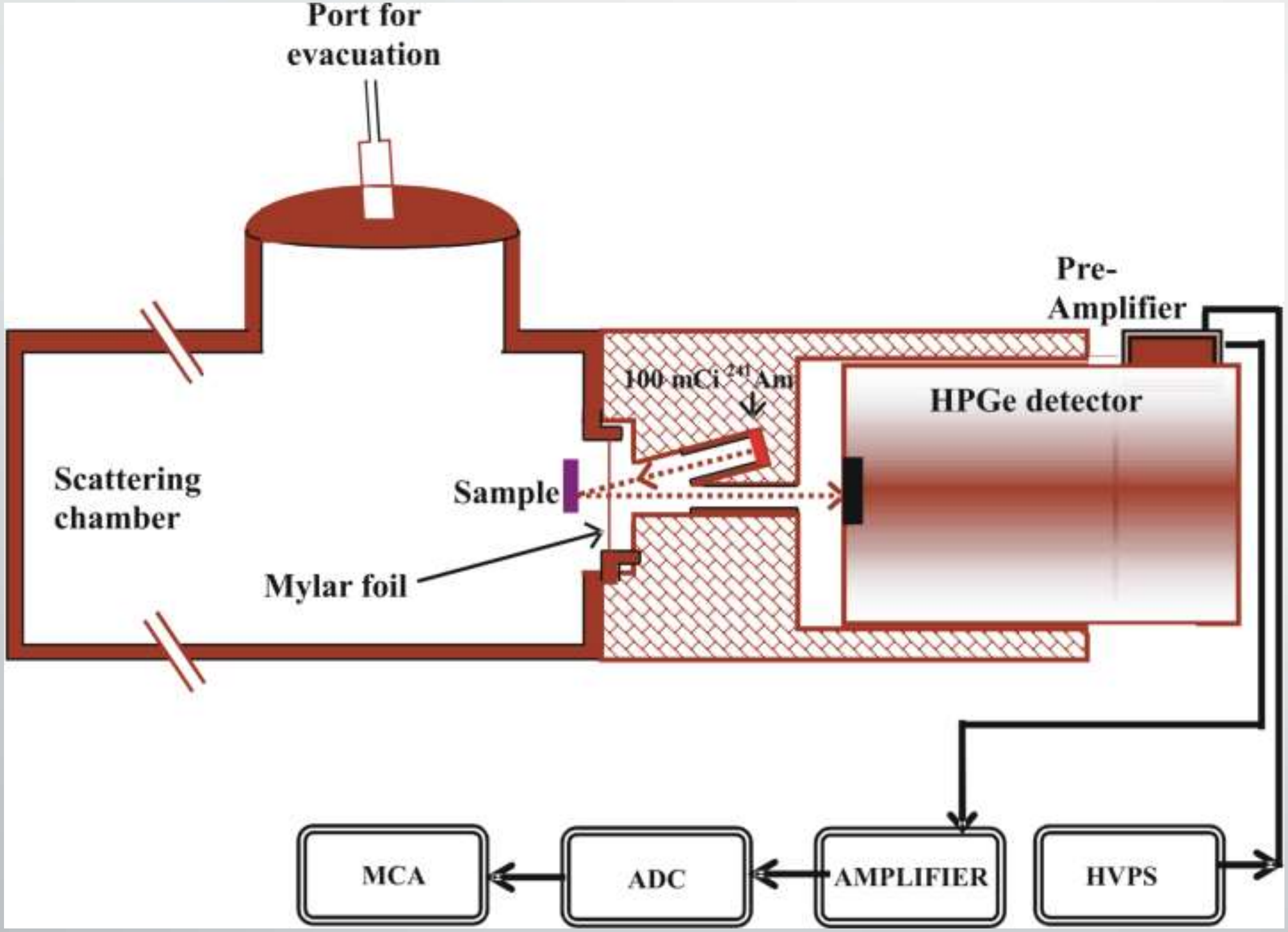
Layout of first Indian 20Ci ^{137}Cs Compton spectrometer in Udaipur



