Chapter 10

PORTFOLIO MANAGEMENT (DIVERSIFICATION, PORTFOLIO RETURN, BETA)

In present day world of security scams, security booms, security price volatility where sensex may fall and rise up to eleven hundred points a day, an investor is highly confused as to the selection of securities for the purpose of purchase and sale. In investment decisions, an investor, whether an individual or a firm or institution wants to know : which shares are to be selected and why? How much should be invested in each type? How could one minimise risk? One needs to resolve all those and similar issues in order to maximise return and minimise risk. Portfolio management helps in this respect. Hence, individual investors, institutional investors, professionals, corporate executives etc. would be keenly interested in learning and relearning the portfolio management procedures and norms associated with trading in securities in the fast changing world.

WHAT IS PORTFOLIO MANAGEMENT?

In finance, the term 'portfolio' refers to the 'collection of assets', held by an individual or institution purely for investment purposes. The assets may be cash, financial assets (shares, debentures / bonds other securities), gold, paintings, antiques and real assets. In the portfolio context assets are held for investment purposes and not for 'consumption' purposes. ***A portfolio is the combination of different investments that constitute an investor's total holdings.*** A portfolio may be defined as a ***'basket or combination of securities'***. Thus, if one invests in cement business, he is creating a portfolio of businesses. Similarly, if one invests in REL, HUL and INFOSYS, he is building a portfolio of shares. The need to construct a portfolio arises because it is not desirable for any investor to invest all his funds in the individual security or asset. The investor wants to spread risks by diversification.

Basically, portfolio management involves a proper decision making as to what to purchase and what to sell. It requires detailed risk and return analysis and proper money management in terms of investments in a basket of assets, the basic objective being reduction of risk and maximisation of returns. Thus, portfolio management deals with the study of risk-return analysis for individual securities and entails choosing the best set of portfolios to maximise the returns of the rational investor. It refers to the investment of funds in efficient combination of securities. In other words, *planning one's portfolio as per risk-return profile and managing it efficiently so as to secure highest return for lowest risk at a particular level of investment is called portfolio management.*

Portfolio management is a dynamic concept and requires continuous and systematic analysis, judgement and operations. In this context, *investment / portfolio management may be defined as the process of construction, revision and evaluation of a portfolio to obtain maximum returns commensurate with the risk preference or tolerance of the investor.* Thus, portfolio management involves the following activities :

- ***Construction of portfolio*** based upon the data base of the investor, his objectives, constraints, preferences for risk and return etc. It consists of portfolio analysis, selection and execution.
- ***Monitoring / reviewing of portfolio*** from time to time in light of changing market conditions. Accordingly changes are incorporated in the portfolio.
- ***Evaluation of the portfolio*** in terms of targets set for risk and return and making adjustments accordingly.

PROCESS OF PORTFOLIO MANAGEMENT

Portfolio management is a continuous process which involves a set of complex activities that need to be properly planned and managed. The following steps are involved in portfolio management.

Define and Prioritise the Objectives

What are the objectives for making an investment? Is it current income requirement or capital appreciation or both? What about the safety of the principal? An investor must define the objectives and prioritise them. For example, for a conservative investor, the safety of the principal may be more important than, say current income and /or capital appreciation. For another investor, capital appreciation may be the top priority.

Identify Constraints

No one can take an investment decision in a situation where there will be no constraint : Identification of these constraints is crucial for the construction of a portfolio. Some important constraints include :

- Liquidity or marketability of investment.
- Investment horizon or period after which investment will be liquidated. It affects maturity profile of assets.
- Tax considerations (rates and tax shelter).
- Individual needs and preferences related to age, family, source of income and wealth.
- Regulatory requirements applicable to institutional and professional investors only.

Select the Appropriate Mix

Earlier we have examined the risk return relationship and diversification. An appropriate asset mix must be selected with regard to the risk and return. This is the most important decision in portfolio management. The combinations of stocks (shares of companies and mutual funds) and bonds (fixed income securities) that are acceptable may have to be determined judiciously. The selection of an appropriate mix also involves fundamental analysis and technical analysis (discussed in last two chapters). There is an intrinsic value of a security. This value depends on the underlying fundamental factors relating to the company, the industry and the economy. Technical analysis aims at gauging the prevailing mood of investors and the relative strengths of supply and demand.

Determine Portfolio Strategy

After the selection of an appropriate asset mix, a portfolio strategy needs to be formulated. One may apply an active portfolio strategy or a passive portfolio strategy. *Active portfolio strategy* aims at earning superior risk-adjusted returns through market timing, security selection, sector rotation or some suitable combination of these. *Passive portfolio strategy* strives to hold a broadly diversified portfolio in order to maintain a pre-determined level of risk exposure.

Execute the Portfolio

This is the most important phase of portfolio management. It is concerned with implementing the portfolio plan by buying or selling a given amount in specified securities. This practical step has a tremendous impact on the investment results, because the purchase of over priced stocks results in loss and the purchase of under priced shares in profits. Hence, the investor should try to purchase the stocks when the market is falling.

Re-balancing or Revision of the Portfolio

Stock prices fluctuate frequently and in response to such fluctuations in stock prices, periodic re-balancing of the portfolio is required. Such a re-balancing may involve a shift from stocks to bonds or vice versa As a result of the revision, the value of the portfolio and its composition (that is, the proportions of stocks and bonds) would change.

