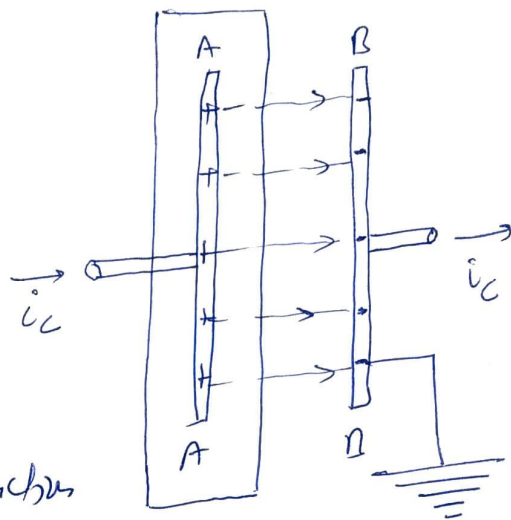


Displacement Current

Kirchoff's Law $\rightarrow$ The sum of ^{incoming} currents on any point in a steady state close circuit is equal to the sum of outgoing current from that point. (11)

This law does not apply to a Capacitor being charged.

Current i_c enters in plate AA, but no current coming out. Similarly conductor



Current i_c coming out from plate BB, but no current is entering.

According to Maxwell $\rightarrow$ When a capacitor is charged the charge on its plate increases slowly due to conduction current. The intensity of electric field between the plates of capacitor increases with increase in charge. The rate of increase of intensity of electric field was ~~defined~~ related to equivalent current density.

If $\sigma \rightarrow$ uniform charge density on the plates of capacitor

$$E = \frac{\sigma}{\epsilon_0} = \frac{Q}{\epsilon_0 A} \quad \text{--- (1)}$$

If $dQ \rightarrow$ increase in charge Q in time ' dt '

$$dE = \frac{dQ}{\epsilon_0 A} = \frac{I dt}{\epsilon_0 A} \quad \text{--- (2)} \quad I \rightarrow \text{conduction current}$$

$$\frac{dE}{dt} = \frac{I}{\epsilon_0 A} \quad (3) \quad \frac{I}{A} = \frac{\partial D}{\partial t}$$

(12)

If we define the equivalent current density J_D between the plates;

$$J_D = \epsilon_0 \frac{dE}{dt} \quad (4)$$

The total equivalent current between the plates

$$I_D, \quad I_D = J_D A$$

$$I_D = J_D A = \epsilon_0 \frac{dE}{dt} A \quad (5)$$

From eq (3) & (5) $J_D = I$

This equivalent current $\rightarrow$ Displacement Current

The magnetic field is produced by this current in similar way as conduction current I_C .

$\frac{I}{A} = \frac{\partial D}{\partial t} \rightarrow$ density of some current which corresponds to the "current" which flows in the space, even in vacuum, between the parallel plates of a

Capacitor when the charged plates are connected by a wire, thus completing the conduction current

Idea of the relative size of the two types of current in conductors

Consider a copper wire in which electric field

$$E = E_0 e^{-i\omega t}$$

$$J = \sigma E = \sigma E_0 e^{-i\omega t}$$

$$\Rightarrow |J|^2 = \sigma^2 |E_0|^2$$

$$\text{and } D = \epsilon_0 E_0 e^{-i\omega t}$$

$$\frac{\partial D}{\partial t} = -i\omega \epsilon_0 E_0 e^{-i\omega t}$$

$$\left| \frac{\partial D}{\partial t} \right|^2 = \omega^2 \epsilon_0^2 |E_0|^2 \Rightarrow \left| \frac{J}{\frac{\partial D}{\partial t}} \right| = \frac{\sigma}{\omega \epsilon_0} \text{ for copper}$$

$\sigma = 5.9 \times 10^7$, $\Rightarrow \frac{\sigma}{\omega \epsilon_0} \sim \frac{10^{19}}{\omega} \rightarrow$ ratio is very large for all frequencies.