B. L. Ahuja, M. Sharma, S. Mathur, Nucl. Instrum. & Meth. Phys. Res. B 244 (2006) 419–426

B. L. Ahuja, R. Joshi, J. Sahariya, J. Exp. Nanosci., iFirst (2012) 1-8.

First–ever lowest intensity 100 mCi ^{241}Am Compton spectrometer

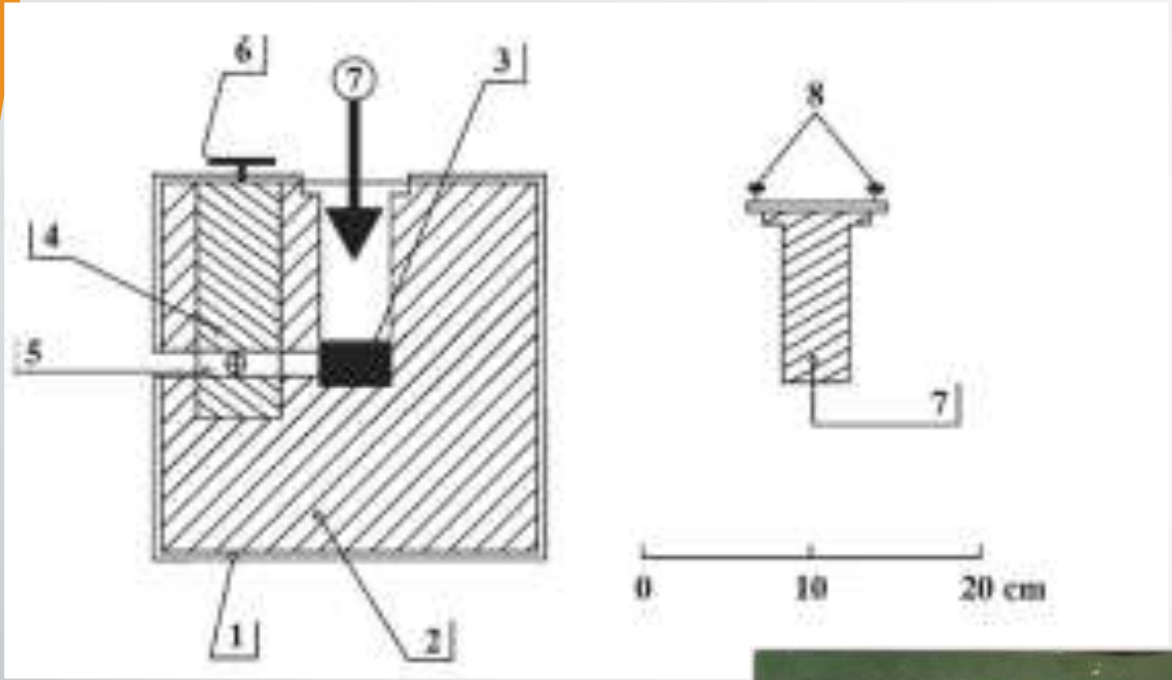


B. L. Ahuja ,V. Sharma, A. Rathor, A. R. Jani and B. K. Sharma, Nucl. Instrum. & Meth. Phys. Res. B 262 (2007) 391-398

Possible gamma-ray sources

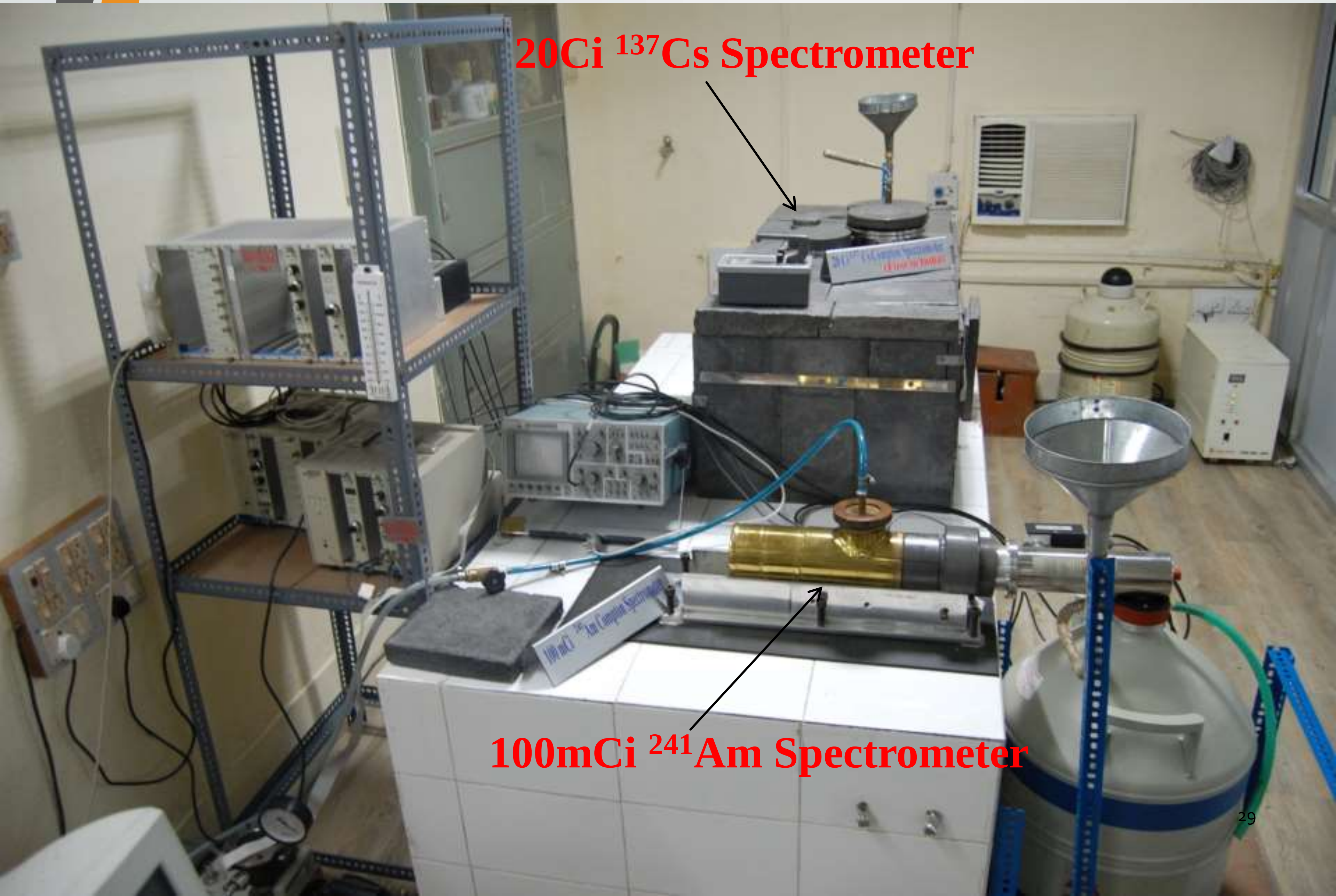
Source	Half life	Energy	Remarks
^{241}Am	432.2 yrs.	59.54 keV	Long half life
^{198}Au	2.7 days	412 keV	Very short half life, possible near nuclear reactor
^{51}Cr	27.70 days	320 keV	Very short half life & very expensive
^{123}Te	104 days	159 keV	Very short half life & very expensive
^{137}Cs	30 yrs.	661.65 keV	Very high energy