Evaluate and Monitor the Performance

There should be a provision for the periodic evaluation of a portfolio, that is, its risk and return. Is the portfolio return commensurate with the risk? If not, how can one increase the return with a given degree of risk? Or, keeping the return constant, is it possible to reduce the risk? These and many other questions need to be answered for evaluating the performance of a portfolio. The results of analysis need to be continuously monitored and reported to improve the quality of portfolio management.

DIVERSIFICATION

The Principle of Allocating the fund among several

An intelligent investor does not confine his investments to only one security; instead he invests in several other securities to avoid harmful consequences of wrong decision. ***The principle of allocating the funds among the several eligible securities is known as diversifications.***

The objective of diversification is to reduce the instability of return. So in an efficient capital market the investors should not hold all their eggs in one basket. He should have a well diversified portfolio. *More the number of securities in the portfolio, higher is the degree of diversification.* Diversification may take any of the following forms–

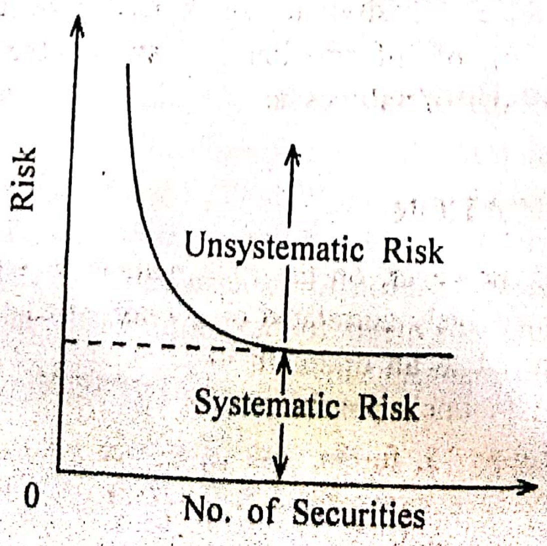
- ***Different Assets*** *e.g.* gold, bullion, real estate, government securities etc.
- ***Different Instruments*** *e.g.* shares, debentures. Bonds, etc.
- ***Different Industries*** *e.g.* textiles, IT, pharmaceuticals, etc.
- ***Different Companies*** *e.g.* new companies, new company's product etc.

Proper diversification involves two or more companies / industries whose fortune fluctuate independent of one another or in different directions. One single company / industry is always more risky than two companies / industries. Two company's in textile industry are more risky than one company in textile and one in IT sector. Two companies / industries which are similar in nature are more risky than two in dissimilar industries.

EFFECT OF DIVERSIFICATION

Diversification helps reducing variability of returns and reducing risk of total investment. The diversification works because returns and prices of all securities do not move exactly together. Variability in one security will be off set by the reverse variability in some other security, hence the overall risk of the investor will be less and less affected. The total risk arising from a portfolio of an investor can be classified as discussed earlier into two components. One is market related risk which can not be diversified at all which is called ***non-diversificable or systematic risk,*** and another component which can be eliminated through diversification is called ***diversifiable or unsystematic risk.*** A portfolio manager seeks to eliminate unsystematic risk by proper diversification though he can not eliminate systematic risk.

The effect of diversification on the risk of portfolio is represented grophically as under–



Relationship between Risk and Diversification

TYPE OF DIVERSIFICATION

Diversification of portfolio can be categorised in the following two types which are briefly discussed below :

- Naïve or simple diversification
- Markowitz diversification

NAIVE OR SIMPLE DIVERSIFICATION

The naive kind of diversification is also known as simple diversification. Here, the securities are selected at random and no analytical procedure is followed. This is familiar *'do not put all your eggs in one basket'* approach. It involves as many baskets as possible and includes as many companies as possible and as many industries as possible in one's portfolio. It is believed that probability of reducing risk is more with a random selection as the statistical error of choosing wrong companies will come down due to randomness of selection (a statistical technique). The principle believes in the possibilities of lowering the risk to even zero, if there are adequate number of companies and industries. But investment into many assets leads to the following problems :

(1) ***Purchase of bad stocks :*** While buying stocks at random, sometimes, the investor may purchase certain stocks which will not yield the expected return.

(2) ***Difficulty in obtaining information :*** When there are too many securities in a portfolio, it becomes difficult for the portfolio manager to obtain detailed information about their performance. In the absence of information, he may not provide right advice as to what to buy and what not to buy.

(3) ***Increased transaction cost :*** Some cost such as brokerage security transaction tax (STT) etc. has to be incurred whenever a stock is to be purchased. Purchasing stocks in small quantities frequently involves higher transaction costs than the purchase of large quantity in one go.

(4) ***Increased research cost :*** Before the purchase of stocks, detailed anslysis as economic and technical performance of individual stock has to be carried out. This requires collecting and processing of information and storing the same. These procedures involve high costs in terms of salaries to be paid to the analysts who are specialised people in this field.

MARKOWITZ DIVERSIFICATION

Markowitz emphasised, however, on the right number of securities as well as on right kind of securities which are negatively correlated or not correlated at all. According to him, the unsystematic risk can be reduced to an optimum level or even can be reduced to zero if 10-15 common stocks are added in one's portfolio.

