$\hat{j}\hat{j}$ - Coupling : Ideal case of $\hat{j}\hat{j}$-coupling for two electrons.

The spin s_i^* of each electron is quantized with respect to its own l_i^* to form a resultant j_i^*, such that $\hat{j}_i$ takes half-integer values only.

Two j^*s are in turn quantized with respect to each other to form a resultant J^*, such that J takes integral values only.

For example, we consider configuration pd, with $s_1 = \frac{1}{2}$, $l_1 = 1$, $s_2 = \frac{1}{2}$ and $l_2 = 2$. For the p electron, $j_1 = \frac{1}{2}$ or $\frac{3}{2}$ and for d electron $j_2 = \frac{3}{2}$ or $\frac{5}{2}$.

Combining these four values in all possible ways;

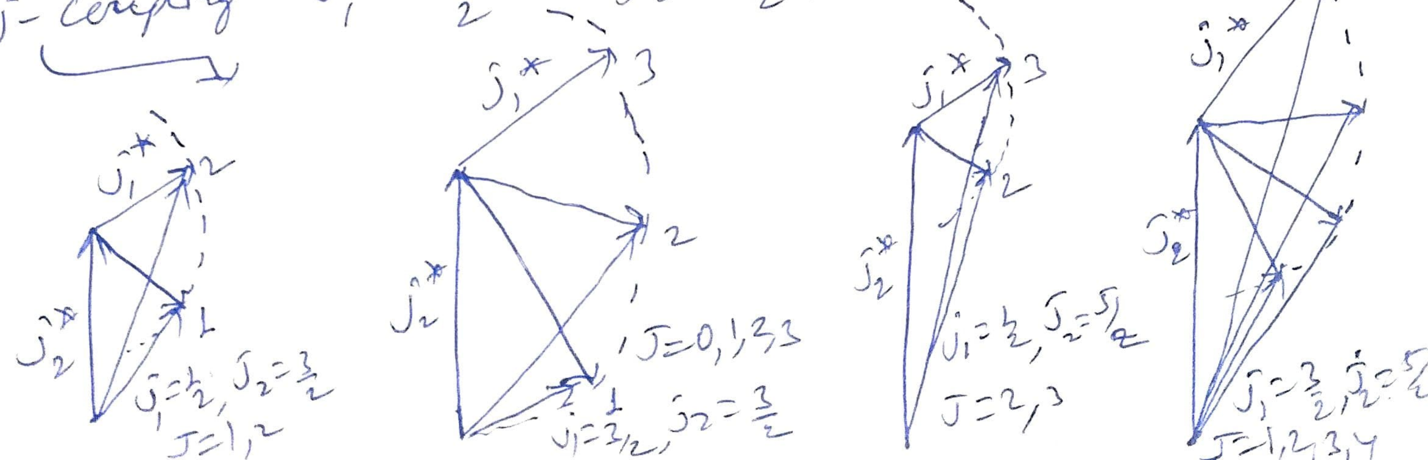
$j_1 = \frac{1}{2}$, $j_2 = \frac{3}{2}$; $J = 1$ and 2

pd Configuration $j_1 = \frac{3}{2}$, $j_2 = \frac{3}{2}$; $J = 0, 1, 2, 3$

Vector diagrams of two valence electrons in jj-coupling

$j_1 = \frac{1}{2}$, $j_2 = 5/2$; $J = 2$ and 3

$j_1 = \frac{3}{2}$ and $j_2 = 5/2$; $J = 1, 2, 3, 4$



Same number of terms arrived at on the LS-coupling, and they have exactly the same set of resultant J values.

<u>Selection Rules</u> :- An extension of the selection rules of hydrogen and the alkali metals to two-electron systems, one electron spectra may be thought of as special cases.

With two electrons taking part in producing the various types of terms, transitions may occur in which two electrons jump simultaneously with the emission of a single radiated frequency.

Selection rules for two-electron systems in general may be written as

$$\Delta l_1 = \pm 1 \quad \text{and} \quad \Delta l_2 = 0, \pm 2 \qquad (1)$$

If a single electron jumps, the l value of one changes by <u>unity</u>, and the other by <u>zero</u>.

If a double electron jump occurs, the l value of one changes by unity and the other by zero or by two.

There are no restrictions on the total quantum number n of either electron. For the various terms arising from all possible electron configurations the further restrictions are divided into two parts.

