$$\omega_{l^*} = B\frac{e}{2m_0} \quad \text{and} \quad \omega_{s^*} = B\frac{e}{2m_0}2 \quad \text{---(1)}$$

The ratio between the magnetic and mechanical moment for the spin of the electron is twice the orbital ratio.

s^* should (classically) precess twice as fast as l^*. Multiplying each of these angular velocities by the projection of the angular momentum on $B(H)$ we get the first two terms of the energy

$$\Delta E_{lB} = B\frac{e}{2m_0} l^* \frac{h}{2\pi}\cos(l^*B)$$

$$= B\frac{e}{2m} m_l \frac{h}{2\pi} \quad \text{---(2)}$$

$$\Delta E_{sB} = 2B\frac{e}{2m} s^* \frac{h}{2\pi}\cos(s^*B)$$

$$= 2B\frac{e}{2m} m_s \frac{h}{2\pi} \quad \text{---(3)}$$

The sum of these two energies accounts for the main energy shift from the unperturbed energy level and is

$$\Delta E = \Delta E_{lB} + \Delta E_{sB}$$

$$= (m_l + 2m_s) B\frac{eh}{4\pi m_0} \quad \text{---(4)}$$

$(m_l + 2m_s) \longrightarrow$ strong field quantum number

In terms of frequency change

$$\Delta\nu = 4(m_l \pm 2m_s)\frac{eB}{4\pi m_0} \quad \text{---(5)}$$

The shift in terms of wave numbers

$$\Delta\bar{\nu} = \Delta(m_l + 2m_s)\frac{eB}{4\pi m_0 c} \quad \text{--- (6)}$$

In Lorentz unit of $\frac{eB}{4\pi m_0 c}$

$$\Delta\bar{\nu} = \Delta(m_l + 2m_s)\,L \;\; cm^{-1} \quad \text{--- (7)}$$

To this magnetic energy the small correction term due to interference between l^* and s^* must be added. These two vectors precess independently around B (H), each motion produces magnetic field at the electron which perturbs the motion of other.

This interaction energy is small as compared with that due to the external field, is of the same order of the magnitude as the fine-structure doublet separations in field-free space

It is given by the Γ factor

$$\Gamma = -\Delta E_{l,s} = a\, l^* s^* \cos(l^* s^*) \quad \text{--- (8)}$$

where

$$a = \frac{R\alpha^2 Z^4}{n^3 l(l+\frac{1}{2})(l+1)} \; cm^{-1} \quad \text{--- (9)}$$

In field-free space, the angle between l^* and s^* is constant and the cosine term $\cos(l^* s^*)$ is evaluated easily. In the present case → the angle is continually changing so the average value of the cosine must be calculated

From trigonometry, it may be shown that s^* and l^* precessing independently with fixed angles around a third direction B (H)

$$\cos(l^* s^*) = \cos(l^* B) \cdot \cos(s^* B) \quad (10)$$

$$\Rightarrow \Gamma = -\Delta E_{l,s} = a\, l^* \cos(l^* B)\, s^* \cos(s^* B) \quad (11)$$

these are just projections of l^* and s^* on B, so that

$$-\Delta E_{l,s} = a\, m_l m_s = \Gamma \quad (12)$$

So the total energy shift becomes

$$T = T_0 - (m_l + 2m_s)L - a\, m_l m_s \quad (13)$$

$T_0 \rightarrow$ term value of the hypothetical center of gravity of the fine-structure doublet.

Now since $\Delta m_l = 0$ or ± 1 and $\Delta m_s = 0$

So $\Delta(m_l + 2m_s) = 0$ or ± 1

$\Rightarrow$ We get three different frequencies $\rightarrow$ result is normal Zeeman triplet as before.