Biological shielding of source

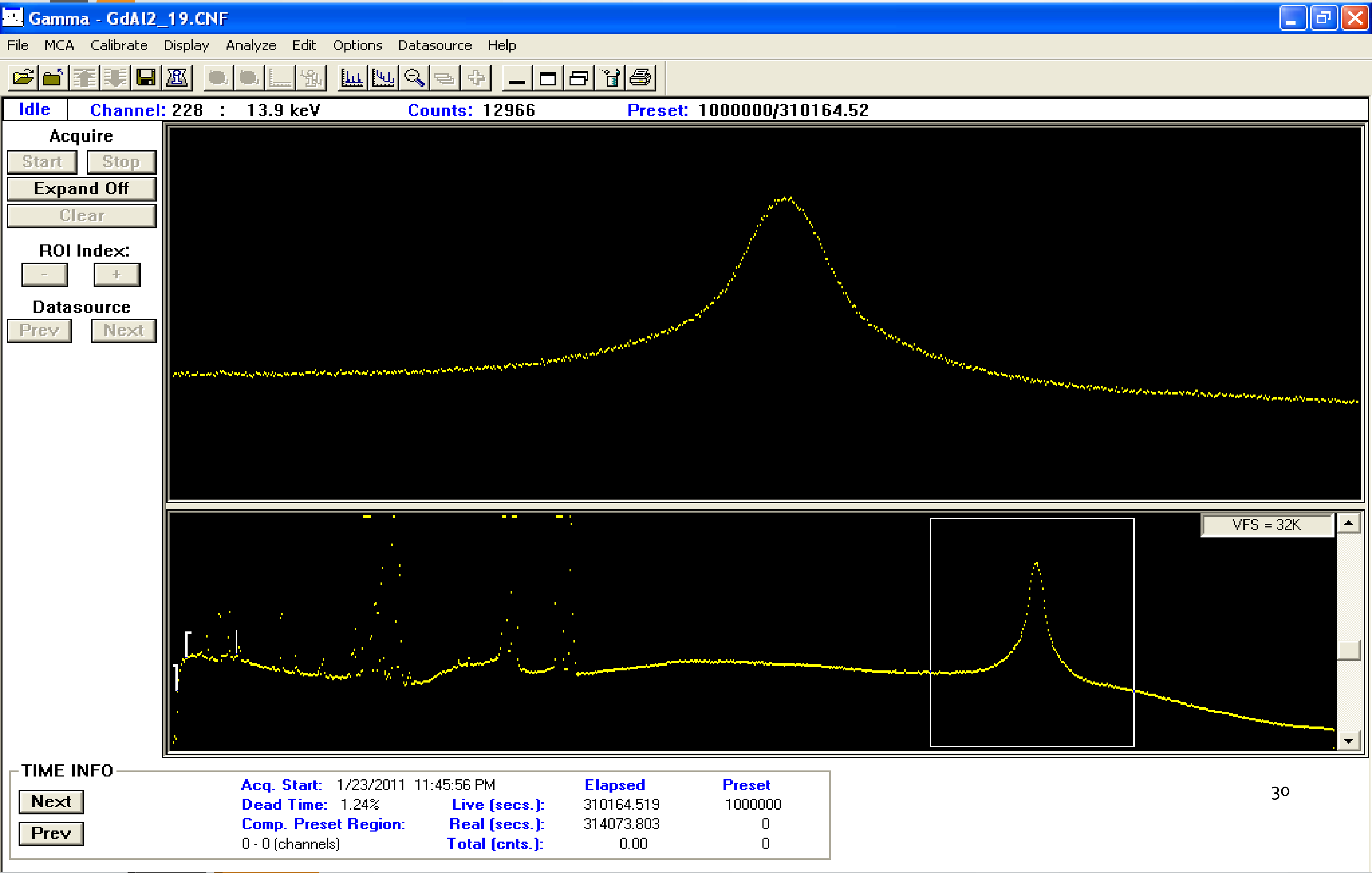



LEAD

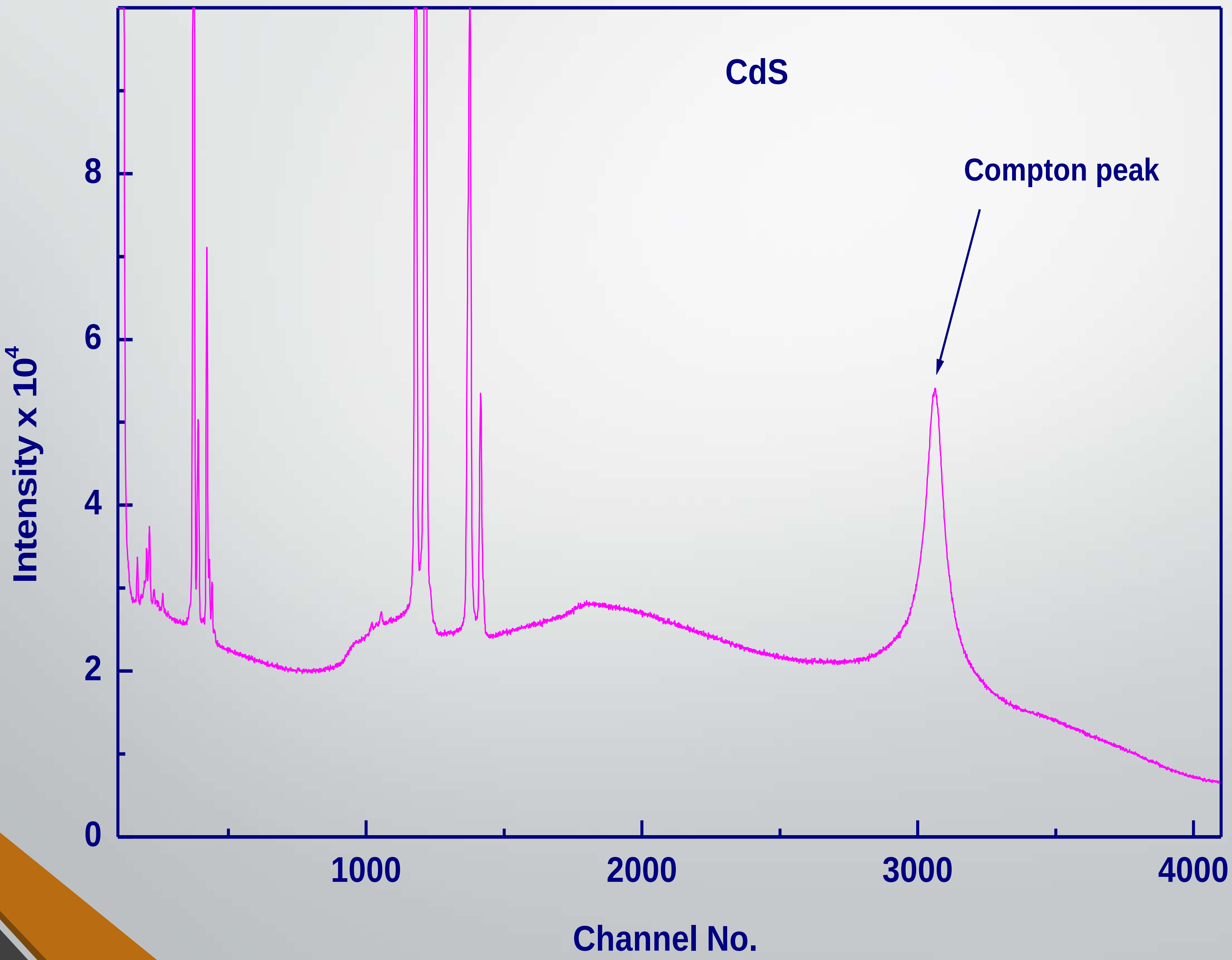
Laboratory view of Compton spectrometers



