Branch and Bound Method :

It divide (Branches) the feasible region into smaller subset and then examine each of them successively until a feasible solution that gives optimal value of the objective fun." is obtained at each branch we establish limit for each set referred to as boundary ing

The procedure for this method

Step 1: obtained the optimal solution to the given problem ignoring the restriction of integer

Step 2: Test the integrability of the optimal solution obtained in step 1. their are two cases

Case-a If the solution is an integer the current solution is optimum to the given L.P.P

Case-b If the solution is not integer go to next step.

Step 3: Considering the value of the objective function as upper bound obtained the lower bound by rounding the integeral value of the decision variable

Step 4: Let the optimum value x_j^* of the variable x_j is not integer then subdivide

the given L.P.P into two problems:
sub problem 1: Given L.P.P with an additional constraint $x_j \leq [x_j^*]$

sub problem 2: Given L.P.P with an additional constraint $x_j \geq [x_j^*]+1$ where $[x_j^*]$ largest integer less than and equal to x_j

Step 5: Solve ~~to~~ the two sub problem obtained in step 4 there may be arise 3 cases
case-I if the optimum solution of the 2 sub problem are integral then the required solution is one that gives largest values of z
case-II if the optimum solution of one sub problem is integral and the other subproblem has no feasible optimum sol.^{n} the required sol.^{n} is the ~~since~~ same as that of the subproblem having integer valued sol.^{n}
case-III if the optimum sol: of one subproblem is integral while that of other is not integer value recard the integer solution and partition the sub-problem with non-integer values.
Step 6: Repeat step 3-5 until all integer value solution are recorded.
Step 7: Select the sol.^{n} amongs the recorded integer value sol.^{n} that gives an optimum values of z

11/Jan/01

Q. 06

Using Branch and Bound method to solve the I.P.P

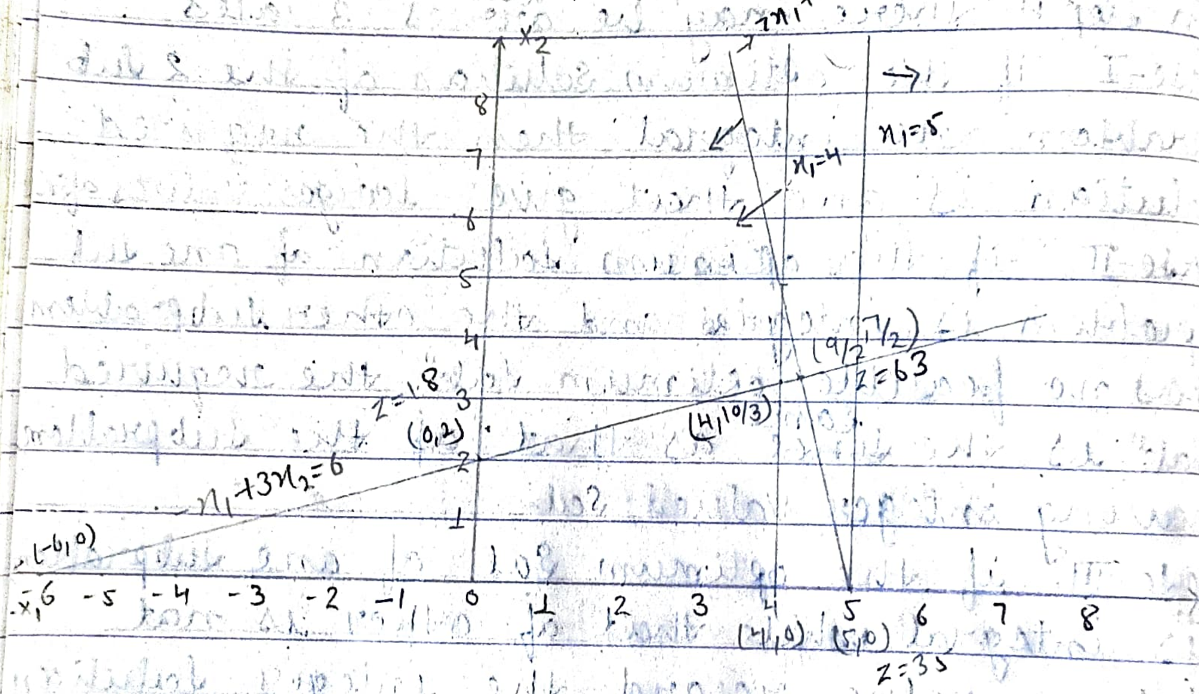
$$\left.\begin{aligned} \text{Max } Z &= 7x_1 + 9x_2 \\ -x_1 + 3x_2 &\leq 6 \\ 7x_1 + x_2 &\leq 35 \end{aligned}\right] \text{— (A)}$$

and $x_1, x_2 \geq 0$ and are integer.