But if the number exceeds 15, further risk cannot be reduced. Moreover diversification cannot reduce systematic risk. Due care and skill is required in the selection of stocks. Only those stocks should be selected which vary with regard to risk and return. The investor can minimise the total risk by investing in such a diverse set of stocks because they may differ in the degree of risk and may have positive and negative covariance.

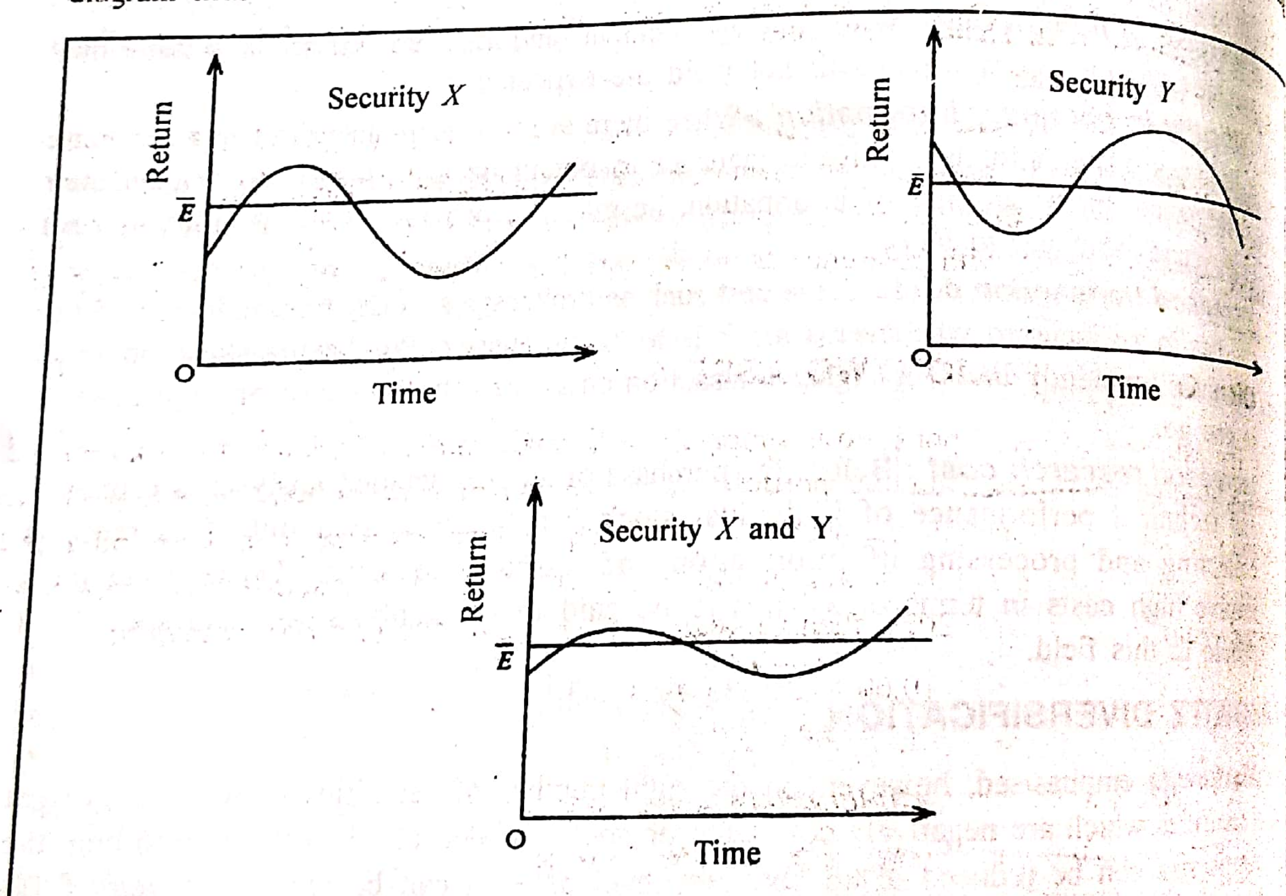
choose negative Corr

CORRELATION BETWEEN SECURITIES

Investment in a number of securities also cannot guarantee reduction of unsystematic diversifiable risk. An investor has to ensure that he chooses negatively correlated securities to benefit from diversification. In order to diversify risk for the creation of an efficient portfolio (one that allows the firm to achieve the maximum return for a given level of risk or to minimize risk for a given level of return), the concept of correlation must be understood.

Correlation is a statistical measure that indicates the relationship, if any, between series of numbers representing anything from cash flows to test data. ***If the two-series move together, they are positively correlated; if the series move in opposite directions, they are negatively correlated.*** The existence of perfectly correlated-especially negatively correlated securities is very rare.

In order to diversify portfolio risk and thereby reduce the investor's overall risk, the securities that are best combined or added to existing portfolio of securities are those that have a negative (or low positive) correlation with existing securities. ***By combining negatively correlated securities, the overall variability of returns of risk can be reduced.*** The following diagram illustrates the result of diversifying to reduce risk.



Reduction of Risk through Diversification

It shows that a portfolio containing the negatively correlated securities X and Y, both having the same expected return Ē also has the return E, but less risk (*i.e.* less variability of return) than either of the securities taken separately. This type of risk is sometimes described as diversifiable or alpha risk.

... of securities the overall variability of Returns of Risk can be Reduced.

