How H can be conserved & yet not be the total energy & vice-versa.

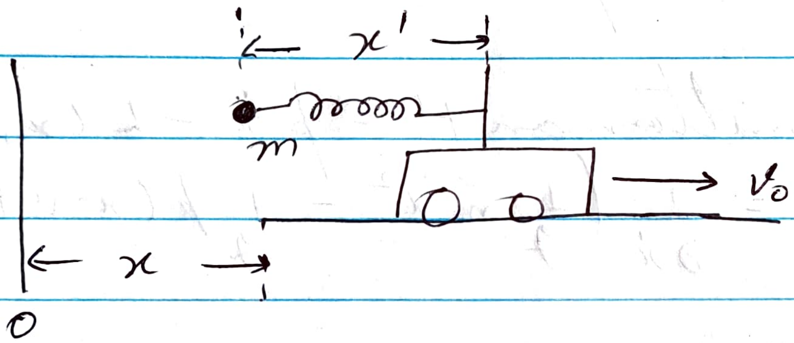
(1) The precise way to obtain ~~a~~ L, the lagrangian is: $L = T - V$

(2) Thus a change of the generalized co-ordinates may change the functional appearance of L but not it's magnitude.

(3) However, $\because H$ depends in both magnitude & functional form on the co-ordinates, a change of co-ordinates can lead to a completely different Hamiltonian

Example:

(1) Consider a 1-d system consisting of a point mass 'm' attached to a spring of force constant k. The other end of the spring is attached to a massless cart.



(2) Let the cart be moved by an external source with uniform speed v_0.

(3) Considering the movement to be along the x-direction, let 'x' be the generalized co-ordinate for an observer at O.

(4) The Lagrangian $L(x, \dot{x}, t) = T - V$

$$T = \frac{1}{2} m \dot{x}^2 \quad \& \quad V = \frac{1}{2} k (x - v_0 t)^2$$

(5) For an observer at O, in a fixed time 't', the spring moves forward through a distance of ~~xo~~ $v_0 t$ & $\therefore$ the displacement of the spring alone

is $(x - v_0 t)$ $\therefore$ ~~$\mathring{x}$~~ $\therefore$ ~~$T = \frac{1}{2} m\left(\frac{d(x - v_0 t)}{dt}\right)$~~

~~$\frac{1}{2} m\dot{x}^2$~~ & $V = \frac{1}{2} k (x - v_0 t)^2$

$\therefore$ the equation of motion is:

$m\ddot{x} + k(x - v_0 t) = 0$ which is an S.H.M + linear displacement.

(b) $\therefore$ The Hamiltonian $H = p\dot{x} - L(x, \dot{x}, t)$

But $p = \frac{\partial L}{\partial \dot{x}} = \frac{\partial}{\partial \dot{x}}\left(\frac{1}{2} m\dot{x}^2 - \frac{1}{2} k(x - v_0 t)^2\right)$

$= m\dot{x}$

$\therefore H = m\dot{x}^2 - \left[\frac{1}{2} m\dot{x}^2 - \frac{1}{2} k(x - v_0 t)^2\right]$

$= \frac{1}{2} m\dot{x}^2 + \frac{1}{2} k(x - v_0 t)^2 = T + V.$ — (1)

$\Rightarrow$ By choosing x as a generalized co-ordinate, H represents the total energy. But H is now an explicit fn. of t.

$\Rightarrow \frac{\partial H}{\partial t} = -\frac{\partial L}{\partial t} \neq 0$ — (2)

If H is conserved $\frac{\partial H}{\partial t} = 0$.

$\Rightarrow$ in this case B

while H represents the total (mechanical) energy it is not conserved.

Instead, let x' be generalized co-ordinate. Now the oscillation of the spring will only involve x' while the

K.E $= \frac{1}{2} m\dot{x}^2 = \frac{1}{2} m(\dot{x}' + v_0)^2$

$$\therefore L' = \frac{1}{2} m(\dot{x}' + v_0)^2 - \frac{kx'^2}{2} = \frac{1}{2} m(\dot{x}'^2 + v_0^2 + 2\dot{x}' v_0) - \frac{kx'^2}{2} \quad \text{— (1)}$$

$$\therefore H' = m\dot{x}'^2 - \frac{m\dot{x}'^2}{2} - m\dot{x}' v_0 - \frac{mv_0^2}{2} + \frac{kx'^2}{2}$$

$$= \left(\frac{1}{2} m\dot{x}'^2 + \frac{1}{2} kx'^2\right) \cancel{- m\dot{x}' v_0} - \frac{mv_0^2}{2} \quad \text{— (3)}$$

If T ∵ in terms of x' alone,

$T' = \frac{1}{2} m\dot{x}'^2$ & $V' = \frac{1}{2} kx'^2$,

$H' \neq T' + V'$ But represents is conserved ∵ (3) is not explicitly time dependent.

The reason is the excess energy which flows into the system to keep the cart moving which is considered in (3) but not in (2).