Additian af a new nan-negative variable

Let us cansider a L.P.P

$$\text{Max } Z = CX$$

$$\text{s.t } AX \leq b$$

$$\text{and } X \geq 0$$

Let optimal basic feasible salutian X_B has obtained,

Naw, if same new nan-negative variable x_{n+1} having optimality variable a_{n+1} and carrespanding coefficient variable c_{n+1} is added to the given prablem, then

$$X_{n+1} = B^{-1} a_{n+1} \quad \text{and}$$

$$\Delta_{n+1} = Z_{n+1} - C_{n+1}$$

where B is current optimal basic then there are two optimal basis

i) $\Delta_{n+1} \geq 0$ ii) $\Delta_{n+1} < 0$

1. $\Delta_{n+1} \geq 0$

The current optimal basic salution remain optimal far the new salutian

2. $\Delta_{n+1} < 0$

Applied the usual simplex method in arder to find the optimal basic feasible salutian

Q. consider to addition of a new variable x_5 to the foll" L.P.P (called the original primal)

$$\text{Max } z = 3x_1 + 5x_2$$

$$\text{s.t } x_1 + \quad + x_3 = 4 \Rightarrow x_1 + 0\cdot x_2 + 1\cdot x_3 + 0\cdot x_4 = 4$$

$$3x_1 + 2x_2 + x_4 = 18 \Rightarrow 3x_1 + 2x_2 + 0\cdot x_3 + 1\cdot x_4 = 18$$

$$\text{and } x_1, x_2, x_3, x_4 \geq 0$$

So that its revised form becomes:

$$\text{Max } z = 3x_1 + 5x_2 + 7x_5$$

$$\text{s.t; } x_1 + \quad + x_5 = 4$$

$$3x_1 + 2x_2 + x_4 + 2x_5 = 18$$

$$\text{and } x_1, x_2, \ldots x_5 \geq 0$$

Here $a_5 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$

sol:

by simplex method the final table is

		c_j	3	5	0	0
	C_B	X_B	x_1	x_2	x_3	x_4
x_3	0	4	1	0	1	0
x_2	5	9	3/2	1	0	1/2
$z=45$		$z_j - c_j$	9/2	0	0	5/2

the optimal sol" is

$$x_1 = 0, \; x_2 = 9, \; x_3 = 4, \; x_4 = 0$$

$$\text{Max } z = 45$$

$$\boxed{X_{n+1} = B^{-1} a_{n+1}} \quad \text{where } n = 4$$

$$X_5 = B^{-1} a_5$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & 1/2 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

$$= \begin{pmatrix} 1\times 1 + 0\times 2 \\ 0\times 1 + 1/2\times 2 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

So our new table including the new vector x_5 is

	c_B	c_j / X_B	3 / x_1	5 / x_2	0 / x_3	0 / x_4	7 / x_5	
x_3	0	4	1	0	1	0	$\boxed{1}$	$4/1 = 4 \leftarrow$
x_2	5	9	3/2	1	0	1/2	1	9/1
		$z_j - c_j$	9/2	0	0	5/2	$-2 \uparrow$	

$R_2 \to R_2 - R_1$

	c_B	c_j / X_B	3 / x_1	5 / x_2	0 / x_3	0 / x_4	7 / x_5
x_5	7	4	1	0	1	0	1
x_2	5	5	1/2	1	-1	1/2	0
		$z_j - c_j$	13/2	0	2	5/2	0

Since all $\Delta_j \geq 0$

So the new optimal solution

$x_1 = 0,\ x_2 = 5,\ x_3 = 0,\ x_4 = 0,\ x_5 = 4$

Max. $Z = 25 + 28$

$= 53$

Q. Let the optimal simplex table for a maximize problem (with all the ~~constant~~ constraints with $\leq$ type

	C_B	c_j / X_B	5 / x_1	12 / x_2	4 / x_3	0 / x_4	$-M$ / x_5-(A_1)
x_2	12	8/5	0	1	$-1/5$	2/5	$-1/5$
x_1	5	9/5	1	0	7/5	1/5	2/5
$z=\frac{141}{5}$	z_j-c_j		0	0	3/5	29/5	$M-2/5$

where x_4 is slack and A_1 is an artificial variable. Let a new variable $x_5 \geq 0$ be introduced in the problem with a cost 30 assigned to it in the objective function. Also suppose that the coefficient of x_5 in the two constant are 5 and 7 resp. Discuss the effect of this addition of a variable on the optimal solution to the given problem

$X_5 = X_{4+1}$

sol:

$$B^{-1} = \begin{pmatrix} 2/5 & -1/5 \\ 1/5 & 2/5 \end{pmatrix}, \quad a_5 = \begin{pmatrix} 5 \\ 7 \end{pmatrix}$$

$$X_5 = B^{-1} a_5$$

$$= \begin{pmatrix} 2/5 & -1/5 \\ 1/5 & 2/5 \end{pmatrix} \begin{pmatrix} 5 \\ 7 \end{pmatrix}$$

$$= \begin{pmatrix} 2/5 \times 5 - 1/5 \times 7 \\ 1/5 \times 5 + 2/5 \times 7 \end{pmatrix} = \begin{pmatrix} 2 - 7/5 \\ 1 + 14/5 \end{pmatrix}$$

$$= \begin{pmatrix} 3/5 \\ 19/5 \end{pmatrix}$$

		c_j	5	12	4	0	30	-M	Min $\frac{X_B}{x_j}$, $x_j \geq 0$
	C_B	X_B	x_1	x_2	x_3	x_4	x_5	A_1	
x_2	12	8/5	0	1	-1/5	2/5	3/5	-1/5	8/3
x_1	5	9/5	1	0	7/5	1/5	$\boxed{19/5}$	2/5	9/19
	$z_j - c_j$		0	0	3/5	29/5	-19/5 ↑	M-2/5	

$R_2 \to \frac{5}{19} R_2$ $\qquad$ $R_1 \to R_1 - 3/5 R_2$

		c_j	5	12	4	0	30	-M
	C_B	X_B	x_1	x_2	x_3	x_4	x_5	A_1
x_2	12	25/19	-3/19	1	-8/19	7/19	0	-5/19
x_5	30	9/19	5/19	0	7/19	1/19	1	2/19
	$\Delta_j = z_j - c_j$		1	0	2	6	0	M

$-\frac{1}{5} - \frac{3}{5} \times \frac{7}{19} = \frac{-19-21}{19 \times 5} = \frac{-\cancel{40}^{8}}{19 \times \cancel{5}}$ $\qquad$ $\frac{2}{5} - \frac{3}{5} \times \frac{1}{19} = \frac{38-3}{19 \times 5} = \frac{\cancel{35}^{7}}{19 \times \cancel{5}}$

$8/5 - \frac{3}{5} \times \frac{9}{19} = \frac{152-27}{19 \times 5} = \frac{\cancel{125}^{25}}{19 \times \cancel{5}}$ $\qquad$ $-\frac{1}{5} - \frac{3}{5} \times \frac{2}{19} = \frac{-19-6}{19 \times \cancel{5}}^{25}$

So the new optimal basic feasible solution becomes

$x_1 = 0, \; x_2 = 25/19, \; x_3 = 0, \; x_4 = 0, \; x_5 = 9/19$

with $\text{Max } z = 5x_1 + 12x_2 + 4x_3 + 30x_5$

$\text{Max } z = 12 \times \frac{25}{19} + 30 \times \frac{9}{19}$

$= 30$