Addition of a new constraint:

It may or may not effect the feasibility of the current optimal solution for this it is sufficient to check whether the new constaint is satisfied by the current optimum solution or not.

If it is satisfies the addition of constant has no effect on the current optimum solution it remains feasible as well as optimal

If the constraint is not satisfied the current solution becomes infeasible so apply dual simplex method is to find the new optimal solution

Q. Discuss the effect of addition of new constraint $x_2 \leq 10$ on the optimality of the optimum sol.^{n} to the L.P.P

$$\text{Max. } z = 3x_1 + 5x_2$$

$$\text{s.t } \quad x_1 \leq 4$$

$$3x_1 + 2x_2 \leq 18$$

$$\text{and } \quad x_1, x_2, x_3, x_4 \geq 0$$

(ii) if instead $x_2 \le 6$ is added to the problem then discuss the post optimality analysis.

Sol: After introducing the slack variable x_3 & x_4

$$\text{Max } z = 3x_1 + 5x_2 + 0.x_3 + 0.x_4$$

$$\text{s.t. } x_1 + 0x_2 + x_3 + 0x_4 = 4$$

$$3x_1 + 2x_2 + 0x_3 + x_4 = 18 \quad -(1)$$

$$\text{and } x_1, x_2, x_3, x_4 \ge 0$$

The optimal simplex table

		C_j	3	5	0	0
	C_B	X_B	x_1	x_2	x_3	x_4
x_3	0	4	1	0	1	0
x_2	5	9	3/2	1	0	1/2
$\Delta_j = z_j - c_j$			9/2	0	0	5/2

since all $\Delta_j \ge 0$
so the optimal soln is

$$x_1 = 0, x_2 = 9, x_3 = 4, x_4 = 0$$

$$\text{Max } z = 45$$

Case-(i) when the new constraint $x_2 \le 10$ is added to given problem we observe that the constraint is satisfies by the current optimal soln, so there is no effect of adding the constraint $x_2 \le 10$

$$x_1 + x_3 = 4$$
$$3x_1 + 2x_2 + x_4 = 18$$

case - II

Suppose the constraint $x_2 \le 6$ is added then it is not satisfies by the current optimal sol.ⁿ

Hence the current optimal sol.ⁿ is no longer feasible so the new eq.ⁿ $x_2 + x_5 = 6$ — (2) is must be added to the optimal table. where x_5 is a slack variable.

To do so we need to eliminate the basic variable x_2 from the new eq.ⁿ this is done by.

eq. (1) divided by 2 and subtract (2).

$$3/2\, x_1 + x_2 + 0.x_3 + 1/2\, x_4 = 9 \quad -(3)$$
$$x_2 + x_5 = 6$$

eq (2) - (3)

$$-3/2\, x_1 + 0.x_2 + 0.x_3 - 1/2\, x_4 + x_5 = -3$$

So the new table is

	c_j	3	5	0	0	0
c_B	X_B	x_1	x_2	x_3	x_4	x_5
x_3 0	4	1	0	1	0	0
x_2 5	9	3/2	1	0	1/2	0
x_5 0	-3	$\boxed{-3/2}$	0	0	-1/2	1
$z = 45$	$z_j - c_j$	9/2	0	0	5/2	0

By Dual simplex Method

$$X_{Br} = \min [X_{Bi} : x_{Bi} < 0]$$

$= \text{Min}[-3] = -3 = x_{r3}$

$$\frac{\Delta_K}{x_{rK}} = \underset{j}{\text{Max}}\left[\frac{+\Delta_j}{x_{rj}} : x_{rj} < 0\right]$$

$$= \text{Max}\left[\frac{+9/2}{-3/2}, \frac{5/2}{-1/2}\right]$$

$$= \text{Max}\left[-3, -5\right] = -3 = \frac{\Delta_1}{x_{31}}$$

		c_j	3	5	0	0	0	
	c_B	x_B	x_1	x_2	x_3	x_4	x_5	
x_3	0	2	0	0	1	$-1/3$	$2/3$	
x_2	5	6	0	1	0	0	1	$R_3 \to -\frac{2}{3}R_3$
x_1	3	2	1	0	0	$1/3$	$-2/3$	
	$z_j - c_j$		0	0	0	1	3	

$R_1 \to R_1 - R_3$

$R_2 \to R_2 - 3/2\, R_3$

all $\Delta_j \geq 0$

The new optimal feasible sol^n is

$x_1 = 2,\ x_2 = 6,\ x_3 = 2$

$\text{Max. } Z = 3 \times 2 + 5 \times 6$

$= 6 + 30$

$= 36$

from (2) Q.

