- Modified Distribution (MODI) Method:

Step 1: For every occupied cell in the initial solution, make an equation like this,

$$R_1 + K_1 = 5$$
$$R_2 + K_1 = 8$$
$$R_2 + K_2 = 4$$
$$R_3 + K_2 = 7$$
$$R_3 + K_3 = 5$$

Putting $R_1 = 0$,

$$K_1 = 5$$

Putting $K_1 = 5$,

$$R_2 = 8 - 5 = 3$$

Putting $R_2 = 3$,

$$K_2 = 4 - 3 = 1$$

Putting $K_2 = 1$,

$$R_3 = 7 - 1 = 6$$

Putting $R_3 = 6$,

$$K_3 = 5 - 6 = -1$$

Step 2: For every vacant cell, calculate improvement index

Improvement Index (II) = Cost - R - K

DB = $4 - R_1 - K_2$
= $4 - 0 - (-1) = 3$.

DC = $3 - R_1 - K_3$
= $3 - 0 - (-1) = 4$

EC = $3 - R_2 - K_3$
= $3 - 3 - (-1) = 1$

FA = $9 - R_3 - K_1$
= $9 - 6 - 5 = -2$

Step 3: Select the largest negative II cell as incoming variable.

31/07/21 ∴ FA is the new incoming variable.

Step 4: i) Trace the closed path for FA.

Rules →

a.) Every vacant cell has a closed path which is unique but two versions are there:
- clockwise
- anticlockwise

b.) Only horizontal and vertical movement is allowed, not diagonal.

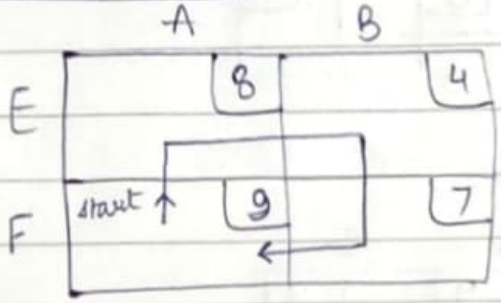
c.) Turning is allowed only at the occupied cells.

d.) If the vacant cell is surrounded by occupied cell in all 3 sides, that will be the path.

In figure, 4 vacant cells and has 1 unique path. Movement is only through row or column

→ FA is surrounded by 3 occupied cells.

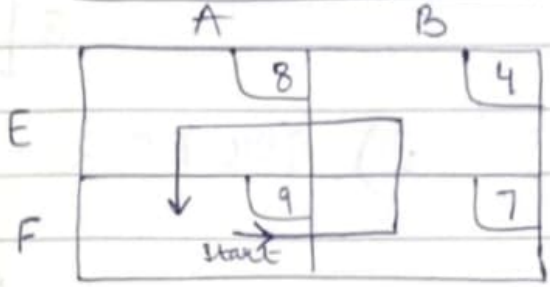
Clockwise (FA)



FA → EA → EB → FB

FA → DA (Only one known then every cell is vacant).

Anti-clockwise (FA)



FA → FB → EB → EA

FA → FC (EC, DC are vacant, no direction).

∴ only one path is available.

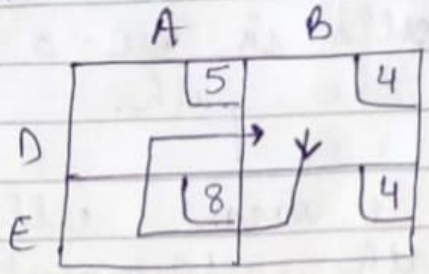
★ Choose clockwise everytime.

Eg.: Additional

Any vacant cell occupied by 3 sides, follow Step 4. [clockwise]

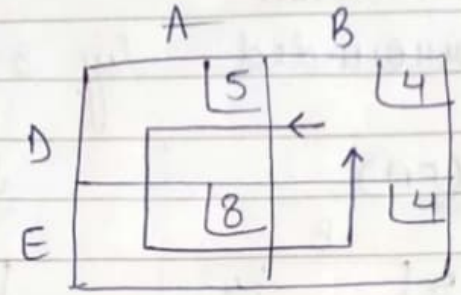
i) For DB :

Clockwise



DB → EB → EA → DA

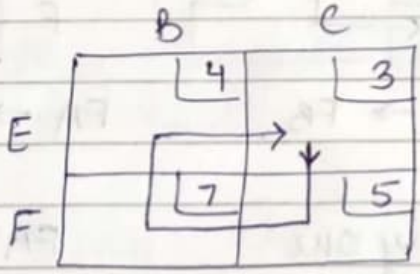
Anticlockwise



DB → DA → EA → EB .