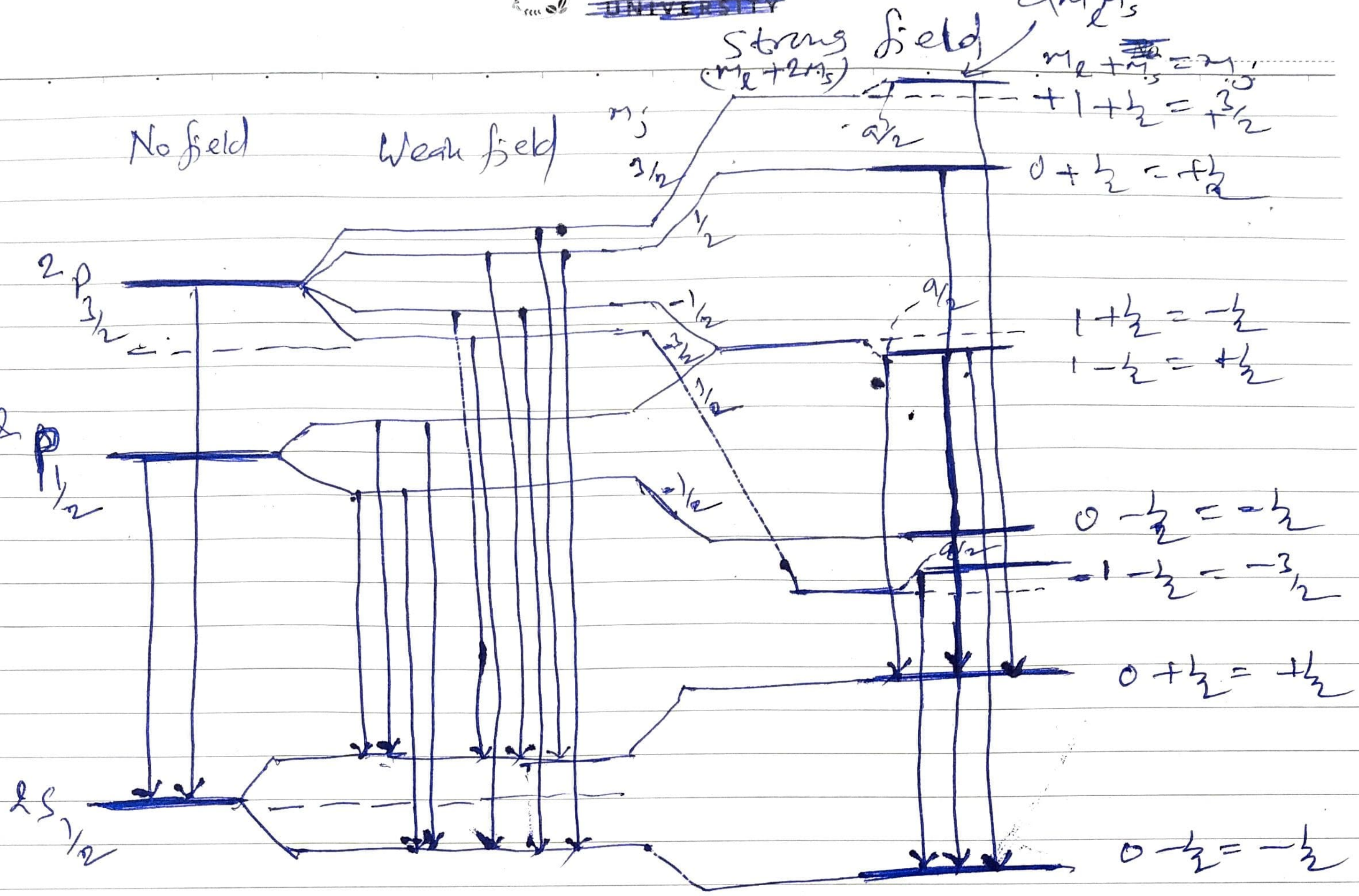
We consider example of a principal series doublet $2P_{3/2} \rightarrow {}^2S_{1/2}$ and ${}^2P_{1/2} \rightarrow {}^2S_{1/2}$

In the strong field, P level is split up to six levels, because for a particular value of $l(l+1)$, m_l has $(2l+1)$ values (here $1, 0, -1$) and for each value of m_l, m_s has two values $(+\frac{1}{2}, -\frac{1}{2})$

Weak- and Strong-field Energies for a Principal-Series Doublet

No field		Weak field (Zeeman Effect)			Strong field (Paschen-Back Effect)				
Term	Γ	m_j	g	$m_j g$	m_l	m_s	$m_j = m_l + m_s$	$m_l + 2m_s$	$a m_l m_s$
$^2P_{3/2}$	$+a/2$	$+3/2$	$4/3$	$+6/3$	$+1$	$+1/2$	$+3/2$	$+2$	$+a/2$
		$+1/2$		$+2/3$	0	$+1/2$	$+1/2$	$+1$	0
		$-1/2$		$-2/3$	-1	$+1/2$	$-1/2$	0	$-a/2$
		$-3/2$		$-6/3$	$+1$	$-1/2$	$+1/2$	0	$-a/2$
$^2P_{1/2}$	$-a$	$+1/2$	$2/3$	$+1/3$	0	$-1/2$	$-1/2$	-1	0
		$-1/2$		$-1/3$	-1	$-1/2$	$-3/2$	-2	$+a/2$
$^2S_{1/2}$	0	$+1/2$	2	$+1$	0	$+1/2$	$+1/2$	$+1$	0
		$-1/2$		-1	0	$-1/2$	$-1/2$	-1	0

The tabulated values are shown schematically in the following figure. At the left the undisturbed fine-structure levels and the observed transitions are shown. The weak-field Zeeman levels are shown next. In the strong field the Paschen-Back levels are shown with and without, the small $l^* s^*$ coupling correction $a m_l m_s$.



Energy levels for a principal series doublet starting with no field at the left and ending with a strong field (Paschen-Back effect) at the right.