Deletion of existing variable :

Let the existing variable x_p is deleted from the given L.P.P

$$\text{Max } z = CX$$
$$\text{s.t } AX \leq b$$
$$\text{and } X \geq 0$$

then we want to investigate the effect of this deletion on the optimality of the current optimal basic feasible solution X_B

There are two cases

(i) $x_p \in X_B$

(ii) $x_p \notin X_B$

$x_p \notin X_B$

(ii) it makes no difference i.e no effect on the solution

$x_p \in X_B$

(i) it ~~second~~ First case let $x_p = x_{Br} \in X_B$ then assign a largest negative cost $-M$ to x_p then applying simplex method to obtain the new optimal solution ($\Delta_j \geq 0$)

For case-(ii)

consider the previous example in which if we delete x_3 then it doesn't make any difference in the optimal solution, As it doesn't belongs to X_B

For case -(i)

consider the L.P.P

Q. (2)

$$\text{Max. } Z = 3x_1 + 4x_2 + x_3 + 7x_4$$

s.t. $8x_1 + 3x_2 + 4x_3 + x_4 \le 7$

$2x_1 + 6x_2 + x_3 + 5x_4 \le 3$

$x_1 + 4x_2 + 5x_3 + 2x_4 \le 8$

and $x_1, x_2, x_3, x_4 \ge 0$

find the optimal solution of this L.P.P and let the variable x_4 be deleted from the L.P.P

Sol:

$$\text{Max. } Z = 3x_1 + 4x_2 + x_3 + 7x_4 + 0x_5 + 0x_6 + 0x_7$$

s.t. $8x_1 + 3x_2 + 4x_3 + x_4 + x_5 = 7$

$2x_1 + 6x_2 + x_3 + 5x_4 + x_6 = 3$

$x_1 + 4x_2 + 5x_3 + 2x_4 + x_7 = 8$

and $x_1, x_2, \dots x_7 \ge 0$

The first simplex table is

basic variable	c_B	c_j / X_B	3 / x_1	4 / x_2	1 / x_3	7 / x_4	0 / x_5	0 / x_6	0 / x_7	
x_5	0	7	8	3	4	1	1	0	0	7/1
x_6	0	3	2	6	1	$\boxed{5}$	0	1	0	3/5
x_7	0	8	1	4	5	2	0	0	1	8/2
$z_j - c_j \rightarrow$			−3	−4	−1	−7 ↑	0	0	0	

$R_2 \rightarrow R_2/5$, $R_1 \rightarrow R_1 - R_2$, $R_3 \rightarrow R_3 - 2R_2$

	c_B	c_j / X_B	3 / x_1	4 / x_2	1 / x_3	7 / x_4	0 / x_5	0 / x_6	0 / x_7	
x_5	0	32/5	$\boxed{38/5}$	9/5	19/5	0	1	−1/5	0	32/38
x_4	7	3/5	2/5	6/5	1/5	1	0	1/5	0	3/2
x_7	0	34/5	1/5	8/5	23/5	0	0	−2/5	1	34
$z_j - c_j$			−1/5 ↑	22/5	2/5	0	0	7/5	0	

$R_1 \rightarrow \frac{5}{38} R_1$, $R_2 \rightarrow R_2 - \frac{2}{5} R_1$

$= \frac{83}{19}$

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$R_3 \to R_3 - \frac{1}{5} R_1$

	C_B	C_j / X_B	3 / x_1	4 / x_2	1 / x_3	7 / x_4	0 / x_5	0 / x_6	0 / x_7
x_1	3	16/19	1	9/38	1/2	0	5/38	-1/38	0
x_4	7	5/19	0	21/19	0	1	-1/19	4/19	0
x_7	0	126/19	0	59/38	9/2	0	-1/38	-15/38	1
		$Z_j - C_j \to$	0	169/38	1/2	0	1/38	53/38	0

The variable to be deleted is a basic variable so assign a cost $-M$ to x_4 and consider the optimum simplex table as a initially simplex table for the L.P.P

	C_B	C_j / X_B	3 / x_1	4 / x_2	1 / x_3	$-M$ / x_4	0 / x_5	0 / x_6	0 / x_7	Min
x_1	3	16/19	1	9/38	1/2	0	5/38	-1/38	0	32/9
x_4	$-M$	5/19	0	$\boxed{21/19}$	0	1	-1/19	4/19	0	5/21
x_7	0	126/19	0	59/38	9/2	0	-1/38	-15/38	1	252/59
		$Z_j - C_j$	0	$\frac{-125}{38} - \frac{21}{19}M$ ↑	1/2	0	$15/38 + \frac{M}{19}$	$\frac{-3}{38} - \frac{4M}{19}$	0	

$R_3 \to R_3 - \frac{59}{38} R_2$ $\qquad$ $R_1 \to R_1 - 9/38\, R_2$

	C_B	C_j / X_B	3 / x_1	4 / x_2	1 / x_3	$\cancel{-M}$ / $\cancel{x_4}$	0 / x_5	0 / x_6	0 / x_7
x_1	3	11/14	1	0	1/2	X	1/7	-1/14	0
x_2	4	5/21	0	1	0	$\cancel{19/21}$	-1/21	4/21	0
x_7	0	263/42	0	0	9/2	X	+1/21	-29/42	1
		$Z_j - C_j$	0	0	1/2	X	5/21	23/42	0

since all $\Delta_j \geq 0$

so new optimal soln.

$$x_1 = \frac{11}{14}, \quad x_2 = \frac{5}{21}, \quad x_3 = 0, \quad x_4 = 0$$

$$\text{Max } Z = \frac{3 \times 11}{14} + \frac{4 \times 5}{21}$$

$$= \frac{139}{42}$$