The creation of portfolio by combining two perfectly correlated securities cannot reduce the portfolio's overall risk below the risk of the least risky project, while the creation of a portfolio combining two securities that are perfectly negatively correlated can reduce the portfolio's total risk to a level below that of either of the component securities, which in certain situations may be zero. *Combining securities with correlations falling between perfect positive correlation (i.e. a correlation coefficient of +1) and perfect negative correlation (i.e. a correlation coefficient of –1), can therefore reduce the overall risk of a portfolio.*

QUANTIFICATION OF PORTFOLIO RISK AND RETURN

As discussed in the beginning of this chapter, a portfolio is defined as a basket or collection of securities. The need to construct a portfolio arises because it is not desirable for any investor to invest all his funds in the individual security or asset. He wants to spread risk by diversification. But, while constructing his portfolio, an investor is faced with the problem of choosing a few among a large number of securities. He would like to select the most desirable securities and allocate his available funds over these securities in the most rational way. His choice or decision will depend upon the risk-return relationship of individual securities and that of the portfolio. It should be noted that the risk-return relationship of a portfolio differs from those of the individual securities combined in it. Hence, how does one compute the risk and return of a portfolio is being explained hereunder.

A portfolio may consist of two securities or more than two securities. The basic nature of analysis is same in both cases, but the quantum of date required and the number of calculations involved are very much different. Hence, portfolio management in the light of risk and return of individual investments with reference to two-security and three-security portfolio is being discussed :

RETURN OF PORTFOLIO (TWO ASSETS)

The expected return of portfolio is simply the weighted average of the returns of individual securities that are comprised in the portfolio. The weights of each security is equal to the proportion of funds invested in the security. For example, if an investor invests his 60% and 40% of his total funds in equity shares of M&M Ltd. and LML Ltd. respectively, which have expected return of 25% and 20%, then the expected return of the portfolio would be :

$$\bar{R}_p = (0.60 \times .25) + (.40 \times .20)$$
$$= .15 + .08 = 23\%$$

The formula used for portfolio return is–

$$\bar{R}_p = [(W_X \times \bar{R}_X) + (W_Y \times \bar{R}_Y)]$$

where,

$\bar{R}_p$ = Expected return of portfolio

$W_X\ W_Y$ = Proportion of funds invested in security X and security Y

$\bar{R}_X\ \bar{R}_Y$ = Expected return of security X and security Y

$W_X + W_Y$ = 1

Illustration 1 : Mr. Ajay has invested Rs. 50,000 in a portfolio of shares. He has invested in shares of ABB Ltd. and the balance in HCL Ltd. The expected return from these two comp are 15% and 12% respectively. Find out the expected return in percentage and in absolute am

Solution : Denoting shares of ABB Ltd. as X and shares in HCL Ltd. as Y–

$$\bar{R}_p = [W_X \times \bar{R}_X] + [W_Y \times \bar{R}_Y]$$
$$= (.30 \times .15) + (.70 \times .12)$$
$$= .0450 + .0840 = .1290 \text{ or } 12.90\%$$

Absolute Return $= 50{,}000 \times 12.90\% = \text{Rs. } 6{,}450$

RISK OF PORTFOLIO (TWO ASSETS)

The risk (as measured by standard deviation) is not simply the weighted average of standard deviation of individual securities in the portfolio. The portfolio's risk will be sma than the weighted average of the standard deviation of the assets. Therefore, in order to f out the risk of the portfolio, the riskiness of each security *vis-a-vis* the overall portfolio is be considered. This requires the incorporation of how the return of a security moves with return of other securities in the portfolio. This can be studied with the help of co-variance. C*o-variance of two securities is a measure of their co-movement.* It expresses the degree to whi the securities vary together. Therefore, *the risk of a portfolio is measured in terms of c variance of its returns.* The co-variance or relationship between two securities X and Y can calculated as under–

$$Cov_{xy} = \frac{\Sigma[(R_x - \bar{R}_x)(R_y - \bar{R}_y)]}{N}$$

Where,

Cov_{xy} = Co-variance between X and Y

R_x = Return of security X

$\bar{R}_x$ = Expected return of security X

R_y = Return of security Y

$\bar{R}_y$ = Expected return of security Y

N = Number of observations

Alternatively

$$Cov_{xy} = r_{xy}\,\sigma_x\,\sigma_y$$

Where,

r_{xy} = Co-officient of correlation between X and Y

σ_x = Standard deviation of return from X

σ_y = Standard deviation of return from Y

If the return of two securities moves in the same direction, co-variance would be positive otherwise negative. If there is no pattern of movement in returns of securities, co-variance would be close to zero. *Co-variance can be used to find out co-efficient of correlation (r) between the returns of securities* which is as follows–

$$r_{xy} = \frac{Cov_{xy}}{\sigma_x\,\sigma_y} \quad \text{Or} \quad \frac{Cov_{xy}}{\sqrt{Var_{(x)}}\sqrt{Var_{(y)}}}$$

how the Return of a security moves with the Return of other security of Portfolio

It should be noted that the **co-variance is an absolute measure, whereas the co-efficient of correlation is a relative measure.** Co-efficient of correlation varies from (–1) to (+1) that means the risk of a portfolio can be reduced to 0 by combing these securities.It can be interpreted as follows–

If r_{xy} = 1 No systematic risk can be diversified *i.e.* diversification does not reduce risk

If r_{xy} = –1 All unsystematic risk can be diversified *i.e.* risk of a portfolio can be reduced to zero.