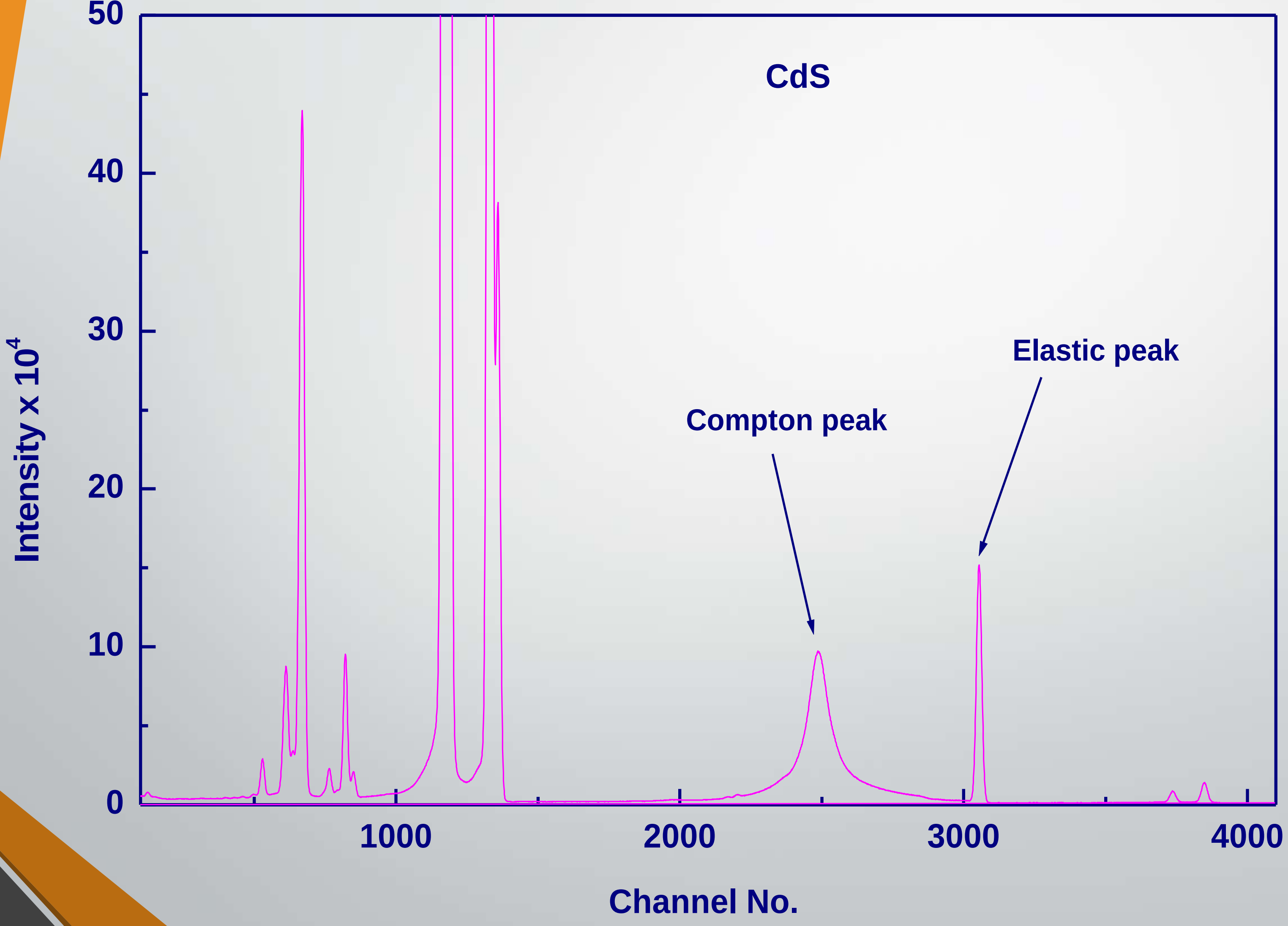
Data visualization
^{137}Cs Compton spectrometer