Sol:

Using Graphical method the optimal sol of I.P.P is $x_1 = 9/2$ and $x_2 = 7/2$.

$\text{Max } Z^* = 63$



Since the solution is not integer value so we choose x_1 i.e $4 \leq x_1 \leq 5$

$[x_1^*] = 4$

Now we divide the L.P.P into 2 subproblem

Subproblem 1: $x_j \leq [x_j^*]$

$x_1 \leq 4$ with (A)

Subproblem 2: $x_j \geq [x_j^*] + 1$

$x_1 \geq 5$ with (A)

The optimum feasible solution of subproblem 1 is $x_1 = 4$, $x_2 = 10/3$ with $z^* = 58$

and the optimum solution of subproblem 2 is $x_1 = 5$, $x_2 = 0$ with $z^* = 35$.

Since the subproblem 1 is not integer value but the subproblem 2 is integer value so their is no need for further branching for subproblem 2.

For the subproblem 1 we consider x_2 i.e

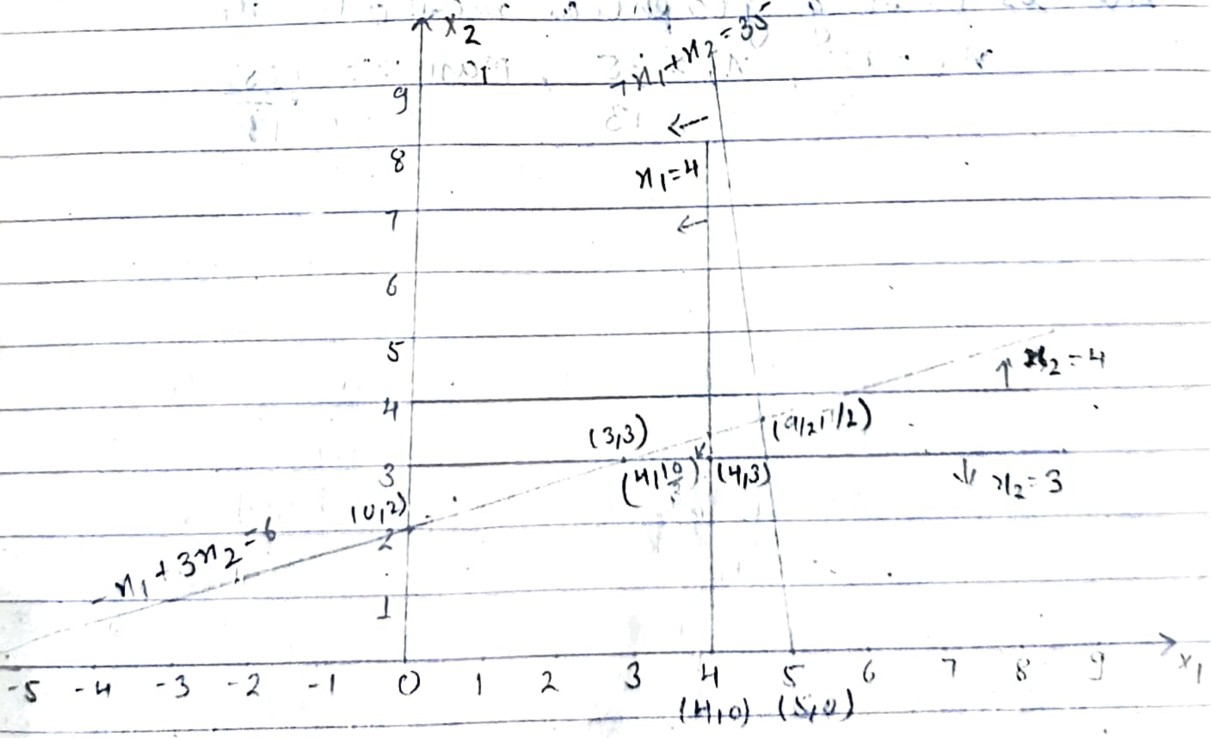
$3 \leq x_2 \leq 4$

$[x_2^*] = 3$

so partition or divide subproblem 1 into 2 subproblem.

