

# Time Estimates and Critical path in Network

An Activity time is a <sup>forecast</sup> forecast of the time an Activity is expected to take from its starting point to its completion.

The main objective of the time analysis is to prepare a planning schedule of the project.

## Notation and their meaning

Activity (i-j)       $(i) \rightarrow (j)$

Activity (i-j) is tail event i and head event j

$T_E$  or  $E_i$

earliest occurrence time of event i

$T_L$  or  $E_j$

latest allowable occurrence time of event j

$D_{ij}$

estimated completion time of activity (i,j)

$(E_s)_{ij}$

earliest starting time of activity (i,j)

$(EF)_{ij}$

earliest finishing time of activity  $(i,j)$

$(LS)_{ij}$

latest starting time of activity  $(i,j)$

$(LF)_{ij}$

latest finishing time of activity  $(i,j)$

Forward pass computations (for earliest event time)

The forward pass computations yield the earliest start and earliest finish time for each activity  $(i,j)$

earliest starting time of activity  $i-j$  is the earliest event time of the tail end event  $i$  i.e

$$(ES)_{ij} = E_i$$

earliest finish time of activity  $(i,j)$  is the earliest starting time plus the activity time

$$\begin{aligned}(EF)_{ij} &= (ES)_{ij} + D_{ij} \\ &= E_i + D_{ij}\end{aligned}$$

earliest event time of the event  $j$  is the maximum of earliest finish time of all the activity ending into that event

$$E_j = \max_i [(E_F)_{ij}]$$

for all immediate predecessor of (i, j)

$$= \max_i [E_i + D_{ij}]$$

Backward pass computations (for latest allowable time)

The latest event time  $L$  indicates the time by which all the activity entering into that event must be completed without delaying the completion of the project.

Latest finish time for activity (i-j) is equal to the latest event time for event j

$$(LF)_{ij} = L_j$$

Latest starting time of activity (i-j) is equal to the latest completion time of act. (i-j) minus the activity time i.e

$$(LS)_{ij} = (LF)_{ij} - D_{ij}$$

Latest event time for event i is the minimum of the latest start time of all activity originating from that event i.e

$$L_i = \min_j [(LS)_{ij}]$$

Determination of float and slack time:

There are mainly 3 kinds of float

1. Total float
2. Free float
3. Independent float

## 1. Total float

The amount of time by which the completion of an act. could be delayed beyond the earliest expected completion time without effecting the overall project duration time. mathematically the total float of an activity (i-j) is the difference between the latest start time and Earliest start time of <sup>that</sup> activity.

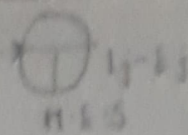
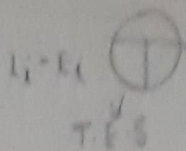
$$\begin{aligned}(T_f)_{ij} &= (L_s)_{ij} - (E_s)_{ij} \\ &= (L_j - D_{ij}) - E_i\end{aligned}$$

## 2. Free float

The time by which the completion of an activity can be delayed beyond the earliest finish time without effecting the earliest start of the succeeding activity, so the free float mathematically represented as

$$\begin{aligned}(F_f)_{ij} &= (E_j - E_i) - D_{ij} \\ &= \text{Total Float} - \text{head event slack}\end{aligned}$$

free float can take value from zero up to total float but it cannot exceed total float.



### 3. Independent float

The amount of time by which the start of an activity can be delayed without affecting the earliest start time of any immediately following activity, assuming that the preceding activity has finished at its latest finish time, Mathematically represented by

$$(If)_{ij} = (E_j - L_i) - D_{ij}$$

Free float - total event slack

The negative independent float is always taken zero

$$\text{Independent float} \leq \text{free float} \leq \text{Total float}$$

### Interfering float

Interfering float is that part of float which cause a reduction in the float of successor activity, mathematically it is a difference between the latest finish time of an activity and the earliest starting time of the <sup>next</sup> activity or zero which is larger