Representative Raw Data (CdS) using ^{137}Cs Compton spectrometer



Representative Raw Data (CdS) using ^{241}Am Compton spectrometer



Control of disturbing factors: Data

Raw data

Background correction

Detector efficiency correction

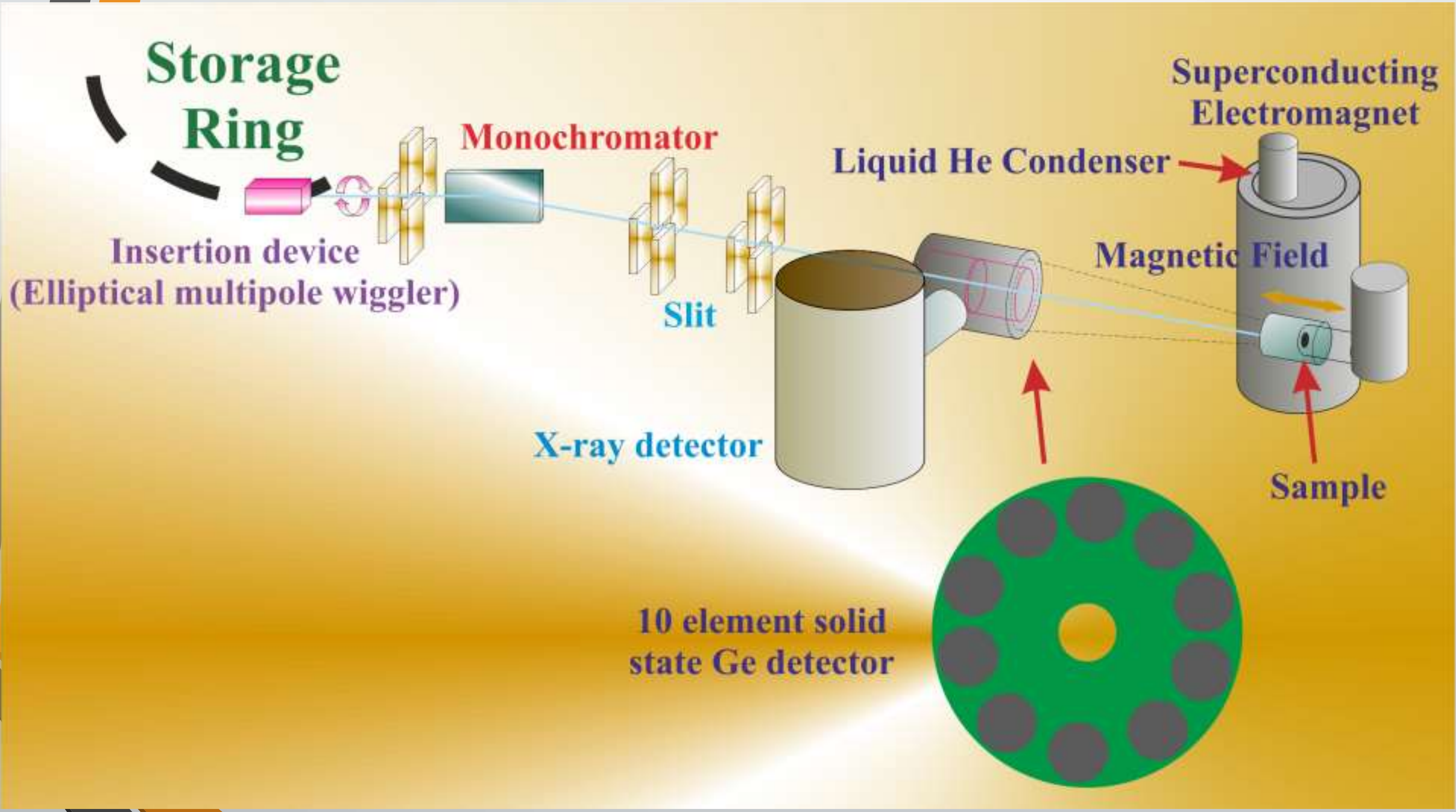
Absorption correction

Compton cross-section correction