Ampere's Law and Maxwell's Modification

(13)

Ampere's Law The line integral of magnetic field is equal to μ_0 times the I_{enc} current in the closed surface

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc} \quad \text{--- (1)}$$

The magnetic field can be established by

- (i) Conduction current
- (ii) Changing electric field or displacement current from Faraday's Law

$$\mathcal{E}_{ind} = \oint \vec{E} \cdot d\vec{l} = (-) \frac{d\phi_m}{dt}$$

Similarly, relation of changing electric flux ψ and magnetic field (a)

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \epsilon_0 \frac{d\psi}{dt}$$

So Ampere's Law

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \epsilon_0 \frac{d\psi}{dt} + \mu_0 I_{enc} \quad \text{--- (2)}$$

$$\text{Electric flux } \psi = \int \vec{E} \cdot d\vec{s}$$

$$\frac{d\psi}{dt} = \frac{d}{dt} \left[\int \vec{E} \cdot d\vec{s} \right]$$

$$= \int \left[\frac{d\vec{E}}{dt} \cdot d\vec{s} \right]$$

From eqⁿ (2)

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \epsilon_0 \int \left(\frac{d\vec{E}}{dt} \right) \cdot d\vec{s} + \mu_0 I_{enc}$$

Modified Ampere's Law

Maxwell eqⁿ for time dependent ~~electric~~ electromagnetic field (14)

in Vacuum

Gauss Law $\oint \vec{E} \cdot d\vec{s} = \frac{1}{\epsilon_0} \sum q_i \Rightarrow \oint \vec{E} \cdot d\vec{s} = \frac{1}{\epsilon_0} Q ; Q = \sum q_i$

from divergence theorem

$$\begin{aligned} \oint \vec{E} \cdot d\vec{s} &= \int \text{div } \vec{E} \, dV \\ &= \int \nabla \cdot \vec{E} \, dV \quad \text{--- (1)} \end{aligned}$$

$$Q = \int_V \rho \, dV \Rightarrow \int \nabla \cdot \vec{E} \, dV = \frac{1}{\epsilon_0} \int \rho \, dV$$

$$\therefore \boxed{\nabla \cdot \vec{E} = \frac{1}{\epsilon_0} \rho} \quad \text{--- (2)}$$

In vacuum $\boxed{\nabla \cdot \vec{B} = 0} \quad \text{--- (3)}$

Integral form of Faraday's Law

$$\oint \vec{E} \cdot d\vec{l} = (-) \frac{d\Phi_B}{dt} = (-) \int \frac{d\vec{B}}{dt} \cdot d\vec{s}$$

from Stokes's theorem

$$\oint \vec{E} \cdot d\vec{l} = \oint \nabla \times \vec{E} \cdot d\vec{s}$$

$$\Rightarrow \oint \nabla \times \vec{E} \cdot d\vec{s} = (-) \int \frac{d\vec{B}}{dt} \cdot d\vec{s}$$

$$\Rightarrow \boxed{\nabla \times \vec{E} = - \frac{d\vec{B}}{dt}} \quad \text{--- (4)}$$

The Modified Ampere's Law

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc} + \mu_0 \epsilon_0 \int \frac{\partial \vec{E}}{\partial t} \cdot d\vec{s}$$

from Stokes's theorem $\oint \vec{B} \cdot d\vec{l} = \oint \nabla \times \vec{B} \cdot d\vec{s}$

$$I_{enc} = \int \vec{J} \cdot d\vec{s}$$

$$\Rightarrow \boxed{\nabla \times \vec{B} = \mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}} \quad \text{--- (5)}$$

Maxwell's eqⁿ in vacuum (Differential form)