### F.T Event Slack:

For any given event the event slack is defined as the difference between the latest event and earliest event time, mathematically for activity (i-j)

$$\text{Head event slack} = L_j - E_j$$

$$\text{Tail event slack} = L_i - E_i$$

On the float can be represented in terms of Head and tail event slack

$$\text{Total float} = L_j - E_i - D_{ij}$$

$$\text{Free float} = E_j - E_i - D_{ij}$$

$$= (L_j - E_i - D_{ij}) - (L_j - E_j)$$

$$= \text{Total float} - \text{Head event slack}$$

$$\text{Independent float} = E_j - L_i - D_{ij}$$

$$= (E_j - E_i - D_{ij}) - (L_i - E_i)$$

$$= \text{free float} - \text{Tail event slack}$$

Conclusion :

Total float value : If the total float value is negative then resources are <sup>not</sup> adequate and activity may not finish in time

2. Total float value zero ~~that~~ then the resources are just sufficient to complete the activity

3. If the total float value are positive then there are extra resources

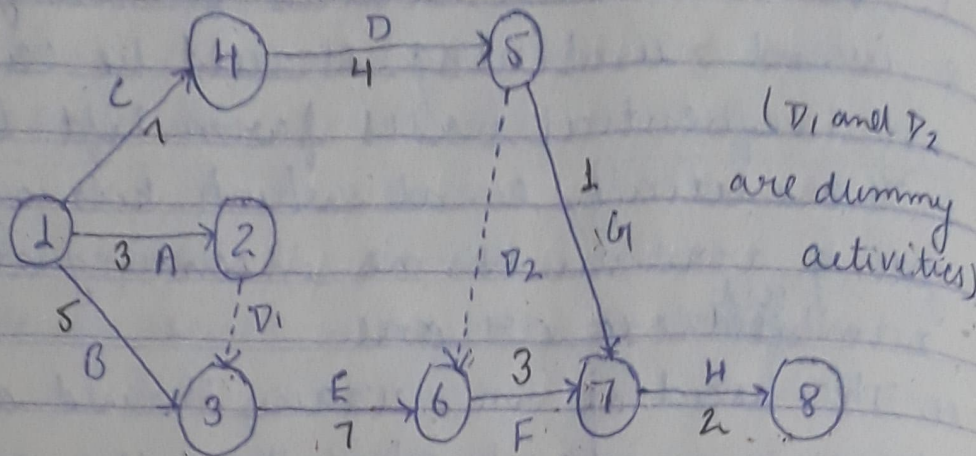
Fulkerson's Rule for Numbering the events:

1. Event number should be unique
2. Event numbering should be carried out in a sequential basis from left to right
3. The initially event which has all out going arrows with no incoming arrow is numbered <sup>and</sup> zero or one.
4. The head of an arrow should always bear a number higher than the assigned at the tail of the arrow, for example, 1-2, 2-3, 2-4, 3-7 except than not 8-7, 9-3
5. Loops should be left ~~as~~ in the sequence of event numbering to avoid subsequent inclusion of an activity if necessary

Q. Consider a project with the following precedence relationship.

| Activity | Time duration | Immediate Predecessor |
|----------|---------------|-----------------------|
| A        | 3             | -                     |
| B        | 5             | -                     |
| C        | 7             | -                     |
| D        | 4             | C                     |
| E        | 7             | A, B                  |
| F        | 3             | E, D                  |
| G        | 1             | D                     |
| H        | 2             | F, G                  |

Construct a network diagram corresponding to above project



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Critical path:

A path in a network is a sequence of activity which have to be completed sequentially due to precedence requirement.

Time duration:

Time duration of the path is the sum of activity times of activity forming of path.

The path or paths with the largest time duration in the network is called critical paths.

All the activities on the critical path are called critical activities.

Referring to the above network diagram path AEFH will take

$$AD, EFH = 3 + 7 + 3 + 2 = 15 \text{ units time}$$

$$CD_2FH = 7 + 4 + 3 + 2 = 16 \text{ unit time}$$

$$CD_6FH = 7 + 4 + 1 + 2 = 14 \text{ unit time}$$

$$BEFH = 5 + 7 + 3 + 2 = 17$$

path BEFH having largest duration is called critical path, The second longest path is called subcritical path  $CD_2FH$  is called subcritical path having next longest duration of 16 units

Critical path in terms of latest event time and earliest event time

Critical event:

The event with zero slack times all are called critical events

Critical Activity:

The activities with zero total float are known as critical activity in other words an activity is said to be critical if a delayed in its start will cause a further delayed in a completion date of entire project

Critical path:

The sequence of critical activity in a network is called a critical path.