If r_{xy} = 0 No correlation exists between the returns of security X and security Y.

STANDARD DEVIATION OF PORTFOLIO (σ_p)

The total risk in a portfolio is the standard deviation of the portfolio. ***The variance of the portfolio (σ_p^2) or standard deviation of the portfolio (σ_p)*** can be calculated with the help of co-variance by applying the following formula–

$$\sigma_p^2 = W_x^2 \sigma_x^2 + W_y^2 \sigma_y^2 + 2W_x W_y \sigma_x \sigma_y r_{xy}$$

$$\sigma_p = \sigma_p^2$$

Where,

σ_p = Standard deviation of portfolio consisting securities X and Y.

W_x W_y = Proportion of funds in security X and security Y.

σ_x, σ_y = Standard deviations of returns of security X and security Y.

r_{xy} = Co-efficient of correlation between security X and security Y.

Illustration 2 : The risk and return characterstics of equity shares of two companies are shown below :

	X Ltd.	Y Ltd.
Expected return ($\bar{R}$)	12%	20%
Standard deviation (σ)	3%	7%

An investor plans to invest 80% of its avaibale funds in X Ltd. and 20% in Y Ltd. The co-efficient of correlation between the returns of the shares of above two companes is +1.0.

Find out the expected returns and variance of the portfolio of shares of X Ltd. and Y Ltd.

Solution

The Expected Return of Portfolio

$$\bar{R}_p = [W_X \times \bar{R}_X] + [W_Y \times \bar{R}_Y]$$

$$= (.8 \times 12\%) + (.2 \times 20\%)$$

$$= 9.6 + 4.0$$

$$= 13.6\%$$

The variance of the portfolio (σ_p^2)

$$(\sigma_p^2) = W_x^2 \sigma_x^2 + W_y^2 \sigma_y^2 + 2W_x W_y \sigma_x \sigma_y r_{xy}$$

$$= (.8^2 \times 3^2) + (7^2 + .2^2) + 2 \times .8 \times .2 \times 3 \times 7 \times 1$$

$$= 5.76 + 1.96 + 6.72$$

$$= 14.44$$

Standard Deviation of the portfolio

$$(\sigma_p) = \sqrt{\sigma_p^2}$$

$$= \sqrt{14.44} = 3.8$$

So, the risk and return of the portfolio are 3.8 and 13.6% respectively.

Illustration 3 : The returns of Security of A and Security of B for the past 6 years are given below :

Year	*Security of A Return (%)*	*Security of B Return (%)*
2003	9	10
2004	5	–6
2005	3	12
2006	12	9
2007	16	15

Calculate the risk and return of portfolio consisting 80% A and 20% B and its coefficient of correlation.

Solution

Calculation of Mean Return and Standard Deviation of Security A :

Year	*Return % (R)*	*$(R - \bar{R})$*	*$(R - \bar{R})^2$*
2003	8	0	0
2004	5	–4	16
2005	3	–6	36
2006	12	3	9
2007	16	7	49
	45		$\Sigma[(R - \bar{R})^2$ 110

Mean Return (R) = 45/5 = 9%

$$\text{Standard Deviation } (\sigma_A) = \sqrt{\frac{110}{5}} = 4.69\%$$

Calculation Mean Return and Standard Deviation of Security B :

Year	*Return % (R)*	*$(R - \bar{R})$*	*$(R - \bar{R})^2$*
2001	10	2	4
2002	–6	14	196
2003	12	4	16
2004	9	1	1
2005	15	7	49
	40		266

Mean Return (R) = 40/5 = 8%

$$\text{Standard Deviation } (\sigma_B) = \sqrt{\frac{266}{5}} = 7.29\%$$

Return of portfolio (R_p) = $[W_A \times \bar{R}_A] + [W_B \times \bar{R}_B]$
= $(0.80 \times 9) + (0.20 \times 8) = 7.2 + 1.6 = 8.8\%$

Risk of portfolio $(\sigma_p)^2$ = $W_A^2 \sigma_A^2 + W_B^2 \sigma_B^2 + 2W_A W_B \sigma_A \sigma_B r_{AB}$
= $(0.80^2 \times 4.69^2) + (0.20^2 \times 7.29^2) + (2 \times 0.80 \times 0.20 \times 4.69 \times 7.29 \times 0.491)$
= $(0.64 \times 22.00) + (0.04 \times 53.14) + 5.37$
= $14.08 + 2.13 + 5.37 = 21.58$

∴ $\sigma_p = \sqrt{\sigma_p^2} = \sqrt{21.58} = 4.645\%$

Analysis– Security A has a higher historic level of return and lower risk as compared to Security B.