Multiple scattering correction

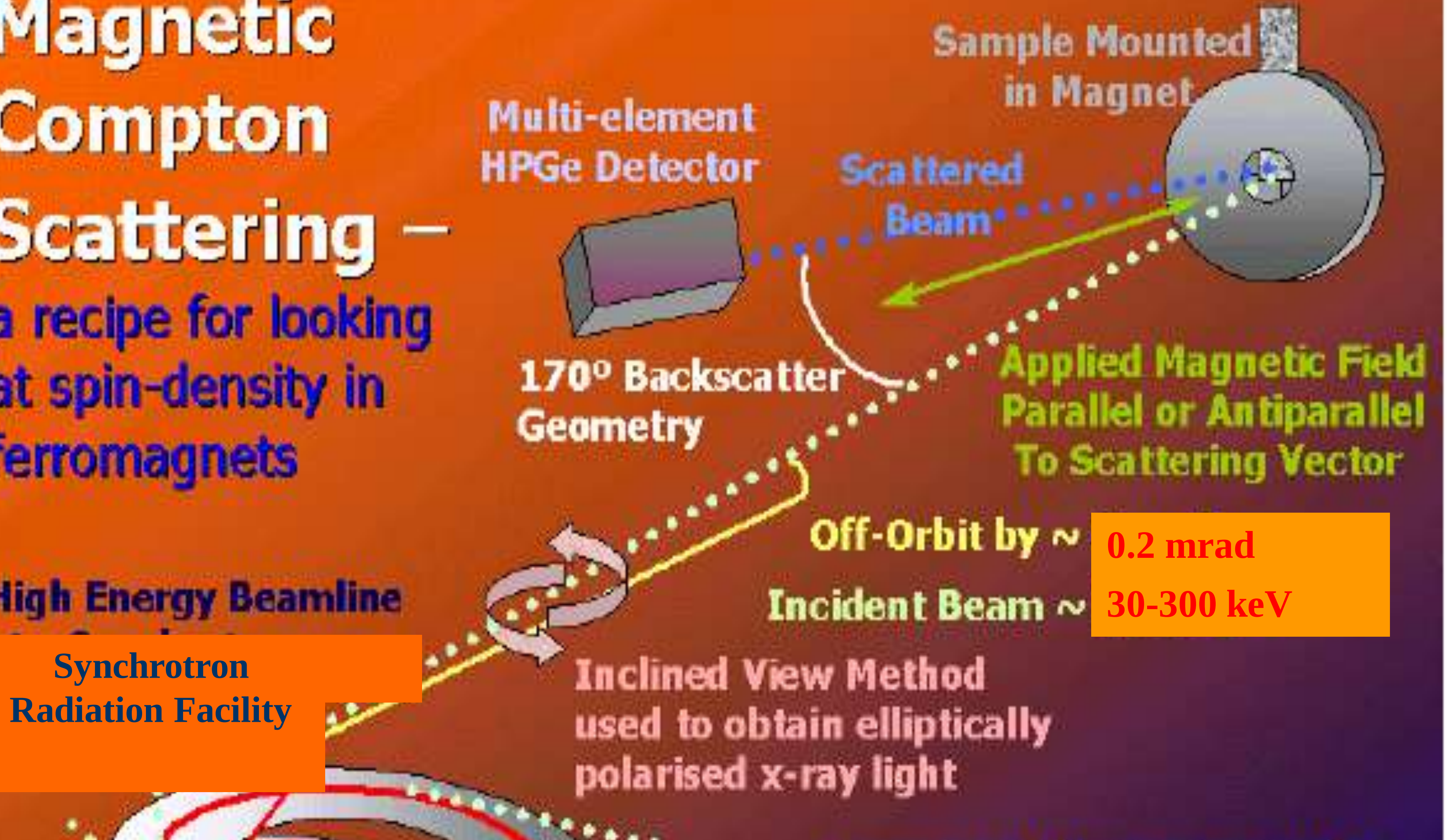
True experimental Compton profile

Schematic layout of magnetic Compton spectrometer available at SPring-8, Japan



Magnetic Compton Scattering –

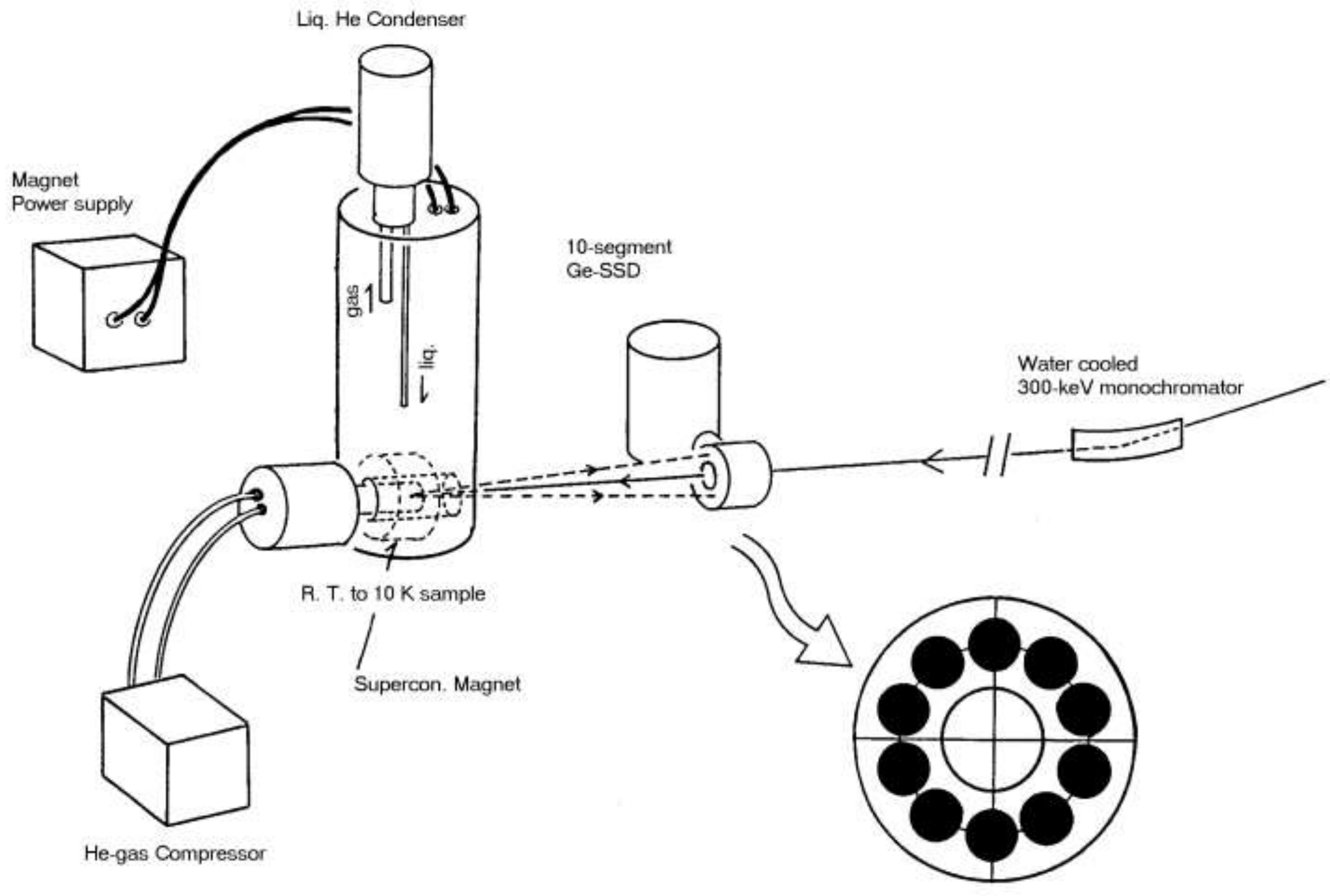
a recipe for looking at spin-density in ferromagnets