3 subproblem 1: with $x_2 \leq 3$

4 subproblem 2: with $x_2 \geq 4$



Since the subproblem 4 has no feasible solution and the subproblem 3 has the optimal solution $x_1=4, x_2=3$ with $z^*=55$.

Since the subproblem 2 and 3 gives us integer value solution but the optimum Sol.^{n} obtained by subproblem 3 so the required optimal solution is $x_1=4, x_2=3$ with $z^*=55$

Q. Solve the I.P.P using branch and Bound Method

(A) →

$$\text{Max. } z = 2x_1+3x_2$$

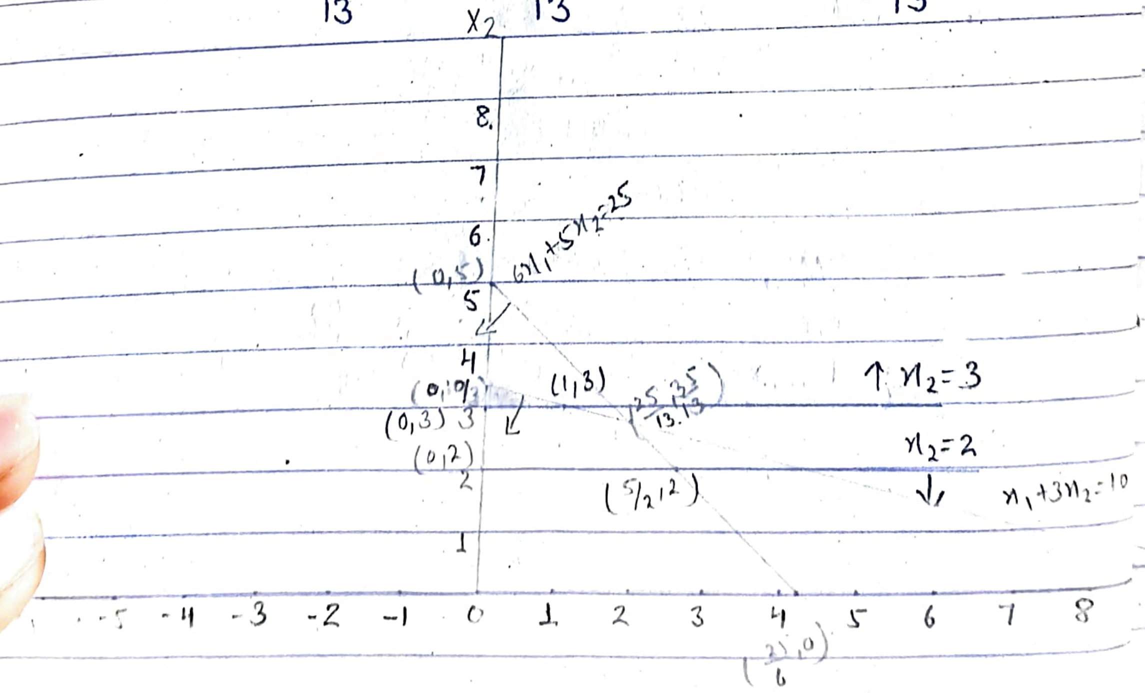
$$\text{s.t. } 6x_1+5x_2 \le 25 \quad \left(\tfrac{25}{6},0\right), (0,5)$$

$$x_1+3x_2 \le 10 \quad (10,0)\ (0,10/3) \qquad \left(\tfrac{25}{13}, \tfrac{35}{13}\right)$$

and $x_1, x_2 \ge 0$ and integer

Sol: Ignoring restriction of integer the optimum Sol.^{n} is (using graphical method)

$$x_1 = \frac{25}{13}, \quad x_2 = \frac{35}{13}, \quad \text{max. } z = \frac{155}{13}$$



since both x_1 and x_2 are fractional the variable which is choosen $\max\left(\frac{25}{13}, \frac{35}{13}\right) = \frac{35}{13}$ which is corresponding to x_2, so select x_2 for branching