A. For LS- coupling the further restrictions are;

$$\Delta S = 0 \qquad \text{---(2)}$$
$$\Delta L = 0, \pm 1$$
$$\Delta J = 0, \pm 1 \ (0 \rightarrow 0 \text{ excluded})$$

B. for jj- coupling the further restrictions are

$$\left.\begin{array}{l} \Delta j_1 = 0 \\ \Delta j_2 = 0, \pm 1 \end{array}\right] \text{or vice versa} \qquad \text{---(3)}$$
$$\Delta J = 0, \pm 1 \ (0 \rightarrow 0 \text{ excluded})$$

Lande's factor for Two Valence Electron System

Lande's g factor in L-S coupling

for two electron, there will be four mechanical moments s_1^*, s_2^* and l_1^* and l_2^* along with four magnetic moments. In L-S coupling two spins s_1^* and s_2^* are coupled together to form a quantized vector S^* ($s_1^* + s_2^* = S$), giving a resultant spin magnetic moment μ_s is the opposite direction of S^*.

Magnetic moments for the spin motion of the electrons are

$$\mu_{s_1} = 2 s_1^* \frac{h}{4\pi m_0} \text{ and } \mu_{s_2} = 2 s_2^* \frac{h}{4\pi m_0}$$

resultant $$\mu_S = 2\left[s_1^* \cos(s_1^* S^*) + s_2^* \cos(s_2^* S)\right]\frac{eh}{4\pi m_0}$$
$$= 2 S^* \frac{eh}{4\pi m_0}$$

Similarly, the orbital vectors are coupled together ($l_1^* + l_2^* = L$) to form a resultant L^* and about this two orbital vectors l_1^* and l_2^* precess. Due to a single resultant L^*, we get single magnetic moment due to orbital motion.

$$\mu_{l_1} = l_1^* \frac{eh}{4\pi m_0} \text{ and } \mu_{l_2} = l_2^* \frac{eh}{4\pi m_0}$$

resultant

$$\mu_L = [l_1^* \cos(l_1^* L) + l_2^* \cos(l_2^* L)] \frac{eh}{4\pi m_0}$$

$$= L^* \frac{eh}{4\pi m_0}$$

Just as the spin and orbit of a single electron are coupled together to form a resultant j^*, so L^* and S^* are coupled together to form a resultant J^*. Projecting μ_S and μ_L on J^* and adding, we get the total magnetic moment of the atom,

$$\mu_J = \left[L^* \cos(L^* J^*) + 2S^* \cos(S^* J^*)\right] \frac{he}{4\pi m_0}$$

The bracketted term gives μ_J in units of Bohr Magneton $\frac{he}{4\pi m_0}$.

Replacing these terms in the bracket by g times J^*, i.e.

$$gJ^* = L^* \cos(L^* J^*) + 2S^* \cos(S^* J^*)$$

we obtain

$$\mu_J = g J^* \cdot \frac{he}{4\pi m_0}$$

Since the angles between L^*, S^* and J^* are fixed, in LS- coupling, the cosine terms are given by

$$L^* \cos(L^* J^*) = \frac{J^{*2} + L^{*2} - S^{*2}}{2J^*}$$

$$S^* \cos(S^* J^*) = \frac{J^{*2} + S^{*2} - L^{*2}}{2J^*}$$

Substituting these terms, we get the g factor

$$g = 1 + \frac{J^{*2} + S^{*2} - L^{*2}}{2J^{*2}}$$

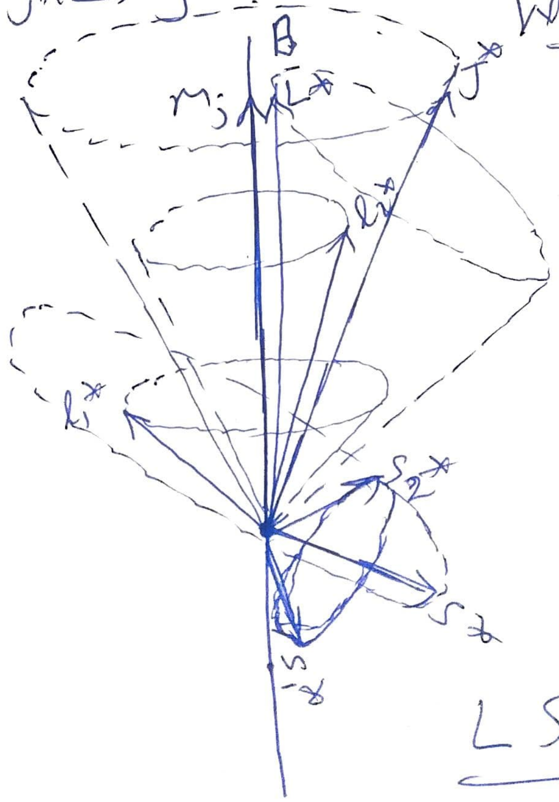
or

$$= 1 + \frac{J(J+1) + S(S+1) - L(L+1)}{2J(J+1)}$$

This is the Lande g factor is exactly the same form for a single electron, except $l^* \to L^*$, $s^* \to S^*$ and $j^* \to J^*$.

Weak field

(Zeeman Effect)



LS- Coupling