(15)

$$(i) \quad \vec{\nabla} \cdot \vec{E} = \frac{1}{\epsilon_0} \rho \quad \text{or} \quad \vec{\nabla} \cdot \vec{D} = \rho$$

$$(ii) \quad \vec{\nabla} \cdot \vec{B} = 0 \quad \text{or} \quad \vec{\nabla} \cdot \vec{H} = 0$$

$$(iii) \quad \vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad \text{or} \quad \vec{\nabla} \times \vec{E} + \mu_0 \frac{\partial \vec{H}}{\partial t} = 0$$

$$(iv) \quad \vec{\nabla} \times \vec{B} = \mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{\partial \vec{D}}{\partial t} \quad \text{or} \quad \vec{\nabla} \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

Integral form

$$\oint \vec{D} \cdot d\vec{s} = \int_V \rho dV$$

$$\oint \vec{B} \cdot d\vec{s} = 0$$

$$\oint \vec{E} \cdot d\vec{l} = (-) \int \frac{\partial \vec{B}}{\partial t} \cdot d\vec{s}$$

$$\oint \vec{H} \cdot d\vec{l} = I + \int \frac{\partial \vec{D}}{\partial t} \cdot d\vec{s}$$

Maxwell eqⁿ in Matter (Dielectric)

if $\rho = 0$ in medium (dielectric)

$$(i) \quad \vec{\nabla} \cdot \vec{E} = 0 \quad (\text{electrostatics})$$

$$(ii) \quad \vec{\nabla} \cdot \vec{B} = 0$$

$$(iii) \quad \vec{\nabla} \times \vec{E} = (-) \frac{\partial \vec{B}}{\partial t}$$

$$(iv) \quad \vec{\nabla} \times \vec{B} = \mu_0 \left(\frac{\partial \vec{E}}{\partial t} \right)$$

Boundary Conditions : We can obtain boundary conditions (16)

The tangential component of Electric field across interface between two medium from Maxwell's eqⁿ (integral form)

$E_{t1} = E_{t2}$ and for magnetic field $H_{t1} = H_{t2}$. From surface integral of the eqⁿ, the normal component boundary conditions are

$$D_{n1} - D_{n2} = \rho_s \text{ and } B_{n1} = B_{n2}$$

In Ideal conditions, (i) Ideal conductor $\sigma = \infty$, current density $\rightarrow J \rightarrow$ limited, $\vec{E} = 0$, from Faraday's law for time varying fields $H = 0$, from Ampere's Circuital law, the limited value of current density $J = 0$, the current in conductor will be surface current, k .

If one of the medium is ideal conductor, the boundary conditions will be,

$$E_{t1} = 0, H_{t1} = k \text{ where } \vec{H}_{t1} = k \times \hat{a}_n$$

$$D_{n1} = \rho_s, B_{n1} = 0$$

Prob. A car ignition coil consists of two insulated coils, one of 16000 turns and the other of 400 turns, wound over each other. The length of each coil is 10 cm and the turns have the radius of 3 cm. A current of 3 A is passed through the primary coil and broken in about 10^{-4} sec. Calculate the voltage induced in the secondary circuit.

Solⁿ The magnetic flux density in the solenoid

$$B = \mu_0 N_1 I_1 \quad \begin{array}{l} N_1 \rightarrow \text{No. of turns per unit length} \\ I_1 \rightarrow \text{Current} \end{array}$$

The magnetic flux through each turn of the top coil is $\mu_0 N_1 I_1 \pi r^2$ and total flux is

$$\phi_2 = \mu_0 N_1 N_2 l \pi r^2 I_1$$

$$\text{Induced emf } \mathcal{E} = - \frac{d\phi_2}{dt} = - \mu_0 N_1 N_2 l \pi r^2 \frac{dI_1}{dt}$$

$$\frac{dI_1}{dt} = 3 \times 10^{-4} \text{ A s}^{-1} = - M \frac{dI_1}{dt}$$

$$\mathcal{E} = 4\pi \times 10^{-7} \times 16 \times 10^4 \times 4 \times 10^3 \times 10^{-4} \pi \times (0.03)^2 \times 3 \times 10^4$$

$$= 6814 \text{ Volts}$$