Q. (i) Discuss the effect of Discrete changes in the activity coefficient a_{ij} of A on the current optimum basic feasible solution

(ii) Let a linear constant $2x_1 + 3x_2 + x_3 + 5x_4 \leq 4$

be added to the constraint to problem check whether any change in the optimal solution of the original problem.
Also discuss the case when the upper limit of the above constraint is reduced by 2 i.e $2x_1 + 3x_2 + x_3 + 5x_4 \le 2$

$a_{12}, a_{22}, a_{32}, a_{13}, a_{23}, a_{33}$, ~~a~~, a_{24}, a_5, a_6

sol:

The optimal table is

	C_j		3	4	1	7	0	0	0
	C_B	X_B	x_1	x_2	x_3	x_4	x_5	x_6	x_7
x_1	3	16/19	1	9/38	1/2	0	5/38	-1/38	0
x_4	7	5/19	0	21/19	0	1	-1/19	4/19	0
x_7	0	126/19	0	59/38	9/2	0	-1/38	-15/38	1
	$z_j - c_j$		0	169/38	1/2	0	1/38	53/38	0

constraint is satisfied by the current optimal solution, so there is no effect of adding the constraint $2x_1+3x_2+x_3+5x_4 \le 4$
also if the constraint is reduced by 2 i.e
$2x_1+3x_2+x_3+5x_4 \le 2$ is added then $3 \le 2$
i.e not satisfies by the current optimal solution!
Hence
the current optimal sol^n is no longer feasible
so the new equation is $2x_1+3x_2+x_3+5x_4+x_8=2$
is must be added to the optimal table, where x_8 is a slack variable
To do so we need to eliminate the basic variable x_1, x_4 and x_7 form the new eq^n this is done by

$$2x_1+6x_2+x_3+5x_4+x_6=3 \quad -(1)$$
$$2x_1+3x_2+x_3+5x_4+x_8=2 \quad -(2)$$
(2) − (1)
$$-3x_2-x_6+x_8=-1$$
$$0x_1-3x_2+0x_3+0x_4+0x_5-x_6+0x_7+x_8=-1$$
So the new table is

		c_j	3	4	1	7	0	0	0	0
	c_B	x_B	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8
x_1	3	16/19	1	9/38	1/2	0	5/38	−1/38	0	0
x_4	7	5/19	0	21/19	0	1	−1/19	4/19	0	0
x_7	0	126/19	0	59/38	9/2	0	−1/38	−15/38	1	0
x_8	0	−1	0	−3	0	0	0	$\boxed{-1}$	0	1
	$z_j-c_j \rightarrow$		0	169/38	1/2	0	1/38	53/38	0	0

Solve this table by dual simplex method

$x_{Br} = \min (x_{Bi} : x_{Bi} < 0)$

$= \min (x_{B4}) = \min(-1) = -1$ so $r = 4$

find entering vector a_k for $r = 4$

$$\frac{\Delta_k}{x_{rk}} = \text{Max}\left[\frac{\Delta_j}{x_{rj}} : x_{rj} < 0\right]$$

$$\frac{\Delta_k}{x_{4k}} = \text{Max}\left[\frac{169/38}{-3} ; \frac{53/38}{-1}\right]$$

$$= \text{Max}\left[-\frac{169}{114}, -\frac{53}{38}\right] = -\frac{53}{38} = \frac{\Delta_6}{x_{46}}$$

So $K = 6$ and our Key element is ~~x_{46}~~ -1

		c_j	3	4	1	7	0	0	0	0
	c_B	x_B	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8
x_1	3	33/38	1	6/19	1/2	0	5/38	0	0	-1/38
x_4	7	1/19	0	9/19	0	1	-1/19	0	0	4/19
x_7	0	267/38	0	52/19	9/2	0	-1/38	0	1	-15/38
x_6	0	1	0	3	0	0	0	1	0	-1
		$z_j - c_j$	0	5/19	1/2	0	1/38	0	0	53/38

all $\Delta_j \geq 0$

So the new feasible Solution is

$x_1 = \frac{33}{38}$, $x_2 = 0$, $x_3 = 0$, $x_4 = \frac{1}{19}$

$$\text{Max. } Z = \frac{33}{38} \times 3 + \frac{7}{19}$$

$$= \frac{113}{38}$$