CORRELATION COEFFICIENT (AB)

$$r_{AB} = \frac{N\Sigma XY - (\Sigma X)(\Sigma Y)}{\sqrt{N\Sigma X^2 - (\Sigma X)^2}\ \sqrt{N\Sigma Y^2 - \Sigma Y^2}}$$

A's return (%)		*B's return (%)*		
X	*X²*	*Y*	*Y²*	*XY*
9	81	10	100	90
5	25	–6	36	–30
3	9	12	144	36
12	144	9	81	108
16	256	15	225	240
ΣX = 45	ΣX² = 515	ΣY = 40	ΣY² + 586	ΣXY = 444

$$= \frac{(5 \times 444) - (45 \times 40)}{\sqrt{5 \times 515) - (45)^2}\ \sqrt{5 \times 586 - (40)^2}}$$

$$= \frac{2,220 - 1800}{\sqrt{2,575 - 2,025}\ \sqrt{2,930 - 1600}} = \frac{420}{\sqrt{550}\ \sqrt{1,330}}$$

$$= \frac{420}{23.452 \times 36.469} = \frac{420}{855.271} = 0.491$$

Alternatively:

Year	*Return (%)*		$(R_A - \bar{R}_A)$	$(R_B - \bar{R}_B)$	$(R_A - \bar{R}_A) \times (R_B - \bar{R}_B)$
	A	*B*			
2001	9	10	0	2	0
2002	5	–6	–4	–14	56
2003	3	12	–6	4	–24

2004	12	9	3	1	
2005	16	15	7	7	3
					49
					84

$$Cov_{AB} = \frac{\Sigma(R_A - \bar{R}_A)(R_B - \bar{R}_B)}{N} = \frac{84}{5} = 16.8$$

$$r_{AB} = \frac{Cov_{AB}}{\sigma_A \sigma_B} = \frac{16.8}{4.69 \times 7.29} = \frac{16.8}{34.19} = 0.491$$

RISK AND RETURN OF PORTFOLIO (MULTI SECURITY)

The basic principle for calculation of expected return and variance of portfolio of mo[re] than two securities is the same as discussed in case of two security portfolio. The expected ret[urn] of the portfolio is the weighted average of the returns of individual securities in the portfolio. The standard deviation or variance of multi-security portfolio depends on the proportion of each security in the portfolio and the variance and co-variance included in the portfolio. But the calculation of variance is more typical as all the securities and their correlation have to be considered. The formula for calculating risk of portfolio consisting *three securities* is given below with an illustration.

$$\sigma_p^2 = W_x^2 \sigma_x^2 + W_y^2 \sigma_y^2 + W_z^2 \sigma_Z^2 + 2\, W_x\, W_y\, r_{xy}\, \sigma_x\, \sigma_y + 2\, W_y\, W_z\, r_{yz}\, \sigma_y\, \sigma_z + 2\, W_x$$
$$W_z\, r_{xz}\, \sigma_x\, \sigma_z$$

Where, W_1, W_2, W_3 = Proporation of amount invested in securities X, Y and Z

$\sigma_x, \sigma_y, \sigma_z$ = Standard deviations of securities X, Y and Z

r_{xy} = Correlation coefficient between securities X and Y

r_{yz} = Correlation coefficient between securities Y and Z

r_{xz} = Correlation coefficient between securities X and Z.

Illustration 4 : A portfolio consists of three securities P, Q and R with the following parameters-

	Security			*Correlation cofficient*
	P	*Q*	*R*	
Expected return (%)	25	22	20	
Standard deviation (%)	30	26	24	
Correlation coefficient : PQ				−0.5
QR				+0.4
PR				+0.6

If the securities are equally weighted, how much is the risk and return of the portfolio of these three securities ?

Solution

$$\Sigma(R_p) = W_p \times (R_p) + W_Q \times (R_Q) + W_R\, (R_R)$$
$$= (25 \times \tfrac{1}{3}) + (22 \times \tfrac{1}{3}) + (20 \times \tfrac{1}{3}) = 22.33\%$$

$$\sigma_p^2 = W_P^2 \sigma_Q^2 + W_Q^2 \sigma_Q^2 + W_R^2 \sigma_R^2 + 2\, W_P\, W_Q\, r_{PQ}\, \sigma_P\, \sigma_Q + 2\, W_Q\, W_R\, r_{QR}\, \sigma_Q\, \sigma_R$$
$$+ 2\, W_P\, W_R\, r_{PR}\, \sigma_P\, \sigma_R$$

Where,

W_1, W_2, W_3 = Proporation of amount invested in securities X, Y and Z

$\sigma_P, \sigma_Q, \sigma_R$ = Standard deviations of securities X, Y and Z

r_{PQ} = Correlation coefficient between securities X and Y

r_{QR} = Correlation coefficient between securities Y and Z

r_{PT} = Correlation coefficient between securities X and Z.

$$\sigma_p^2 = (\tfrac{1}{3})^2 (30)^2 + (\tfrac{1}{3})^2 (26)^2 + (\tfrac{1}{3})^2 (24)^2 + 2(\tfrac{1}{3})(\tfrac{1}{3})(-0.5)(30)(26) + 2(\tfrac{1}{3})(\tfrac{1}{3})(0.4)(26)(24) + 2(\tfrac{1}{3})(\tfrac{1}{3})(0.6)(30)(24)$$

$$\sigma_p^2 = 100 + 75.11 + 64 - 86.67 + 55.47 + 96 = 303.91$$

$$\sigma_p = \sqrt{303.91} = 17.43$$

BETA ESTIMATION

As explained earlier in this chapter, the risk associated with an investment are of two kinds *i.e.* unsystematic and systematic. ***Unsystematic risk,*** which arises on account of firm specific factors and can be eliminated or diversified by investing in a large portfolio of securities. ***Systematic risk*** is associated with market conditions like boom, recession etc.; and is dependent on the market which is unavoidable even by diversification of the portfolio. The systematic risk of an individual security is measured in terms of its sensivity to market movements (based market index like Sensex in India). which is referred to a security's beta denoted by the Greek letter Beta (β). A β of 2 implies that if the market return increases or decreases by 10% over a period, the security return is expected to increase or decrease respectively 20%. Thus, in this case, the security return on an average moves twice of much as the market return. On the other hand β of 0.5 inplies that the security return moves only half as much as the market does. A β of zero characterises a risk free security like a government bond whose return is almost sensitive to the market return.