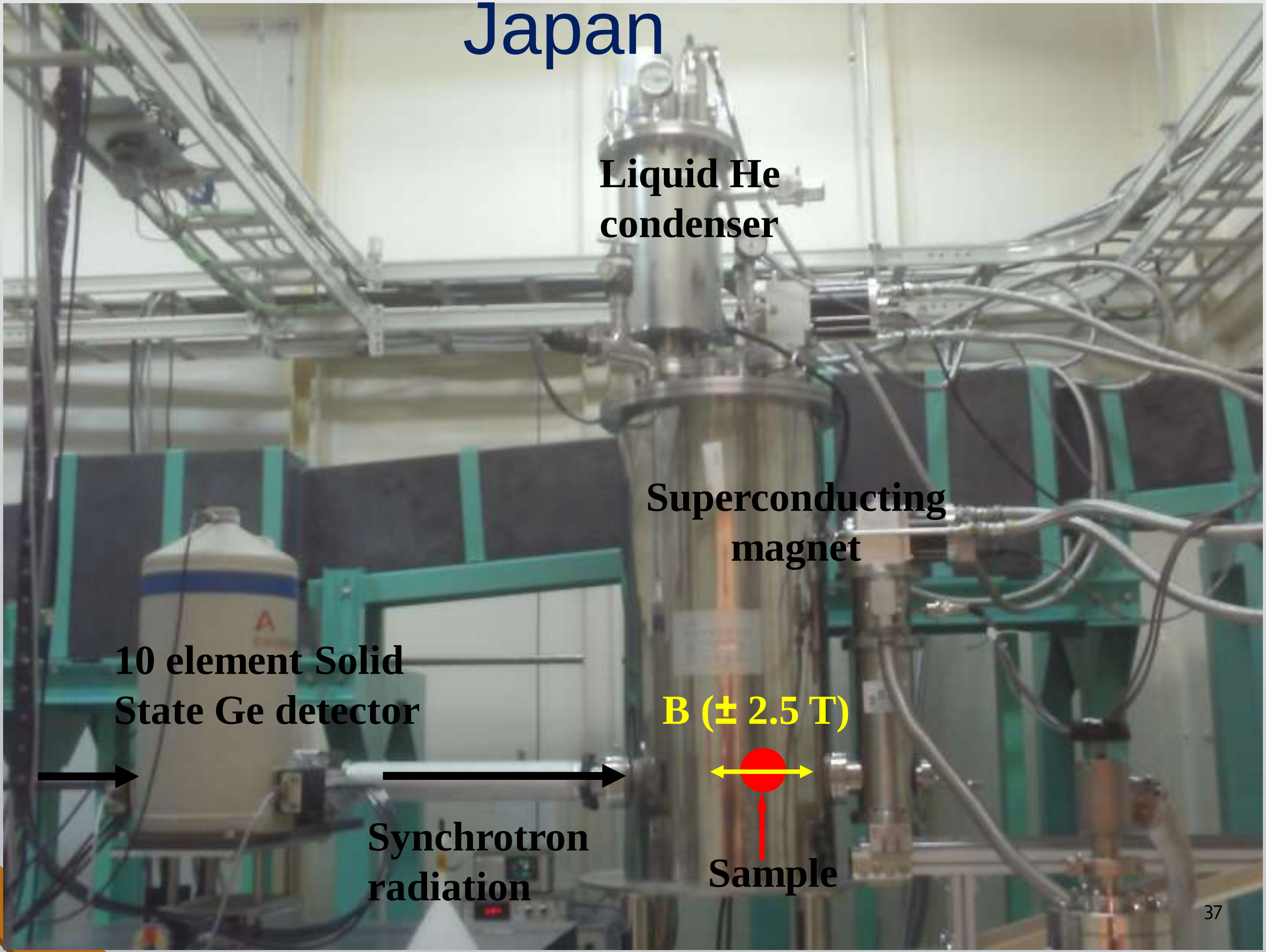
...an unambiguous way to measure the spin component of magnetisation

Experimental set-up for MCP at SPring8, Japan

Y. Kakutani, Y. Kubo, A. Koizumi, N. Sakai, B. L. Ahuja and B. K. Sharma, J. Phys. Soc. Japan *72*, 599 (2003).



Magnetic Compton Spectrometer: SPring-8, Japan



Experimental Features

- Sample size : Approx. 1.2 mm x 1.2 mm x 3.0 mm
- Beam size : 1.1 mm x 1.1 mm
- Incident energy (Si 620 reflection) : 175 keV
- Scattering angle : 178°
- Magnetic field in the sample : + – – + – + + – -----------

where + and – represent the relative directions of magnetic field and [+ being parallel]

- Switching time : 3 to 6 sec
- Dwell time : 60 sec

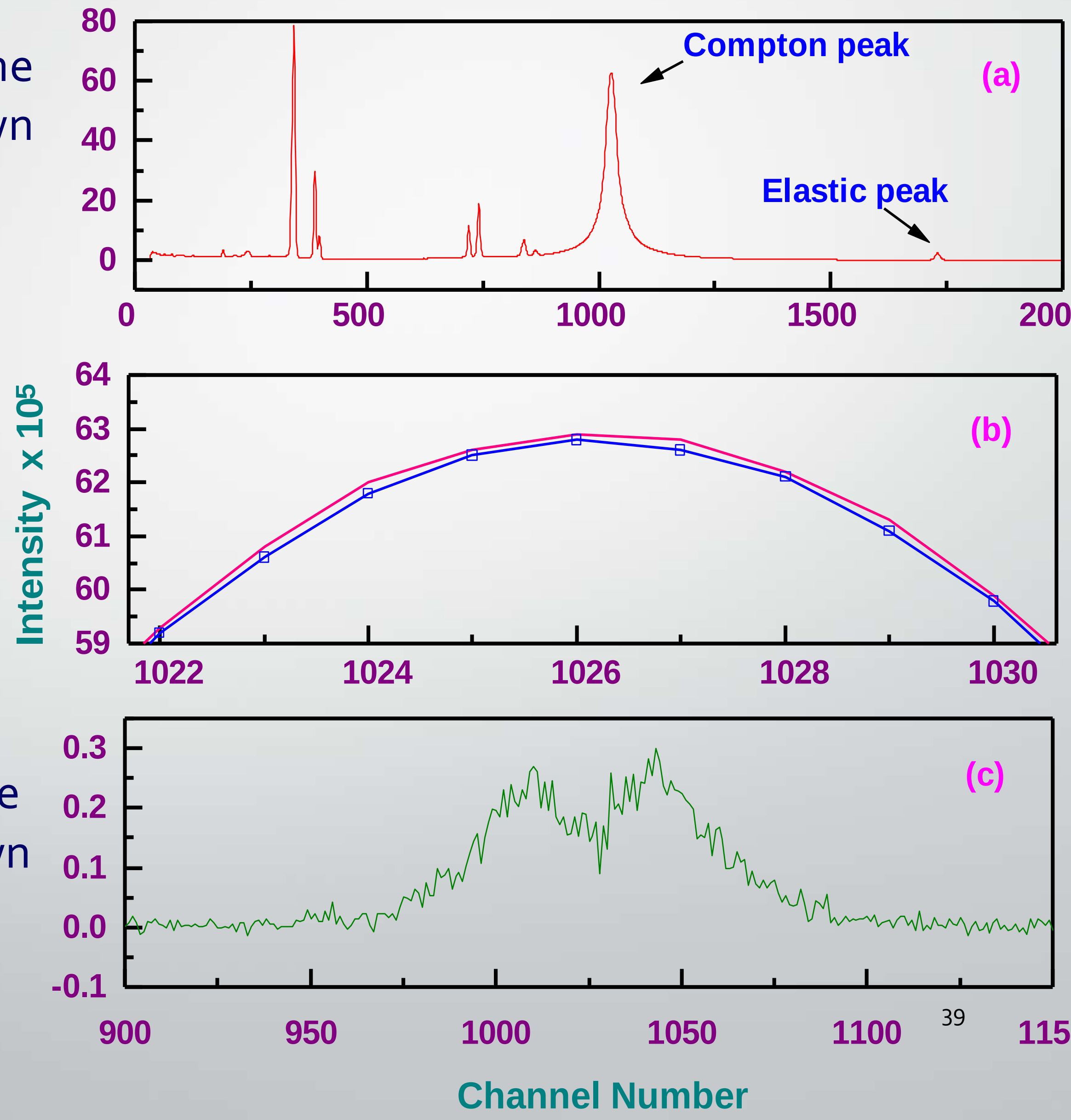
(Peak brightness : 1.378 x 10^{17} ph.s^{-1}.$mrad^{-1}$.mm^{-2} per 0.1% BW from elliptical multipole wiggler)