ii) For EC :

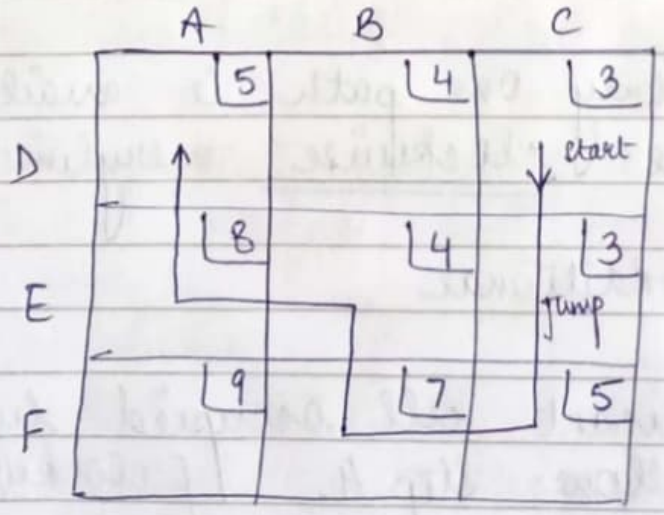
Clockwise



EC → FC → FB → EB

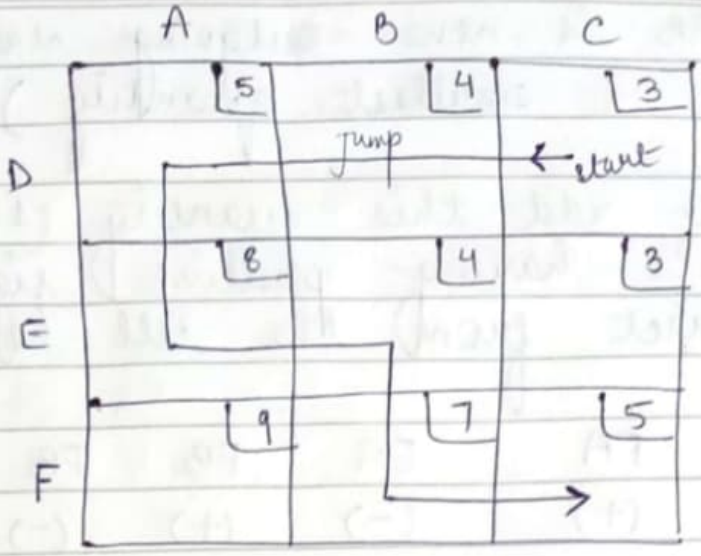
iii) For DC :

Clockwise



DC → EC (jump) → FC → FB → EB → EA → DA .

EC to be jumped because it is vacant



DC → DA → EA → EB → FB → FC.

e.) It is not necessary to turn at every occupied cell. If required, we can jump over it.

Step 4: ii.) Starting from the new incoming cell FA, give (+)(-) sign to all the cells in the path. (Always start with +ve.)

	FA	EA	EB	FB
	(+)	(−)	(+)	(−)
Qty	−	200	−	100

	A	B
E	− 8 200	+ 4 100
F	+ 9	7 − 100

Now find the smallest positive quantity on the cells having -ve signs.

Value of -ve => EA = 200, FB = 100

FB is new outgoing variable (because smallest quantity).

⇒ Now add this quantity (100) to the cells having positive sign and subtract from the cell having -ve sign.

FA	EA	EB	FB
(+)	(−)	(+)	(−)
0	200	100	100
+100	−100	+100	−100
100	100	200	0

Table 2:

		K_1 A	K_2 B	K_3 C	
R_1	D	[5] 100	[4]	[3]	100
R_2	E	[8] 100	[4] 200	[3]	300
R_3	F	[9] 100	[7] −	[5] 200	300
		300	200	200	700

DA and FC are unchanged because not in path of FA.

Calculation of cost = $(100 \times 5) + (100 \times 8) + (200 \times 4) + (100 \times 9) + (200 \times 5)$

= ₹ 4000.

Step 5:

$R_1 + K_1 = 5$
$R_2 + K_1 = 8$
$R_2 + K_2 = 4$
$R_3 + K_1 = 9$
$R_3 + K_3 = 5$

Putting $R_1 = 0$,

$R_1 + K_1 = 5$
$\Rightarrow K_1 = 5$

Putting $K_1 = 5$,

$R_2 + K_1 = 8$
$\Rightarrow R_2 = 3$

Putting $R_2 = 3$,

$R_2 + K_2 = 4$
$\Rightarrow K_2 = 1$

Putting $K_1 = 5$,

$R_3 + K_1 = 9$
$\Rightarrow R_3 = 4$

Putting $R_3 = 4$,

$R_3 + K_3 = 5$
$\Rightarrow K_3 = 1$.