Thus, ***beta (β) is a measure of the volatility of stock price in relation to movement in the stock index of the market.*** If β of a particular share is high, it means its prices increases more, if the market increases. Its price will decrease more, if the market decreases. Therefore, beta is the index of systematic risk the higher the riskiness of a security, the higher the value of its β. A secutiry with a β value greater than 1 is referred to an ***aggressive security***, one with a β value less than 1 is referred to as a ***defensive security*** and one with β value equal to 1 is referred to normal security.

Volicity of Stock Price — Relation to Movement in Stock India of the Market

ASSUMPTIONS

Beta co-efficient is calculated on the following assumptions–

- Investors are rational;
- Investors can differentiate between a risky and risk free security;
- They seek a higher return for a risky security;
- Between two risky securities, the investors will define the premium for one risky security with reference to another.

$\beta = 1$ Normal Secur

$\beta > 1$ Aggressive Security

$\beta < 1$ defensive security

Illustration 5 : The following information is available about two securities A and B :

Particulars	A	B
1. Expected returns	15%	20%
2. Standard Deviation of Expected Returns	12%	10%
3. Beta	0.6	0.9

(i) Which of the two securities is more risky? Why?

(ii) Which of the two securities is more volatile?

Solution

(i) The standard deviation of a set of number is the average variability around the mean. The higher the standard deviation, the higher is the investment risk. Thus, security 'A' is more risky because its standard deviation is higher than 'B'.

(ii) Beta is a measure of performance of a particular security in relation to the general movement of the market. If a security has a beta 1, its rise and fall corresponds exactly with the market. Normally, beta values fall in the range of 0.6 to 1.80. When the market rises of falls sharply and suddenly, the Beta co-efficient of securities is a useful measure to keep tabs on the volatility of a portfolio. As compared to security 'A', security 'B' has a higher Beta, hence security 'B' is more volatile than security 'A'.

CALCULATION OF β (SINGLE SECURITY)

The risk of a share is measured by its standard deviation but in relation to market, it is its co-variance with the market. The β co-efficient is the relative measure of sensivity of an asset's return to change in the return on the market portfolio. Therefore, β is calculated by relating the return of security with the return for the market. Mathematically, the beta of security is the security's co-variance with the market portfolio divided by the variance of the market portfolio. The formula used is as follows :

$$\beta = \frac{Cov_{(a,m)}}{Var_m \text{ or } \sigma_m^2} = \frac{\sigma_a \sigma_m r_{(a,m)}}{\sigma_m^2} = \frac{\sigma_a r_{(a,m)}}{\sigma_m}$$

Where,

β = Beta of individual security.

$Cov_{(a,m)}$ = Co-variance between returns of an individual security a, and the returns of the market portfolio m.

Var_m = Variance of retrun of market portfolio (σ_m^2).

σ_a = Standard deviation of an asset or individual security *a*.

σ_m = Standard deviation of the market portfolio *m*.

$r_{(a,m)}$ = Correlation co-efficient between the return of security *a* and the market portfolio *m*.

Co-variance is a measure of variation of one variable (in this case the security return) with another variable (in this case the market or the index return) which is given by–

$$Cov_{(a,m)} = \frac{\Sigma (R_a - \bar{R}_a)(R_m - \bar{R}_m)}{n \text{ or } (n-1)}$$

Where, $\bar{R}_a$ and $\bar{R}_m$ are the averages of the security return (R_a) and market returns (R_m) respectively.

Variance of the market return Var (R_m) is computed as follows–

$$Var(R_m) \text{ or } \sigma_m^2 = \frac{\Sigma(R_m - \bar{R}_m)^2}{n \text{ or } (n-1)} \text{ or } \sigma_m^2$$

Note : *When data or in the form of non-probability distribution (n) is used, but if only sample return data over some past period are available, the σ and Cov is calculated using (n–1).*

Illustration 6 : Given below is information of rates of returns of and data from two companies A and B :

	Year 2004	*Year 2005*	*Year 2006*
Market Return	12.0	11.0	9.0
Company A Return	13.0	11.5	9.8

Required : Determine the beta coefficients of the shares of Company A.

