

Advantages of a Variational principle formulation

" It is possible to extend this to non-mechanical systems.

(2) For eg. consider a Lagrangian L for a system:

$$L = \frac{1}{2} \sum_j L_j \dot{q}_j^2 + \frac{1}{2} \sum_{j \neq k} M_{jk} \dot{q}_j \dot{q}_k - \sum_j \frac{q_j^2}{2C_j} + \sum_j E_j(t) q_j \quad \text{with a dissipation } F_D$$

$$F_D = \frac{1}{2} \sum_j R_j \dot{q}_j^2$$

(3) The resulting Lagrangian's eqns. are: (For a particular j)

$$L_j \frac{d^2 q_j}{dt^2} + \sum_{k \neq j} M_{jk} \frac{d^2 q_k}{dt^2} + R_j \frac{dq_j}{dt} + \frac{q_j}{C_j} = E_j(t) \quad (1)$$

If this is an electrical system, it describes a system of coupled circuits with:

- L_j 's $\rightarrow$ self inductances
- M_{jk} 's $\rightarrow$ mutual inductances
- R_j 's $\rightarrow$ resistances
- C_j 's $\rightarrow$ capacitances
- q_j 's $\rightarrow$ charges $\rightarrow \dot{q}_j \rightarrow$ currents
- E_j 's $\rightarrow$ external e.m.f.s.

11) Alternatively it can describe a set of coupled springs ~~obeying~~ obeying Hooke's law with $L_{j,k}$ & $M_{j,k}$ being masses (inertial terms)

$\frac{1}{C_{j,k}}$ $\rightarrow$ the spring constants &

q_k $\rightarrow$ displacement

& E_j 's $\rightarrow$ time varying forces

The dissipative forces originate from friction.

How H can be conserved & yet not be the total energy & vice-versa.

12) The precise way to obtain a L , the lagrangian is: $L = T - V$

(2) Thus a change of the generalized co-ordinates may change the functional appearance of L but not its magnitude.

(3) However, $\because H$ depends in both magnitude & functional form on the co-ordinates, a change of co-ordinates can lead to a completely different Hamiltonian