Step 6: Calculating Improvement Index for vacant cells.

$DB = 4 - R_1 - K_2$
$= 4 - 0 - 1 = 3$

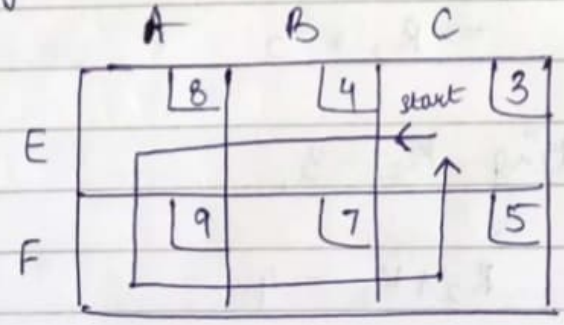
$DC = 3 - R_1 - K_3$
$= 3 - 0 - 1 = 2$

$EC = 3 - R_2 - K_3$
$= 3 - 3 - 1 = -1.$

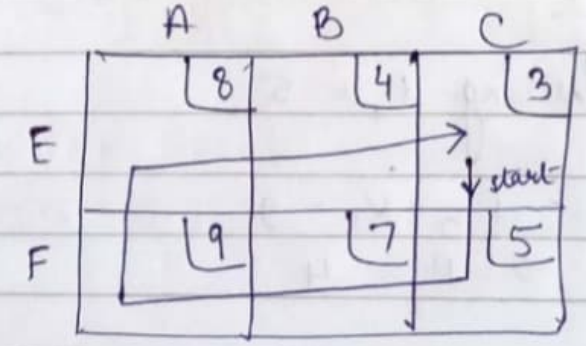
$FB = 7 - R_3 - K_2$
$= 7 - 4 - 1 = 2$

∴ EC is the new incoming variable

Step 7: i) Tracing closed path:



Anticlockwise



Clockwise

ii)

EC → FC → BF (jump)
→ FA → EA → EB. (jump)

	A	B	C
E	100 [8] −	[4]	[3] +
F	100 [9] +	[7]	[5] − 200

	EC →	FC →	FA →	EA
	(+)	(−)	(+)	(−)
Qty	-	200	100	100

Cells having -ve sign → EA = 100, FC = 200.

Smallest positive among these two is 100, i.e. cell EA.

∴ EA is the outgoing variable.

EC	FC	FA	EA
(+)	(−)	(+)	(−)
0	200	100	100
+100	−100	+100	−100
100	100	200	0

Table 3: K_1 K_2 K_3

		A (K_1)	B (K_2)	C (K_3)	
R_1	D	[5] 100	[4]	[3]	100
R_2	E	[8]	[4] 200	[3] 100	300
R_3	F	[9] 200	[7]	[5] 100	300
		300	200	200	700

Calculating cost – $(100 \times 5) + (200 \times 4) + (100 \times 3) + (200 \times 9) + (100 \times 5)$

$= ₹ 3900.$

Step 8:

$R_1 + K_1 = 5$

$R_2 + K_2 = 4$

$R_2 + K_3 = 3$

$R_3 + K_1 = 9$

$R_3 + K_3 = 5$

Putting $R_1 = 0$,

$\Rightarrow 0 + K_1 = 5 \Rightarrow K_1 = 5.$

Putting $K_1 = 5$,

$R_3 + 5 = 9 \Rightarrow R_3 = 9 - 5 = 4.$

Putting $R_3 = 4$,

$R_3 + K_3 = 5$

$4 + K_3 = 5 \Rightarrow K_3 = 1$.

Putting $K_3 = 1$,

$R_2 + K_3 = 3$

$\Rightarrow R_2 = 3 - 1 = 2$.

Putting $R_2 = 2$,

$2 + K_2 = 4$

$\Rightarrow K_2 = 4 - 2 = 2$.

Step 9: Calculating Improvement Index for Vacant cells:

$DB = 4 - R_1 - K_2$

$= 4 - 0 - 2 = 2$

$DC = 3 - R_1 - K_3$

$= 3 - 0 - 1 = 2$

$EA = 8 - R_2 - K_1$

$= 8 - 2 - 5 = 1$

$FB = 7 - R_3 - K_2$

$= 7 - 4 - 2 = 1$.

Since all the Improvement Index is positive, optimal solution has been achieved.

∴ ₹ 3900 is the optimal cost.