Solution

Determination of Beta Coefficients of the Shares of Company A and Company B

Company A

Year	R_a	R_m	$(R_a-\bar{R}_a)$	$(R_m-\bar{R}_m)$	$(R_a-\bar{R}_a)(R_m-\bar{R}_m)$	$(R_m-\bar{R}_m)^2$
1	13.0	12.0	1.57	1.33	2.09	1.77
2	11.5	11.0	0.07	0.33	0.02	0.11
3	9.8	9.0	– 1.63	– 1.67	2.72	2.79
	$\Sigma R_a = 34.3$	$\Sigma R_m = 32.0$			$\Sigma(R_a-\bar{R}_a)(R_m-\bar{R}_m) = 4.83$	$\Sigma(R_m-\bar{R}_m)^2 = 4.67$

$$\bar{R}_a = \Sigma R_a/n = 34.3/3 = 11.43\%$$

$$\bar{R}_m = \Sigma R_m/n = 32.0/3 = 10.67\%$$

$$\sigma_m^2 = \frac{\Sigma(R_m - \bar{R}_m)^2}{n} = \frac{4.67}{3} = 1.557$$

$$Cov_{am} = \frac{(R_a - \bar{R}_a)(R_m - \bar{R}_m)}{n} = \frac{4.83}{3} = 1.610$$

$$\beta_a = \frac{Cov_{am}}{\sigma_m^2} = \frac{1.610}{1.557} = 1.034$$

Illustration 7 : From the following data, compute beta of Security A :

$\sigma_a = 12\%$ $\sigma_m = 9\%$

$r_{am} = +0.72$

Solution

Calculation of beta of Security A :

$$\beta = \frac{\sigma_a \sigma_m r_{am}}{\sigma_m^2} = \frac{12 \times 9 \times 0.72}{9^2} = \frac{77.76}{81} = 0.96$$

Illustration 8 : Covariance of returns for the market with returns for share of company A is 18%. What is the value of beta for share A and what would be its expected return? If–

Risk-free return	10%
Market return	15%
Standard deviation of market returns	30%

Solution

The beta value of share A

$$\beta = \frac{Cov_{(a,m)}}{\sigma_m^2}$$

$$= \frac{0.18}{0.3^2} = \frac{0.18}{0.09} = 2$$

Expected rate of return from share A:

$$\bar{R}_A = 10\% + (15\% - 10\%) \times 2 = 20\%.$$

Illustration 9 : Correlation coefficient between company A returns and market returns 0.6. Calculate the beta factor and expected return of company A share, if–

Risk-free return	10%
Market return	15%
Standard deviation of market returns	8%
Standard deviation of returns from Share A	12%

Solution

$$\beta = \frac{\sigma_a \, r_{(a,m)}}{\sigma_m} \text{ or } \frac{\sigma_a \, \sigma_m \, r_{am}}{\sigma_m^2} = \frac{(12 \times 8 \times .60)}{8^2}$$

$$= \frac{57.6}{64} = 0.9$$

$$\text{Expected return (CAPM)} = R_f + (R_m - R_p)\,\beta$$

$$= 10\% + (15\% - 10\%) \times 0.9 = 14.5\%.$$

CALCULATION OF BETA OF A PORTFOLIO

The β of a portfolio is nothing but the weighted average of the β's of the securities that constitute the portfolio (W × β), the weights being the proportions of investments in the respective securities. For example, if the β of the security X is 2.0 and that of security Y is 0.6 and we hold a proportion of 40% and 60% of the two securities respectively. The following formula can be applied $\beta_p = (W_s \times \beta_s)$ the β of the portfolio will be 1.16 [(2.0 × 0.4) + (0.6 × 0.6)].

Illustration 10 : You have invested the following sums in four securities A, B, C and D.

A : Rs. 10,000; B : Rs. 20,000; C : Rs. 16,000; D : Rs. 14,000.

The values of the β of securities are 0.80; 1.20; 1.40; and 1.75 respectively. Compute beta of the portfolio.

Solution

Computation of Portfolio Beta

Security	Amount Invested (Rs.)	Weighted Investment (%)	β	W×β
A	10,000	0.17	.80	0.1360
B	20,000	.33	1.20	0.3960
C	16,000	.27	1.40	0.3780
D	14,000	.23	1.75	0.4025
Total	60,000	1.00		ΣWβ = 1.3125

Therefore, β of Portfolio is 1.3125

Illustration 11 : Yash Portfolio Ltd. has three investments in its portfolio. Its details are given below:

Investment	$E(R_i)$	β_1	Proportion invested funds
X	14%	1.6	50%
Y	16%	1.2	20%
Z	12%	0.8	30%

Calculate the weighted average of expected return and Beta factor of the portfolio.

Solution

Weighted Average of Expected Return of the Total Portfolio :

$\bar{R}_p$ = (14% × 0.5) + (16% × 0.2) + (12% × 0.3) = 7% + 3.2% + 3.6%= 13.8

Weighted Average Market Sensitivity Index of the Total Portfolio :

β_p = (1.6 × 0.5) + (1.2 × 0.2) + (0.8 × 0.3) = 0.8 + 0.24 + 0.24 = 1.28

UTILITY OF BETA

Beta can be used for stock selection for which general market outlook for the future is assessed. If the market is expected to go up, portfolio with shares having large beta would be constructed. If the market outlook suggests a decline, shares with negative beta or betas less them one are added in the portfolio and shares with larger positive beta are sold. One can not expect the beta to be constant overtime. Therefore, historical beta has to be updated frequently to use for prediction. While interpreting beta, the following ***limitations*** are to be kept in mind–

- ***Not Total Volatility :*** Beta measures the degree of responsiveness of expected return on a given security relative to changes in market movements. It does not indicate total volatility in the expected return on a given security.
- ***Only One Portion :*** Beta measures only one portion of the volatility or returns (caused by systematic risk). It does not measure all the elements that cause the volatility in the returns in an asset *i.e.* total risk.

SPECIAL CARE THAT MUST BE TAKEN IN COMPUTING β (BETA)

There are a few things about β estimation that we must be careful about :