How to extract MCP ?

Energy spectrum from the spin-up and spin-down scatterers .

Same as Fig (a) but with the selected channel numbers viz. 1021 to 1031.

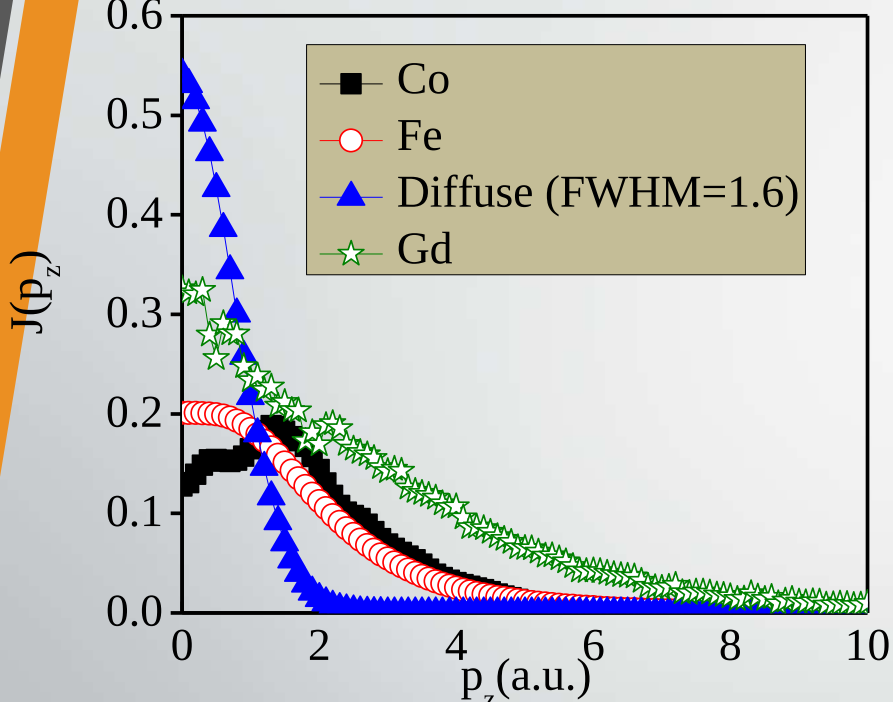
Raw magnetic Compton profile (spin-up data – spin-down data)

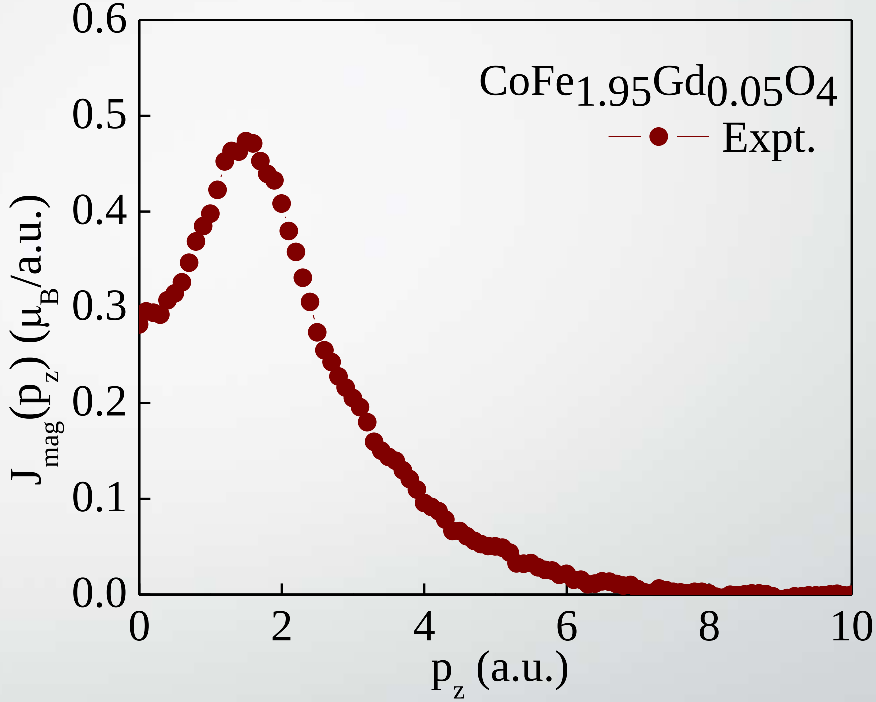


Data Correction

- Beam decay correction
- Background correction (cancels in MCP)
- Absorption correction
- Cross-section correction
- Multiple scattering correction (Small: may not be required)

Representative fitting of MCP





$$S = \sum_{p_z} \left[J_{mag}^{CFGO}(pz) - pJ_{mag}^{Co}(p_z) - q_{mag}^{Fe}(p_z) - rJ_{mag}^{Gd}(p_z) - sJ^{diffuse}(p_z) \right]^2$$


Thank you & Best
of luck !