Integral form of Faraday's law

$$\mathcal{E} = \int_l E \cdot \hat{l}\, dl = (-) \int_s \frac{d\vec{B}}{dt} \cdot d\vec{s} \quad \text{---} (1)$$

Stokes theorem

$$\int \vec{E} \cdot \hat{l}\, dl = \int_s \nabla \times \vec{E} \cdot ds$$

$$\Rightarrow \quad \int_s (\nabla \times E) \cdot ds = -\int \frac{\partial \vec{B}}{\partial t} \cdot d\vec{s}$$

$$\text{or} \quad \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad \text{---} (2)$$

Differential form of Faraday's Law

eq^n (2) and Gauss Law $\nabla \cdot \vec{E} = \rho/\varepsilon_0$ show that electric field has non-conservative part due to changing magnetic flux density & has conservative part due to electric charge density.

These are two kind of sources for electromagnetic fields.

first kind of sources → Energy is conserved in cyclic process

Such kind of conservative or irrotational system are presented by

$$\nabla \cdot E = \rho/\varepsilon_0 \quad \text{in electrostatics}$$

$$\text{where} \quad E = -\nabla\phi$$

No curl exist for such kind of sources

because

$$\nabla \times \vec{E} = \nabla \times \nabla\phi = 0$$

Second kind of Sources → related to such systems in which energy is transferred in cyclic process such as magnetic field of solenoid. Such field are represented by curl → divergence does not exist.

Any vector field can be represented uniquely if its divergence and curl sources exist.

↓ Helmholtz Theorem

$$\nabla \cdot \nabla \times \vec{E} = -\frac{\partial}{\partial t}(\nabla \cdot \vec{B}) = 0$$

$$\Rightarrow \nabla \cdot \vec{B} = 0 \qquad B \rightarrow \text{is solenoidal}$$

Self Inductance and Mutual Inductance

The magnetic flux of a current carrying coil is given by

$$\phi \propto I$$

or

$$\phi = LI \quad \text{--- (1)}$$

↓

Self Inductance of coil

If coil is static but magnetic field or flux changes with time. If current is time dependent flux will also be time dependent $\phi = \phi(t)$

$$\Rightarrow \frac{d\phi}{dt} = \frac{d\phi}{dI}\frac{dI}{dt} = L\frac{dI}{dt} \quad \text{--- (2)}$$

here $\frac{d\phi}{dI} = L$

$$\Rightarrow \varepsilon = -\frac{d\phi}{dt} = -L\frac{dI}{dt} \quad \text{--- (3)}$$

↓

depends on geometry of coil.

Self inductance of a wire in the form of a solenoid will be more than the self inductance of a straight wire.

$L \rightarrow$ also depends on the medium in which coil is placed.

If unit current passes through the coil the magnetic flux passing through coil is equal to its self inductance

$$\phi = L \quad \text{for } I = 1$$

unit of self inductance is <u>1 Henry.</u>

Self inductance of a circuit will be unit if the current in it changes 1 amp / sec and produces 1 volt e.m.f.

If I_1 current is passed through a coil. If a secondary coil is brought near, magnetic flux is produced in second coil ϕ_2

$$\phi_2 = L_{21} I_1 \quad \text{---- (4)}$$

Similarly magnetic flux ϕ_1 related to current in second circuit I_2

$$\phi_1 = L_{12} I_2 \quad \text{---- (5)}$$

Potential energy of system in first condition

$$U_p = - \phi_2 I_2$$

$$= - L_{21} I_1 I_2 \quad \text{---- (6)}$$

Similarly potential energy in second condition

$$U_p = - \phi_1 I_1 = - L_{12} I_2 I_1 \quad \text{---- (7)}$$

From (6) and (7)

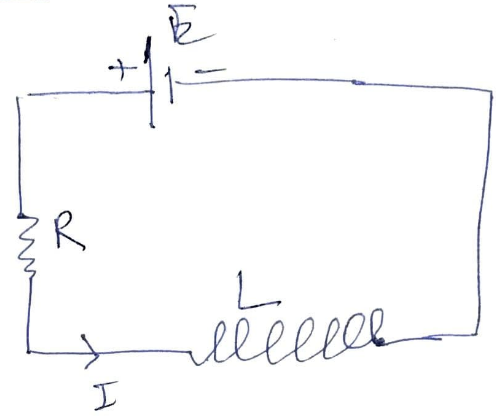
$$L_{21} = L_{12} = M$$

$\downarrow$

<u>Mutual Inductance between two circuits</u>

Energy in Magnetic field

Energy in a field is equal to the work done to establish it.



Magnetic energy stored in an inductor

Let the current in circuit at time t is $I(t)$. The voltage drop across coil is $L\frac{dI}{dt}$.

$\Rightarrow$ e.m.f $\quad \varepsilon = -L\frac{dI}{dt}$

From Ohm Law

$$\varepsilon - L\frac{dI}{dt} = RI \quad \text{---(7)}$$

Work done by e.m.f ε in moving dq charge

$$dW = \varepsilon\, dq = \varepsilon I\, dt$$

$$\Rightarrow \frac{dW}{dt} = \varepsilon I \quad \text{---(8)}$$

$$\Rightarrow \frac{dW}{dt} = \varepsilon I = LI\frac{dI}{dt} + RI^2$$

$$\Rightarrow W = \int_0^T \varepsilon I\, dt = L\int_0^T I\frac{dI}{dt}\,dt + R\int_0^T I^2\,dt$$

$$= \underbrace{\frac{1}{2}LI_T^2}_{\text{energy stored in inductor}} + \underbrace{R\int_0^T I^2\,dt}_{\text{energy dissipation in resistance } R} \quad \text{---(9)}$$