So we divide the given problem into 2 subproblem

Sub. prob. 1 is (A) with $x_2 \leq 2$

Sub. prob. 2 is (A) " $x_2 \geq 3$

Solution of subproblem 1 is $x_1 = \frac{5}{2}$, $x_2 = 2$ with $z = 11$

Solution of subproblem 2 is $x_1 = 1$, $x_2 = 3$ with $z = 11$

since the subproblem

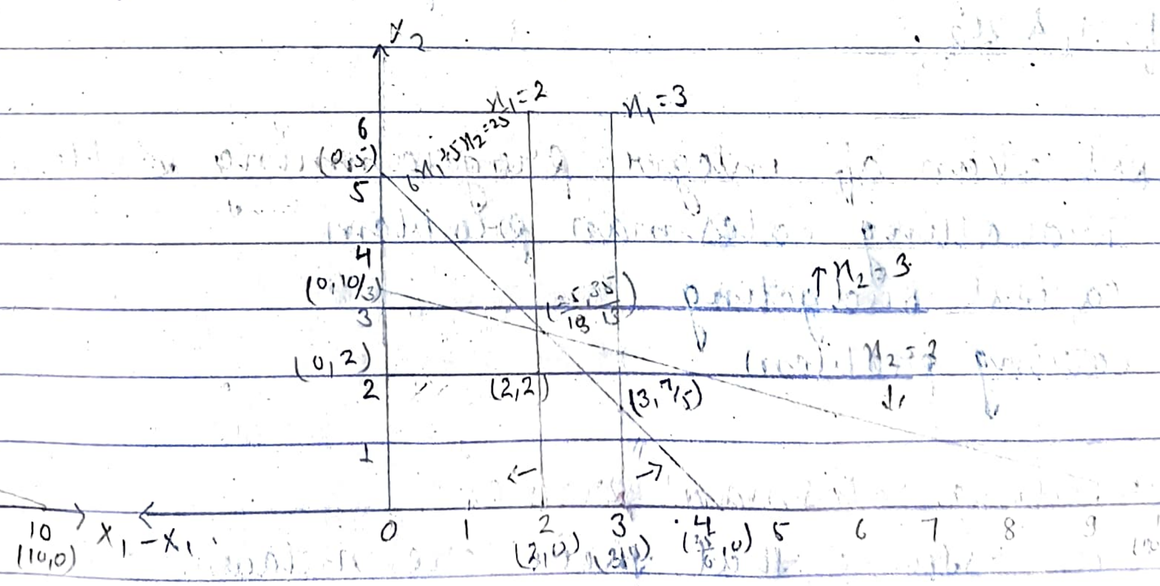
So there is no need for further branching but for the sub problem 1,

since in subprob. 1 we consider x_1 i.e $2 \leq x_1 \leq 3$

So partiation or divide subproblem 1 into 2 subproblem

sub. prob. 3 with $x_1 \leq 2$

Sub. prob. 4 with $x_1 \geq 3$



Solution of subproblem 3 is $x_1 = 2$, $x_2 = 2$
with $z = 10$

Since the solution of sub. prob. 3 is all integer
So there is no need for further branching
Solution of subproblem 4 is $x_1 = 3$, $x_2 = 7/5$
with $z = 10.2$

So there is not integer value, so sub prob. 4 divide into 2 subproblem, we chose x_2 i.e $2 \le x_2 \le 3$
sub problem. 5 with $x_2 \le 2$
Sub problem. 6 with $x_2 \ge 3$

1/Jan/07

Note:
If we have 3-variable in L.P.P then for a initial solution take $x_3 = 0$ and solve the linear cons~~tant~~traint to find the value of x_1 & x_2

Application of integer programming problem
1. Travelling salesman problem
2. capital Budgeting, matching problem, sequencing
3. catering problem, scheduling problem, location problem

Travelling salesman problem
Let us assume that their are n-towns with